Comparative Analysis of Change between Ellipsoidal Height Differences and Equivalent Orthometric Height Difference

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Abstract

Height is an important component in the determination of the position of a point. The study aimed at performing a comparative analysis of change between ellipsoidal height differences and the equivalent orthometric height difference of points. A hi-target Differential Global Positioning System (DGPS) was used to acquire GPS data with an occupation period of thirty (30) minutes on each point, which were processed using Hi-target Geomatics Office (HGO) software to obtain the ellipsoidal heights. An automatic level instrument was used to acquire leveling data, which were processed using the height of collimation method to obtain the orthometric heights. A total of fifty (50) points were occupied as common points for both the GPS and levelling observations at 20-meter intervals. The accuracy of the height difference was determined using standard deviation with the ellipsoidal height difference as 53.59cm and the orthometric height as 53.07cm respectively. A Root Mean Square Error value of 0.0621m was obtained as the accuracy of the change between the two height differences. Statistical analysis using the independent-sample Z test was used to analyze the data at a 5% significant level. The result shows no significant difference in the performance of the two height systems. It is worthy to note that GPS and spirit levelling height differences can be used interchangeably for any heighting in short distances for surveying and engineering applications.

Keywords: GPS, levelling, Ellipsoidal height, Orthometric height, Statistical analysis

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Ghana Journal of Geography Vol. 12(1), 2020 pages 132- 144

https://doi.org/10.4314/gjg.v12i1.7

Introduction

Levelling is a vital operation through which elevation of points or differences in elevation are determined to produce necessary data for mapping, engineering design, and construction. Spirit levelling is based on the fact that the axis of the spirit level is perpendicular to the plumb line and the height difference between two points is obtained as the difference of readings on the level rods settled on those two points (Zarko & Sinisa, 2014). The level position is considered in the middle of those two points. Spirit levelling is a very accurate method especially for short levelling lines (Heiskenen & Moritz, 1967). The height obtained through spirit leveling is known as orthometric height. Orthometric heights are the natural heights above sea level, which means heights above the geoid. They thus have an unequalled geometrical and physical significance (Ayan, 2001; Zarko & Sinisa, 2014).

Global Positioning System (GPS) is a 3-dimensional positioning method that determines coordinates in a global geocentric orthogonal system. For practical reasons the global coordinates X, Y, Z are transformed into ellipsoidal coordinates ϕ, λ, h and eventually into local horizontal coordinates n, e, u (Otaka & Josef, 2004). The heights determined by a Global Positioning System (GPS) are related to the WGS-84 ellipsoid while levelling heights (orthometric, normal) are related to a given vertical datum, which is physically defined (Otaka & Josef, 2004). To obtain accurate elevations with the Global Positioning System (GPS), the geoidal heights in the area must be known and applied. The height obtained through Global Positioning System (GPS) observation is known as the ellipsoidal height which cannot be used directly for practical surveying but needs to be transformed into orthometric heights, being the distance measured along the plumb line between the geoid and a point on the Earth's surface (Ceylan & Baykal, 2008; Atinc & Ramazan, 2019). Global Positioning System (GPS) heighting is considered as a substitute for classical terrestrial height measuring methods in the present time. From a methodological point of view, the determination of height with the help of GPS is more complicated than classical terrestrial methods (Otaka & Josef, 2004).

Badejo *et al.* (2016) investigated the use of ellipsoidal heights in place of orthometric heights for engineering surveys. DGPS and geodetic levelling observations were carried out to obtain the ellipsoidal and orthometric heights for a number of points in the study area. Mean accuracy of 13.2ppm was obtained over a total distance of 139.114km which satisfied the accuracy of third-order levelling which is good enough for engineering surveys. In another study carried out by Audu and Tijani (2017), they compared the elevation differences obtained from the total station and the automatic level instrument. The maximum and minimum difference obtained between the height difference obtained from the two instruments was 62mm and 20mm respectively. Zarko and Sinisa (2014) compared the height differences obtained by spirit levelling and trigonometric levelling methods. They obtained a root mean square error of 0.98mm and concluded that there is no significant difference between the two levelling methods at a 5% significant level.

Although it may seem that the difference in heights derived from both the spirit levelling and the Global Positioning System (GPS) could not be compared, it is possible to state that under some conditions it is possible to compare the two methods, bearing in mind the essential characteristics of spirit leveling and Global Positioning System (GPS), it is possible, under the same restrictions, to state that height differences obtained by GPS and spirit leveling are comparable (Zarko & Sinisa, 2014). Those limitations are firstly related to the GPS leveling method because the different influences considerably limit its accuracy. In this light, the results of height differences obtained by the spirit leveling method could be considered significantly more accurate than those obtained by the GPS method. The main difference between the Global Positioning System (GPS) and the spirit levelling method for height difference determination is in the construction of geodetic instruments used for each, in the measurement methods, and in the influences which affect their accuracy. In this study, the accuracy of the change between ellipsoidal height differences and equivalent orthometric height difference has been determined for fifty (50) common points which were marked in the study area.

Orthometric Height

The orthometric height of a point is the distance H along a plumb line from the point to a reference height. When the reference height is a geoid model, orthometric height is used for practical purposes, that is "height above sea level".

Orthometric height (H) is computed as follows;

$$\mathrm{Hp} = \frac{\mathrm{Cp}}{\mathrm{gp}} \tag{1}$$

Mathematically, the orthometric height is given by the geopotential number (C) divided by the integral mean value of gravity ($\overline{gp}$) taken along the plumbline.

Alternatives to orthometric height include the dynamic height and normal height, and different countries may choose to operate with either of the heighting systems. Gravity is not constant over large areas; as such, the height of a level surface other than the reference surface is not constant, and orthometric heights need to be corrected for that effect, as in Eq. 2 (Heiskanen & Moritz, 2006). Thus, orthometric heights are purely geometric; they are the length of a particular curve (a plumb line), which can be derived through the use of spirit and trigonometric levelling methods, etc. (Heiskanen and Moritz, 1967).

$$\mathrm{OC_{AB}} = \sum_{A}^{B}\left(\frac{\mathrm{g} - \gamma_0}{\gamma_0}\,\delta\mathrm{n} + \frac{\breve{\mathrm{g}} - \gamma_0}{\gamma_0}\,\mathrm{H_A} - \frac{\breve{\mathrm{g}} - \gamma_0}{\gamma_0}\,\mathrm{H_B}\right) \tag{2}$$

Where $\breve{g} = \frac{g_A + g_B}{2}$, g_A and g_B have surface values at ground level.

Ellipsoidal Height

Ellipsoid heights are the straight-line distances normal to a reference ellipsoid produced away from (or into) the ellipsoid to the point of interest. Before GPS, it was practically impossible for anyone outside the geodetic community to determine an ellipsoid height. Now, GPS receivers produce three-dimensional baselines resulting in determinations of geodetic latitude, longitude, and ellipsoid height (Meyer, 2002). Ellipsoid heights are almost never suitable surrogates for orthometric heights because equipotential ellipsoids are not, in general, suitable surrogates for the geoid (Meyer, et al., 2005a; Kumar 2005).

The difference between the ellipsoidal height and the orthometric height is referred to as the geoidal undulation.

The fundamental relationship between these three terms is given thus;

$$h = H + N \tag{3}$$

where; h is the ellipsoidal height, H is the orthometric height, and N is the geoidal undulation.

Study Area

The research was conducted at the Obanla campus of the Federal University of Technology, Akure, Ondo State in the South-Western part of Nigeria. The geographic location lies approximately between latitude 07º 18’ 10.02” N to 07º 18’ 56.04” N and longitude 05º 7’ 54.02” E to 05º 7’ 53.46” E.

Methodology

The methodology adopted in this study includes GPS observation using Hi-target Differential GPS receivers and spirit levelling observation using an automatic level instrument. A total of fifty (50) points were marked and observed for both GPS and levelling on a route of about 1.2 kilometers. In order to minimize the errors introduced by earth curvature and refraction, distances between the test points were made to be 20m. The GPS observed data was processed using the Hi-target geomatics office (HGO) software to obtain the ellipsoidal heights while the spirit levelling data was computed using the height of the collimation method to obtain the orthometric heights on the Microsoft Excel spreadsheet. Finally, a comparative analysis of change between the ellipsoidal height differences and equivalent orthometric height difference is performed.

GPS Observations

The GPS surveys were carried out in order to obtain the ellipsoidal heights of the 50 points marked within the study area. The GPS measurements were taken with Hi-target DGPS receivers using the static method. The reference receiver was set up on a reference station (FUTA SVG/G13/05), a temporary adjustment was performed and all precautions were taken. After the setting operation, the GPS instrument was allowed to track enough satellites for data streaming. The rover receiver was moved from one point to another after carrying out all the necessary settings until all the points were occupied. The occupation period for each point was thirty (30) minutes.

Spirit Levelling Observations

In this study, a closed-loop levelling operation was conducted with an automatic level instrument together with two levelling staffs of three meters long. The automatic level instrument was set-up midway between the survey control point (FUTA SVG/G13/05) and peg 1, the backsight observation was taking on the control point with the help of the levelling staff held over the control point and another levelling staff was held over peg 1 for foresight reading. The instrument was moved to a point between peg 1 and peg 2 at equal interval, the backsight observation was made to peg 1 and the foresight observation to peg 2. This procedure was repeated until the last point was reached and the operation was closed on another survey control point (FUTA/SVG/GPS/14/49) as illustrated in Figure 2.

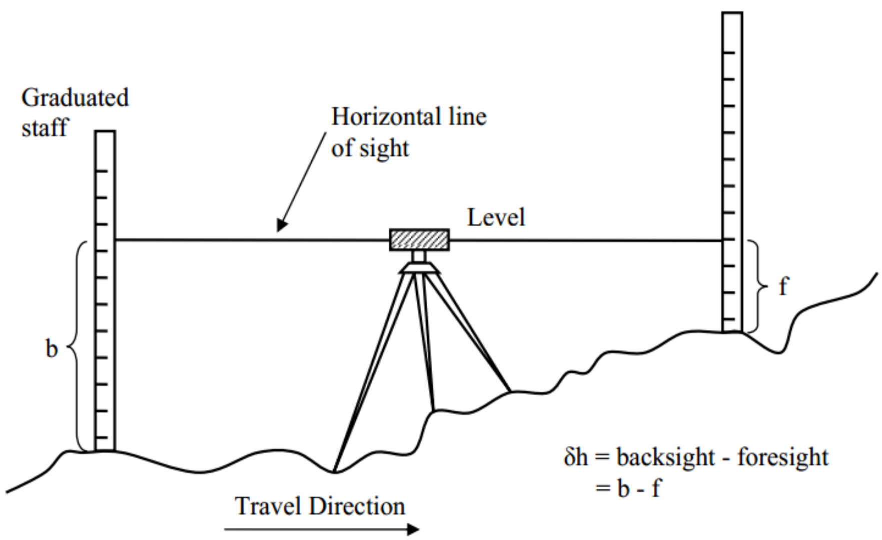

Figure 2. Spirit levelling procedure (Kemboi & Odera, 2016)

Results and Discussion

Presentation of Results

The results obtained from this study are the ellipsoidal heights and equivalent orthometric heights of fifty (50) selected points determined using GPS and spirit levelling observations. In order to compare the results of the change between the ellipsoidal height differences and equivalent orthometric height difference, the height difference between the points was separately determined and the accuracy of the results estimated using the root mean square error (RMSE). Table 1 shows the processed results obtained from the GPS and spirit levelling observations while Table 2 shows the change in height difference between the selected points.

Table 1: Results obtained from GPS and spirit levelling observation

Stations	Easting (m)	Northing (m)	Ellipsoidal Height (m)	Orthometric Height (m)
PT1	735416.4	807701	374.9	374.911
PT2	735416.8	807681.1	374.266	374.277
PT3	735417	807661.1	373.631	373.639
PT4	735417.1	807641.2	372.954	372.97
PT5	735417.4	807621.2	372.269	372.272
PT6	735417.5	807601.3	371.636	371.65
PT7	735417.7	807581.3	370.896	370.92
PT8	735417.9	807561.4	370.138	370.181
PT9	735418.1	807541.5	369.471	369.511
PT10	735418.4	807521.6	368.636	368.681
PT11	735418.6	807501.5	367.518	367.541
PT12	735418.8	807481.5	366.147	366.174
PT13	735419.1	807461.4	365.11	365.148
PT14	735419.5	807441.3	364.532	364.558
PT15	735419.7	807421.2	364.519	364.548

Source: Extracted from GPS and spirit levelling processed data (2020).

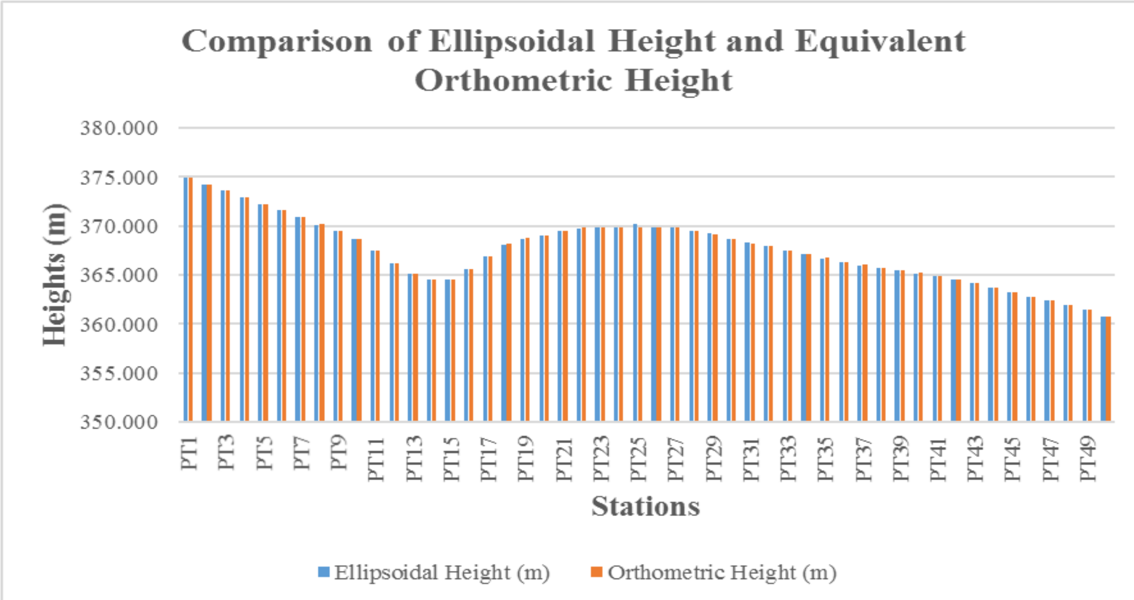

Figure 3: Chart Showing Comparison of Observed Ellipsoidal and Orthometric Heights

Table 2: Change between Ellipsoidal and equivalent Orthometric height difference

Stations	Ellipsoidal Height (h)m	Orthometric Height (H)m	Δh (m)	ΔH (m)	(Δh - ΔH) m
Peg 2	375.539	375.553			
PT1	374.9	374.911	-0.639	-0.642	0.003
PT2	374.266	374.277	-0.634	-0.634	0
PT3	373.631	373.639	-0.635	-0.638	0.003
PT4	372.954	372.97	-0.677	-0.669	-0.008
PT5	372.269	372.272	-0.685	-0.698	0.013
PT6	371.636	371.65	-0.633	-0.622	-0.011
PT7	370.896	370.92	-0.74	-0.73	-0.01
PT8	370.138	370.181	-0.758	-0.738	-0.02
PT9	369.471	369.511	-0.667	-0.67	0.003
PT10	368.636	368.681	-0.835	-0.83	-0.005
PT11	367.518	367.541	-1.118	-1.14	0.022
PT12	366.147	366.174	-1.371	-1.366	-0.005
PT13	365.11	365.148	-1.037	-1.026	-0.011
PT14	364.532	364.558	-0.578	-0.59	0.012
PT15	364.519	364.548	-0.013	-0.01	-0.003

Source: Extracted from results of ellipsoidal and equivalent orthometric height difference (2020).

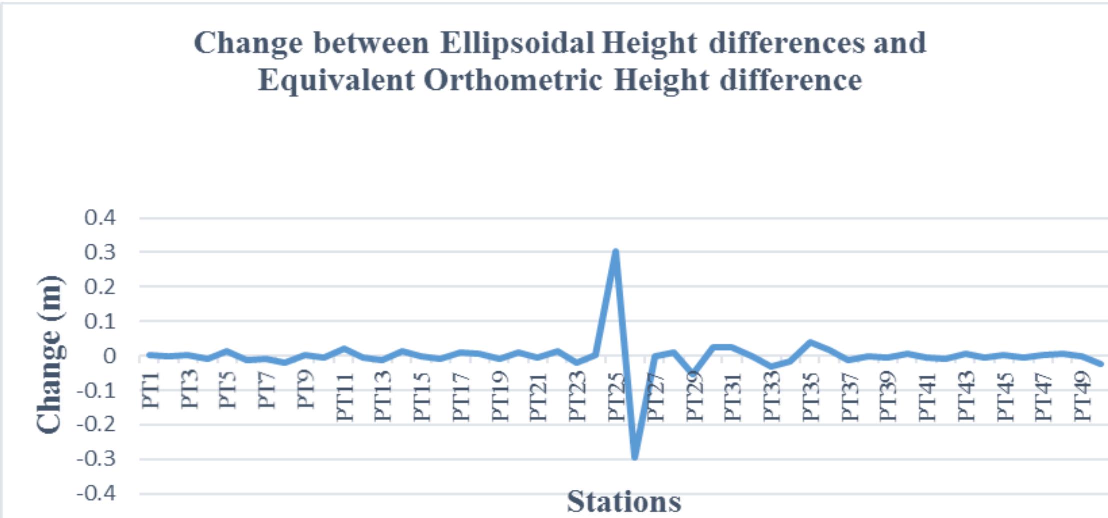

Figure 4: Chart Showing Change between Ellipsoidal Height differences and Equivalent Orthometric Height difference

Root Mean Square Error

In this study, the root mean square error (RMSE) is used to estimate the accuracy of the change between ellipsoidal height differences and equivalent orthometric height difference by squaring the height differences using equation (3)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\Delta h_i - \Delta H_i)^2}{n}} \qquad (3)$$

Where;

n is the total number of points; Δh is the ellipsoidal height difference of point *i* and ΔH is the orthometric height difference of point *i*

RMSE = 0.0621m

Standard Deviation of Differences in Ellipsoidal Height and Orthometric Height Differences

The mean and standard deviation of the ellipsoidal and orthometric heights difference values was calculated. The mean and standard deviation is given by equation (4) and (5) respectively.

$$\overline{X} = \frac{\sum X}{n} \qquad (4)$$

$$S = \sqrt{\frac{\sum (X - \overline{X})^2}{n-1}} \qquad (5)$$

Standard deviation is a key accuracy indicator and for the ellipsoidal heights differences S= 53.59cm while for equivalent orthometric height differences it is S=53.07cm. The standard deviation obtained for the change between the ellipsoidal height differences and the equivalent orthometric height difference is 0.0623m. This implies that both heights can be used interchangeably for surveying measurements in the study area.

The standard deviation value computed and compared within the permissible limits given by the American Society of Photogrammetry and Remote Sensing (ASPRS 1993) specifications is shown in Table 3 for topographic elevation accuracy requirements.

Table 3: ASPRS Topographic Elevation Accuracy Requirement for Well-Defined Points

Contour Interval (m)	Class I (m) High Accuracy/Standard Deviation Accuracy	Class II (m) Standard Deviation	Class III (m) Standard Deviation
0.5	0.08	0.16	0.25
1	0.17	0.33	0.5
2	0.33	0.67	1
4	0.67	1.33	2
5	0.83	1.67	2.5

Source: American Society of Photogrammetry and Remote Sensing (ASPRS 1993)

From Table 3, it can be deduced that both ellipsoidal height and orthometric height can be used interchangeably to produce a topographical plan of 1m contour interval for base maps, survey plans for engineering applications, and environmental applications.

Hypothesis Testing

In this study, we set a hypothesis to test that ellipsoidal height difference (X_1) and orthometric height difference (X_2) are the same and an alternative hypothesis to reject it using the independent sample Z test. The Z-test hypothesis was performed using Microsoft Excel 2010 and is stated below:

Null Hypothesis: $H_0 : X_1 = X_2$ i.e. ellipsoidal height difference is equal to orthometric height difference

Alternative Hypothesis: $H_i : X_1 \neq X_2$ i.e. ellipsoidal height difference is not equal to orthometric height difference

Decision Rule: H_0 may be rejected at 0.05 significant level if $Z > Z_{1-\alpha/2}$, $Z < Z_{\alpha/2} = Z_{0.975} > Z > Z_{0.025} = 1.96 > Z > -1.96$

Decision: H_0 was accepted; since the computed Z was greater than the Z from the table, i.e. 0.998 > -1.96, which implies that there is no significant difference between the ellipsoidal height differences and equivalent orthometric differences.

Discussion of Result

The coordinates, ellipsoidal heights, and equivalent orthometric heights of selected points obtained from GPS post-processing and levelling field book deduction are presented in Table 1. Figure 3 is a chart showing the comparison of the ellipsoidal and orthometric heights. The change between the ellipsoidal and equivalent orthometric height differences is shown in Table 2. The result produces a standard deviation value of 0.5359m for the ellipsoidal height and 0.5307m for the orthometric height while the RMSE value was 0.0621m. The elevation differences computed from ellipsoidal height and those of orthometric height for the series of points differed by an amount ranging from -29.4cm to 30.5cm, i.e. PT26 and PT25 as shown in Figure 4 with a mean difference of 1mm over a total distance of about 1.2km. We suspect outliers in two extreme cases, so, if we remove these values (-29.4cm and 30.5cm), the ranges and differences will be closer. Hence ellipsoidal and orthometric height differences can be substituted for each other.

The Z-test computed and compared with Z-critical values for comparison of the two height differences and hypothesis tests also showed acceptance of the null hypothesis to imply that there is no significant difference between the ellipsoidal height differences and the equivalent orthometric height difference. This implies that both heights can be used interchangeably for surveying measurements within the study area.

Conclusion

The paper has investigated the accuracy of the change between ellipsoidal height difference and equivalent orthometric height difference. A total of fifty (50) selected points within the study area were observed using Hi-target DGPS with an occupation period of thirty (30) minutes on each point and an automatic level instrument to obtain ellipsoidal and orthometric height data. The height difference between the ellipsoidal and orthometric heights was computed, and so was the change between the differences. The result obtained shows a standard deviation of 53.59cm for the ellipsoidal height difference and 53.07cm for the equivalent orthometric height difference. A Root Mean Square Error value of 0.0621m was obtained as the accuracy of the change between the two heights differences. Furthermore, hypothesis testing was performed using the statistical Z-distribution which shows that there is no significant difference between the ellipsoidal height difference and the equivalent orthometric height difference at a 95% confidence level. Considering the result obtained from the analysis, the ellipsoidal and orthometric heights can be used interchangeably without any effect on the accuracy for heighting in a short distance for survey and engineering requirements such as road construction work, cadastral surveys, and land use classification maps. However, further research can be carried out for longer distances in order to check for the accumulation of error in levelling observation and also with a longer time of occupation for the GPS observation.

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