

# ANALYTIC SIGNAL AND REDUCTION TO POLE IN THE INTERPRETATION OF AEROMAGNETIC DATA AT LOW MAGNETIC LATITUDES: A CASE STUDY OF THE MIDDLE BENUE TROUGH, NIGERIA

O. K. LIKKASON

(Received 17 February, 2006; Revision Accepted 3 April, 2006)

## ABSTRACT

The interpretation of magnetic field data at low magnetic latitudes is difficult because the vector nature of the magnetic field increases the complexity of anomalies from magnetic rocks. The most obvious approach to this difficulty is to reduce data to the magnetic pole. In this approach an expression is given of the total field in the theoretical case when the magnetized rock mass is physically carried to the magnetic pole and the direction of remanent magnetization made vertical by the transformation process. On the other hand the use of analytic signal in locating source body characteristics does not depend on the direction of the body magnetization. A synthetic example demonstrates that if the analytic signal calculations are performed on the reduced to the pole anomalies, it produces enhanced results. The analytic signal of reduced to the pole data is calculated for the aeromagnetic anomalies of the Middle Benue Trough, Nigeria. This result indicates that the sedimentary – basement boundary of the studied area is better outlined than the result obtained from the analytic signal operation performed directly on the total magnetic field anomaly map.

**KEYWORDS:** Analytic signal, magnetic latitude, Nigeria, reduction to pole, aeromagnetic data

## INTRODUCTION

Many methods based on the horizontal and vertical derivatives have been developed to process magnetic anomalies. The several methods developed for gravity data cannot be applied to magnetic field. This is because the magnetic sensitivity of the crust is much bigger and complex than its density variation. In addition, the interpretation of magnetic data is more difficult than gravity data because the Earth's field and the magnetization vectors induce a certain degree of asymmetry especially in low magnetic latitude areas.

The most obvious approach to this problem is to reduce the data to the magnetic pole, where the presumably vertical magnetization vector will simplify observed anomalies. The reduction to pole analysis however requires special treatment of N-S trending features in data observed in low magnetic latitudes due to high amplitude corrections of these features (Baranov 1957; Hansen and Pawlowski 1989; Mendonca and Silva 1993). On the other hand, the amplitude of the 3-D analytic signal of the total field produces maxima over magnetic contacts/faults/shears and other geologic features regardless of the direction of magnetization (Nabighian 1972, 1984; Roest et al. 1992; Macleod et al. 1993; Hsu et al. 1996, 1998). Thus the absence of magnetization direction in the shape of analytic signal anomalies is a particularly attractive characteristic for the interpretation of magnetic field data near the magnetic equator.

This paper illustrates, through synthetic example, that the analytic signal amplitude of a magnetic field can be enhanced if the original magnetic field is first reduced to pole and then the former tool applied. Thus the analytic signal of reduction to pole data produces a better result than when just applied to the original field. This procedure was applied to the aeromagnetic anomalies of the Middle Benue Trough, Nigeria.

## THEORY

### (a) Analytic signal

Analytic signal is formed by the horizontal and vertical gradients of the magnetic anomaly. In a 3-D case, the analytic signal  $A$ , in space domain is given by (Nabighian 1972, 1984; Roest et al. 1992) as:

$$\vec{A}(x, y) = \frac{\partial M}{\partial x} \hat{i} + \frac{\partial M}{\partial y} \hat{j} + \frac{\partial M}{\partial z} \hat{k} \quad (1)$$

Where  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  are unit vectors in the x, y, z directions respectively, and M is the magnitude of the magnetic anomaly. In the frequency domain, equation (1) can be written as

$$\hat{i}.F[A(x, y)] = \hat{h}.\vec{\nabla}F[M] + i\hat{k}.\vec{\nabla}F[M] \quad (2)$$

Where  $\vec{\nabla}$  is the gradient operator in the frequency domain  $(ik_x\hat{x} + ik_y\hat{y} + k\hat{z})$ ;  $\hat{i} = \hat{x} + \hat{y} + \hat{z}$  and  $\hat{h} = \hat{x} + \hat{y}$  (Roest et al. 1992) and  $i$  is a complex number. The horizontal and vertical derivatives of the anomaly are the real and imaginary parts of equations (1) and (2) respectively. The amplitude of the analytic signal  $|\vec{A}|$  is

$$|\vec{A}(x, y)| = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2 + \left(\frac{\partial M}{\partial z}\right)^2} \quad (3)$$

The x and y derivatives can be calculated directly from a total magnetic field using a simple 3x3 filter, and the vertical gradient is routinely calculated using the Fast Fourier Transform (FFT) techniques.

Nabighian (1972) has shown that the analytic signal over a 2-D magnetic contact, magnetic sheet, and a magnetic cylinder located at  $x = 0$  and at depth (to top)  $h$  are respectively

$$\left. \begin{aligned} |\vec{A}(x)| &= \alpha \frac{1}{(h^2 + x^2)^{1/2}} \\ |\vec{A}(x)| &= \alpha \frac{1}{(h^2 + x^2)} \\ |\vec{A}(x)| &= \alpha \frac{1}{(h^2 + x^2)^{3/2}} \end{aligned} \right\} \quad (4)$$

Where  $\alpha$ , the amplitude factor is defined as

$$\alpha = 2T \sin d (1 - \cos^2 I \sin^2 J)$$

In which  $T$  = the strength of magnetization vector

$d$  = dip of the body

$I$  = the inclination of the magnetization vector

$J$  = the direction of the magnetization vector

The expressions in equation (4) show that the analytic signal shape can be used to determine the depth to magnetic sources and other source parameters. It is not our intention to do these here.

#### (b) Reduction to the pole

The results of a magnetic survey are difficult to interpret because there is no simple relationship between tectonic features and the magnetic anomalies. In most cases, the magnetic anomalies are not usually centred at the disturbing bodies. Thus the magnetic picture of the tectonic occurrences undergoes a distortion due to the inclination of the magnetizing vector.

The aim of reduction to the pole is to take an observed total field map and produce a magnetic map that would result, had an area been surveyed at the magnetic pole. This reduction to the pole gives an expression of the total field in the theoretical case when the magnetized rock mass is physically carried to the magnetic pole and the direction of remanent magnetization made vertical by the transformation process.

Baranov (1957) calculated a grid operator for convolution with a magnetic anomaly map in the space domain to reduce the map to the pole. Bhattacharyya (1965) makes it clear that reduction-to-the-pole (RTP) can be accomplished by multiplying the Fourier transform of the magnetic map by the frequency response of a filter with a close-form formula. The FFT technology enables very efficient computation for such a reduction. The RTP filter,  $R_0$ , is specified in the frequency domain by the following formula (Baranov 1975, Gunn 1975), when the rock magnetization is taken into consideration:

$$R_0(k_x, k_y) = \frac{k_z^2}{[\lambda_3 k_z - i(\lambda_1 k_x + \lambda_2 k_y)][\vartheta_3 k_z - i(\vartheta_1 k_x + \vartheta_2 k_y)]} \quad (5)$$

Where  $k_x$  and  $k_y$  are wave numbers in the frequency domain,  $k_z = (k_x^2 + k_y^2)^{1/2}$ , and  $(\lambda_1, \lambda_2, \lambda_3)$  and  $(\vartheta_1, \vartheta_2, \vartheta_3)$  are the direction cosines of the rock-magnetization unit vector and the earth's magnetic field unit vector, respectively. Note the  $\lambda_1 = \cos I \cos D$ ,  $\lambda_2 = \cos I \sin D$ ,  $\lambda_3 = \sin I$ , and  $\vartheta_1 = \cos I_0 \cos D_0$ ,  $\vartheta_2 = \cos I_0 \sin D_0$ ,  $\vartheta_3 = \sin I_0$ , where  $(I, D)$  and  $(I_0, D_0)$  are the inclination and declination of the rock magnetization and the earth's magnetic field, respectively. When the rock magnetization and the earth's magnetic field are in the same direction then  $I = I_0$  and  $D = D_0$ . The value  $i = \sqrt{-1}$  is a complex number.

The frequency-domain filter  $R_0$  of dimension  $k \times l$  is then inverse Fourier transformed to the space domain and windowed to dimension  $m \times n$  ( $k \ll m$  and  $l \ll n$ ). Experiments on synthetic magnetic anomaly have indicated the filter size needs to be at least three-quarters of the data size when  $I = I_0 = 60^\circ$  and  $D = D_0 = 15^\circ$  (Lu 1998), with a rectangular window required to properly reduce the magnetic anomaly to the pole.

When the filter size is smaller or when a different window (such as Hamming window) is selected, a dislocation of the resulting maximum and distortion of the resulting anomaly becomes visible (Lu 1998). These requirements vary with inclination and declination of the magnetic field intensity. At higher latitudes (large inclination), a smaller filter size is sufficient. Thus a bigger filter size is required for the low latitude areas. The instability of RTP operator at low magnetic latitude has been addressed and attempts made to overcome it. This instability arises from huge amplification (in equation (5)) of the anomaly along the azimuth of the geomagnetic field that is necessary to perform the operation. Applying a Wiener and azimuthal filters to the observed data that are assumed to be the sum of the signal and noise components can attain the stability. The aim here is to minimize the expected mean-square error between the observations reduced to the pole and the signal at the pole (Hansen and Pawlowski 1989).

In implementation, a  $m \times n$  windowed space-domain filter is padded with sufficient zeros to generate a grid with dimension  $(k+m) \times (l+n)$  equal to the summation of the dimensions of the data grid and the windowed filter grid. The magnetic data to be reduced to the pole are also padded with zeros or extrapolated to generate a grid of the same size. Both extended filter and data grids are Fourier-transformed and multiplied, i.e.

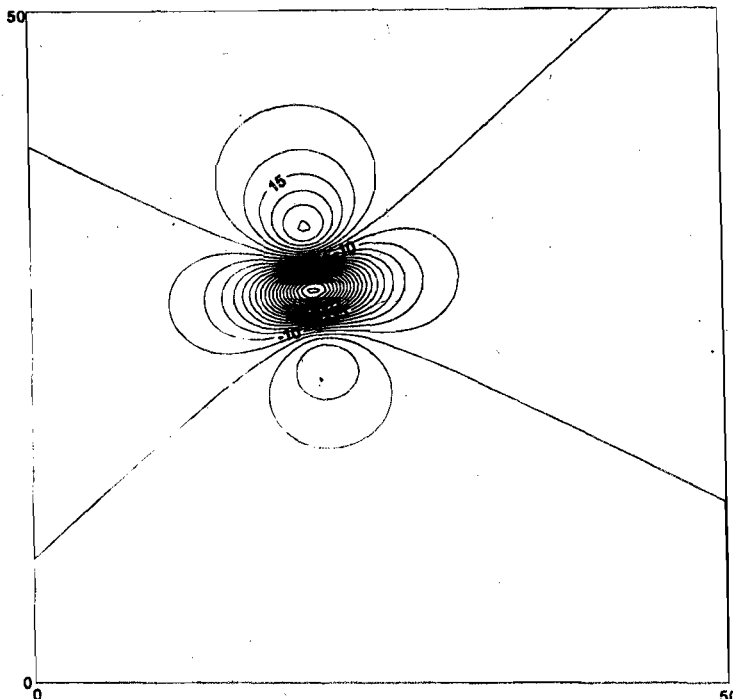
$$P(k_x, k_y) = F(k_x, k_y) R(k_x, k_y) \quad (6)$$

Where  $P$  is the filtered spectrum and  $F$  and  $R$  are the Fourier transforms of the extended data and filter grids respectively. Inverse Fourier transform is then performed on the result followed by trimming back to the original extent of the data to produce the RTP map.

## APPLICATION TO REAL DATA

### 1. Test case

The performance of the analytic signal when applied to reduced-to-pole was tested on the magnetic anomaly of a dipole. The magnetic anomaly of a dipole located at a depth of 4 km at the  $x, y$  coordinates of 20 km each was considered. The inclination and declination of the inducing field are respectively  $6^\circ$  and  $8^\circ$  (i.e. at low magnetic latitudes), where the strength of the inducing field is 33, 510 nT. As for the properties of the dipole: its remanent inclination, declination and magnetization intensity are taken as  $6^\circ$ ,  $8^\circ$  (same as those of the inducing field) and 0.01 A/m respectively while its susceptibility is assumed as  $9.5 \times 10^{-5}$  cgs (0.0012 SI). The magnetic anomaly of this dipole was computed and contoured as map (Fig. 1). The analytic signal and reduced to pole transformations were applied to these data. The peak of the amplitude of the analytic signal map (Fig. 2) coincides with the low of the anomaly map of Figure 1. The RTP map of the dipole field with aforementioned parameters of the direction of the Earth's magnetic field and the body magnetization was produced and displayed (Fig. 3). The analytic signal transformation is again applied to the RTP transformed map (Fig. 3) and this result is shown in Figure 4. The peak amplitude of this analytic signal (Fig. 4) is seen to be more enhanced compared to the one shown in Figure 2 over the same portion. All the anomaly data were computed by means of Phillips (1997) and contoured using the Golden Software surface mapping system (Surfer version 7.02).



**Fig. 1** Magnetic field of the dipole. The inclination and declination of the Earth's field and that of the dipole are taken the same. Their values are respectively  $6^\circ$  and  $8^\circ$ . Magnetic susceptibility and remanent magnetization of the dipole are respectively  $9.5 \times 10^{-5}$  cgs and 0.01 A/m. Dipole location is indicated. Contour interval is 5nT.

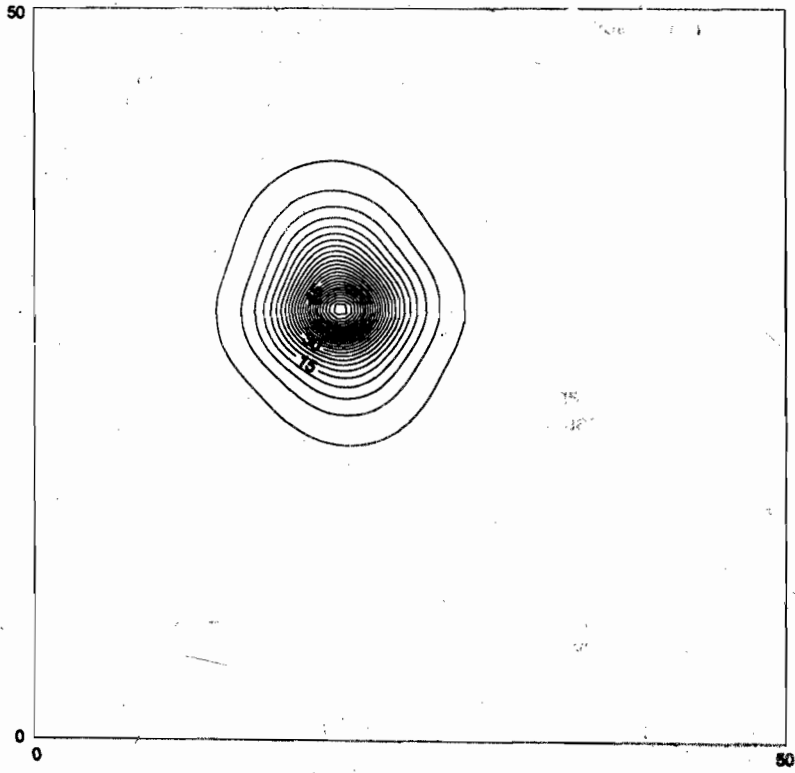


Fig. 2 Analytic signal of the dipole field of Figure 1. Contour interval is  $3nT$ .

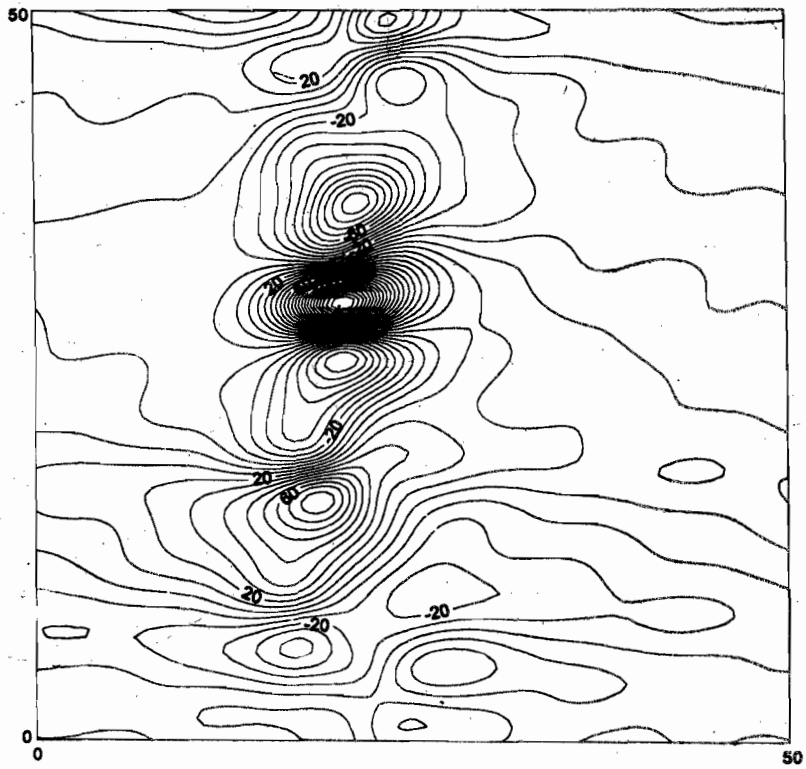


Fig. 3 The reduced to pole map of the dipole field of Figure 1. Contour interval is  $8nT$ .

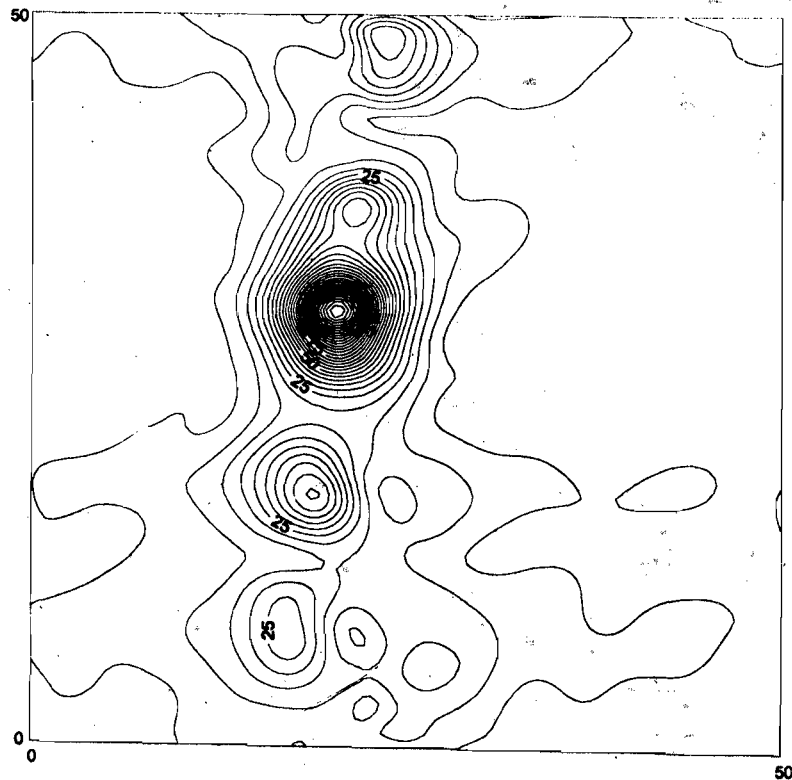


Fig. 4 Analytic signal of the reduced to pole map of Figure 3. Contour interval is 5nT.

## 2. Case study

### (a) Geology of the study area

The Middle Benue Trough of Nigeria (MBT) is part of the entire Benue Trough, Nigeria (Fig. 5a). It lies between Latitudes  $7^{\circ}00'N$  and  $9^{\circ}30'N$  and Longitudes  $9^{\circ}00'E$  and  $11^{\circ}30'E$ . The Benue Trough itself is an intra-cratonic rift structure that extends from the northern limit of the Niger Delta to the southern margin of the Chad Basin (Benkheilil 1982).

Offodile (1976) has described the litho-stratigraphic sequence of the MBT (Fig. 5b). The lithological units of MBT (Fig. 5b) show that the Cretaceous stratigraphy of this geographic division of the Benue Trough comprises the oldest rocks belonging to the Asu River Group: a mixture of shale and siltstones of marine origin, and lava-flows, dykes and sills representing the first middle Albian episode into the Benue Trough. This group, which is believed to be about 3000 m thick, lies unconformably on an older basement complex. The basement complex itself, as it crops along the fringes of the study area, consists of granulitic gneisses, migmatites, Older Granites, Younger Granites, porphyries and rhyolites. Rock units belonging to the Asu River Group outcrop along the axis of the Keana Anticline to the east of the town of Keana (Offodile 1976). The Asu River Group is overlain by the transitional beds of the Awe Formation, which consists of flaggy, whitish, medium to coarse-grained sandstones interbedded with carbonaceous shales or clays from which brine springs issue continuously (Ford 1981, Offodile 1984). The Awe Formation marks the beginning of the regressive phase of the Albian Sea and is overlain by continental fluvialite sands of the Keana Formation.

The Ezeaku, Agwu and Lafia Formations are also present and these represent the Turonian to Early Maastrichtian sediments in the MBT. The Ezeaku Formation comprises essentially of calcareous shale, micaceous fine to medium-grained friable sandstones, and occasional beds of limestone. The Coniacian Agwu Formation consists mainly of black shale, sandstones and local coal seams. The Maastrichtian Lafia Formation is the youngest formation reported in the Middle Benue Trough and consists of coarse-grained ferruginous sandstones, red loose sand, flaggy mudstones and clays (Offodile, 1976).

Major Santonian tectonic events have affected the Albian to Coniacian sediments, producing numerous folds, faults and fractures. These events were accompanied by mafic to intermediate volcanic activity that led to the emplacement of dykes, sills, lavas and tuffs (Short and Stauble 1967, Murat 1972, Benkheilil 1989). Consequently, most parts of the Lower and Middle Benue were uplifted with more than 1000 m of sediments eroded (Agagu and Adighije 1983).

### (b) Aeromagnetic data analysis

An aeromagnetic survey covering almost the entire Nigerian country was carried out during the 1974 – 1976 by three consultants: Hunting Geology & Geophysics Ltd, Fairey Survey Ltd and Polservice PPG, on behalf of the Geological Survey of Nigeria (GSN). The aim of this survey has been to assist in mineral and groundwater development through improved geological mapping. For this study area: MBT, the survey altitude was at 150 m mean terrain clearance (nearly 275 m above the mean sea level) on lines spaced at 2 km flying NNW – SSE direction.

Twenty half – degree square aeromagnetic contour maps, purchased from GSN, were manually digitized along the N-S, E-W directions at 1.5 km interval: digitally merged and corrected for the main field using the International Aeromagnetic Reference

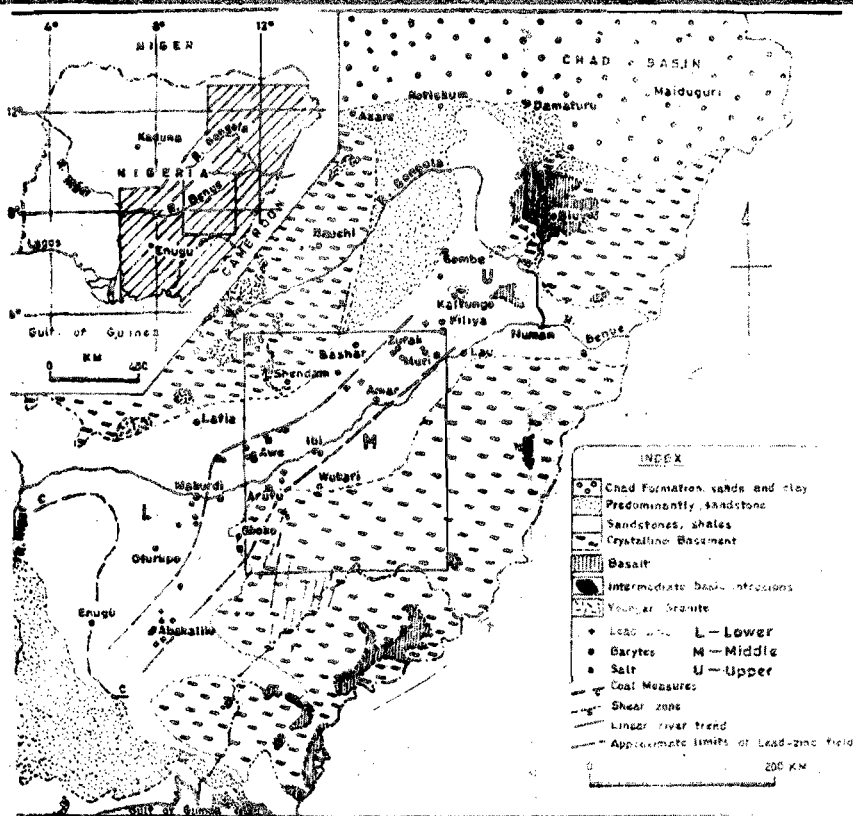


Fig. 5a The generalized geology of the Benue Trough, Nigeria: inset is the location of the Middle Benue Trough (blown up in Figure 5b) (After Reymont 1965, Offodile 1976)

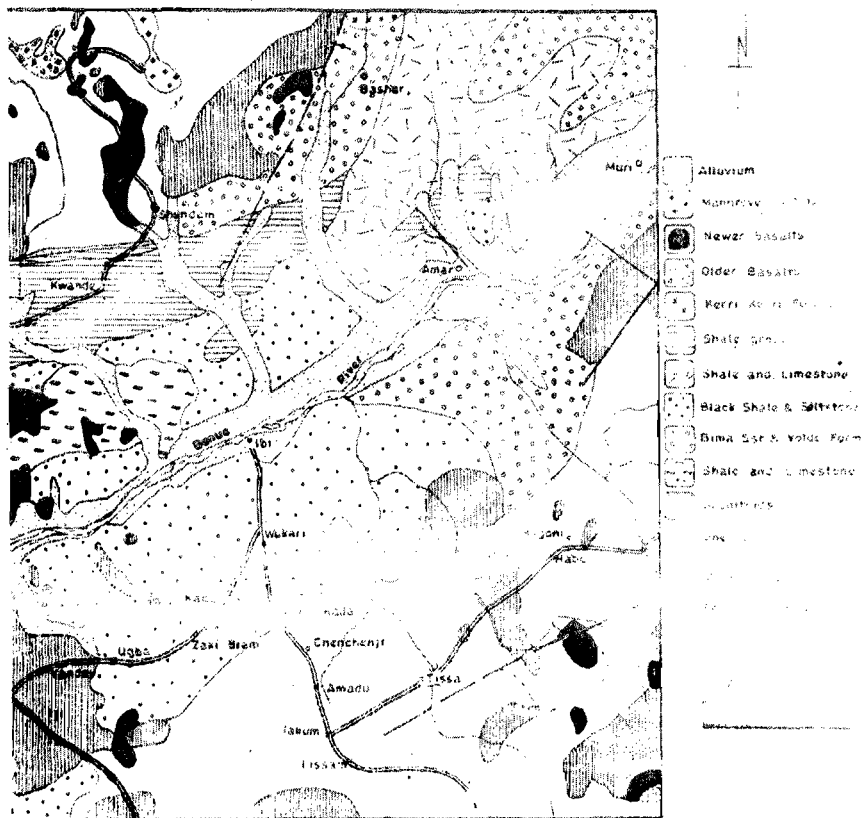
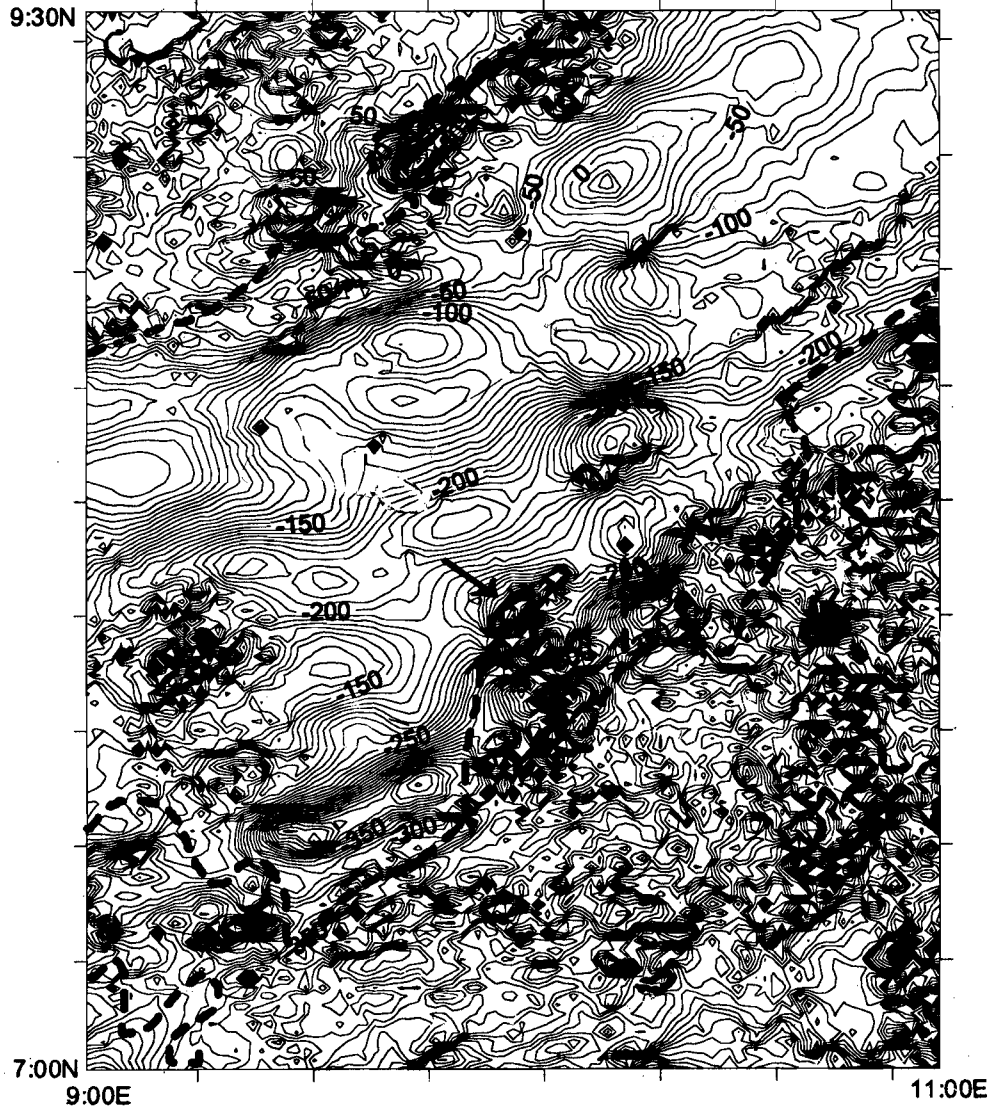


Fig. 5b The detailed geology of the Middle Benue Trough, Nigeria (After Offodile 1976, Geological Survey of Nigeria Map of Nigeria 1994)

Field (IGRF) corresponding to the epoch and altitude of the survey. The resulting magnetic anomaly from this treatment was recontoured using the Golden Software surface mapping system (Surfer version 7.02). This is displayed in Figure 6.

The composite aeromagnetic anomaly map (Fig. 6) shows values ranging from  $-750$  to  $100$  nT and having a northeast trending pattern of low relief magnetic values in the central and southeastern parts of the area, juxtaposed by the high-relief areas in the northwest part. Thus the low gradient and negative contours exist over the sedimentary surface while high gradient and negative contours are dominant towards the southeast edge of the map from the Basement-Cretaceous boundary into the Basement Complex rocks. The NW edge is dominated by a positive low.

The average magnitude of the earth's magnetic field over this area is about  $33510$  nT: its inclination ranges from  $4^{\circ}$ N to  $6^{\circ}$ S and its declination is about  $8^{\circ}$ . This area is clearly in low magnetic latitude, and indeed the  $0^{\circ}$  magnetic latitude passes nearly across the middle of this studied area.

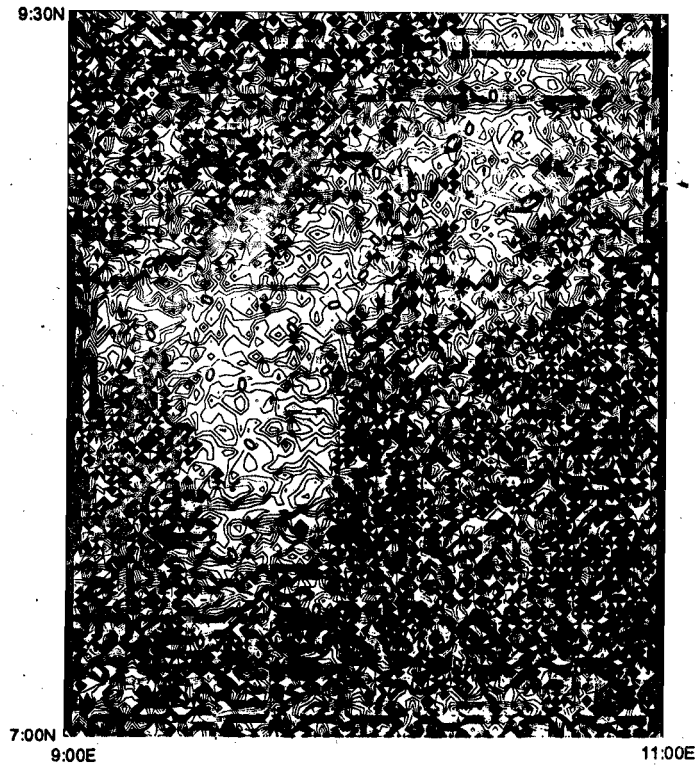


**Fig. 6** Residual total magnetic field anomaly over the Middle Benue Trough, Nigeria. The 1975 IGRF model (epoch date 1<sup>st</sup> January 1974) field has been removed. Approximate sedimentary - basement boundary is indicated in dashed lines. Arrowed is a positive anomaly. Contour interval is 10nT.

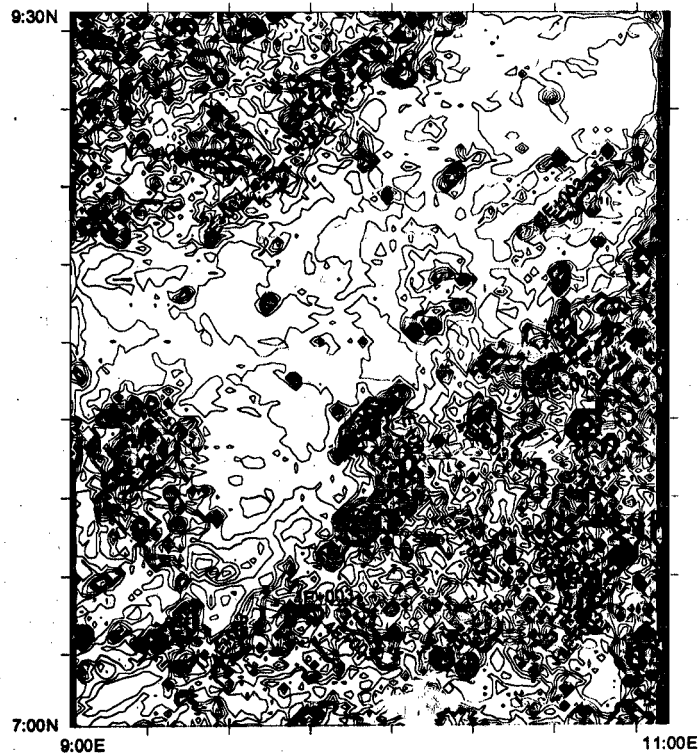
The RTP and the analytic signal maps computed from the original magnetic anomaly map (Fig. 6) are respectively shown in Figures 7 and 8. Both maps (Figs. 7 and 8) attempt to define sets of lineation in the area, but the analytic signal of the RTP map computed and displayed in Figure 9 defines these sets of lineation/fault or edges of units clearer from the pattern of the anomaly peaks. These sets of lineation agree with the known features of the area.

## CONCLUSIONS

Reduction to the pole attempts to simplify the magnetic field by rotating the magnetizing vector to be vertical. However, this technique represents significant problems when the direction of magnetization is not in the same direction as the Earth's field and is particularly crucial and serious when data have been collected at low magnetic latitudes. The computation of the reduction to pole



**Fig. 7** RTP transformation of the aeromagnetic anomaly map of Figure 6 over the Middle Benue Trough, Nigeria. Inclination and declination are respectively  $6^\circ$  and  $8^\circ$ . Contour interval is 1000nT.



**Fig. 8** Output of the analytic signal amplitude (local amplitude) computation of the residual total field magnetic intensity over the Middle Benue Trough, Nigeria. Contour is 700nT/km.





Fig. 9 The analytic signal of the reduced to pole anomalies shown in Figure 7. The defined boundaries are shown in dashed lines. Contour interval is 50000nT/km.

data from the magnetic anomaly is easily accomplished in frequency domain using standard filtering techniques. The problem of amplification of the noise present in the data at low magnetic latitudes is ameliorated by the use of azimuthal filter. This filter acts within a pie-wedge region in the frequency domain centred on the strike of the azimuth and thus acts as a general strike-reject filter, rejecting trends in the direction of azimuth specified in this domain. On the other hand, the analytic signal of the total magnetic field reduces magnetic data to anomalies whose shape can be used to determine the depths of those edges. Thus analytic signal maps magnetization of the ground without regard to direction of magnetization.

The analytic signal transformation of a synthetic magnetic anomaly data to which reduced to the pole analysis is applied produced better results than when performed on the total field anomaly directly. This method was applied to the aeromagnetic field anomaly of the Middle Benue Trough, Nigeria. The regional geology shows an obvious cause of the anomalies as faults/contacts at the sedimentary – basement boundary of this Cretaceous sedimentary cover.

#### REFERENCES

- Agagu, O. K. and Adighije, C., 1983, Tectonic and sedimentation framework of the Lower Benue Trough, S.E. Nigeria, *J. Afric. Earth Sci.*, 1: 267-274
- Baranov, V., 1957, A new method for interpretation of aeromagnetic maps: pseudo-gravimetric anomalies, *Geophysics* 22: 354-383
- Benkheilil, M. J., 1982, Benue Trough and Benue Chain, *Geol. Mag.* 119:155-168
- Benkheilil, M. J. 1989, The origin and evolution of the Cretaceous Benue Trough, Nigeria, *J. Afric. Earth Sci.*, 8: 251-282
- Bhattacharyya, B. K., 1965, Two-dimensional harmonic analysis as a tool for magnetic interpretation, *Geophysics* 30: 829-857.
- Ford, S. O., 1981, The economic mineral resources of the Benue Trough, *Earth Evol. Sciences* 2: 154-163
- Gunn, P. J., 1975, Linear transformations of gravity and magnetic fields, *Geophys. Prosp.* 23: 300-312
- Hansen, R. O. and Pawlowski, R. S., 1989, Reduction-to-the-pole at low latitude by Wiener filtering, *Geophysics* 54: 1607-1613
- Hsu, S. K. Coppens, D. and Shyu, C. T., 1998, Depth to magnetic source using the generalized analytic signal, *Geophysics* 63, 1947-1957
- Hsu, S. K., Sibuet, J. C. and Shyu, C. T., 1996, High-resolution detection of geologic boundaries from potential-field anomalies: an enhanced analytic signal technique, *Geophysics*, 61: 373-386

- Lu, R. S., 1998, Finite-impulse-response reduction-to-the-pole filter, *Geophysics* 63: 1958-1964
- Macleod, I. N., Jones, K. and Dai, T. F., 1993. 3-D analytic signal in the interpretation, of total magnetic field data at low latitudes, *Expl. Geophys.* 24: 679-688
- Mendonca, C. A. and Silva, B. C., 1993, A stable truncated series approximation of the reduction to the pole operator, *Geophysics* 58: 1084 - 090.
- Murat, R. C., 1972, Stratigraphy and palaeogeography of Cretaceous and Lower Tertiary in southern Nigeria, in: *African Geology* (T. F. J. Dessauvage and A. J. Whiteman, Eds.). University Press, Ibadan, 251-269
- Nabighian, M. N., 1972, The analytic signal of two-dimensional magnetic bodies with polygonal cross-section: its properties and use for automated anomaly interpretation, *Geophysics* 37: 507-517
- Nabighian, M. N., 1984, Toward a three dimensional automatic interpretation of potential field data via generalized Hilbert transforms: fundamental relation, *Geophysics*, 49: 957-966
- Offodile, M. E., 1976, A review of the geology of the Benue Valley, In: *Geology of Nigeria* (C. A. Kogbe, Ed.), Elizabethan Publishing Co. Lagos, 319-330
- Offodile, M. E., 1984, The geology and tectonics of Awe brine fields, *J. Afric. Earth Sci.* 2: 191-202
- Phillips, J. D., 1997, Potential-field geophysical software for the PC, version 2.2: U. S. Geol. Surv. Open-File Report 97-725. Available from the following website: <ftp://greenwood.cr.usgs.gov/pub/open-file-reports/ofr-97-0725/pfofr.htm>
- Reyment, R. A., 1965, Aspects of the geology of Nigeria, Ibadan University Press, Ibadan, 184p
- Roest, W. R., Verhoef, J. and Pilkington, M., 1992. Magnetic interpretation using 3-D analytic signal, *Geophysics* 57: 116-125
- Short, K. C. and Stauble, A. J., 1967. Outline of the Niger delta, *Bull. Amer. Assoc. Petr. Geol.* 51: 761-779