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DETERMINATION OF LOCATION AND QUANTITY OF UNDERGROUND MINERAL RESOURCES

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ABSTRACT

Geophysical exploration is a reliable technique for detecting underground mineral resources. Its methods include natural-source method (such as gravity method) and artificial–source method (like seismic method). Various works on these methods, carried out by Kearey and Brooks (1988), Senti (1988), Gumert (1992), Robin (1995) and Kearsley et al. (1998), have shown that the gravity method is a good alternative to the more expensive seismic method. Yet, in Nigeria for instance, seismic method is widely used instead of the gravity method for mineral exploration. This is due, probably, to the inverse problem in the gravity method, which has defied satisfactory solution. This inherent problem is that of determining the most probable location and quantity of the underground mineral resources. This study attempted to determine the parameters (x, y, z), which indicate the location and the parameter (M), which gives the quantity of the underground mineral resources using the concepts of Newton's law of gravitation and Fourier analysis. It was observed that the parameters were found to be precise and satisfactory, hence the solution of the inverse problem in the gravity method of geophysical exploration was achieved at five percent level of significance.

KEYWORDS: Location, Quantity, mineral resources, Newton's law of gravitation, Fourier analysis and Residual gravity anomalies.

INTRODUCTION

Analysis of observed gravity anomalies is particularly suited for locating mass anomalies (mineral resources) beneath the earth's surface at depths of a few kilometres. This can be achieved in two stages:

(a) Separation of gravity anomalies into two components (i.e. regional components and local components). The regional components form the major part of gravity anomalies and provide information on the deep-seated structures of the earth. They are often referred to as the regional gravity anomalies. The local components include the part not covered by the regional components of gravity anomalies and provide information on the density anomalies, often associated with mineral deposits, in the shallow part of the earth. They are usually referred to as the residual gravity anomalies.

(b) Utilization of the residual gravity anomalies to determine the location and quantity of the underground mineral deposits.

Therefore, the presence of residual gravity anomalies in the observed gravity anomalies is an indication of occurrence of mineral resources beneath the earth. There are two broad methods (graphical and analytical methods) for the separation of gravity anomalies into two components to determine the optimum residual gravity anomalies needed for mineral exploration. Various works on these methods are fully discussed in Telford et al. (1990), El-sayed (1993), Reynolds (1998) and Idowu (2005).

The determination of location and quantity of the mineral deposits forms the second (and final) stage of the analysis of gravity anomalies for mineral exploration. Various authors have tried different techniques to accomplish this task. For instance, in the works of Kearey and Brooks (1988), Telford et al (1990) and Reynolds (1998), the plotted positions of horizontal second derivatives of gravity anomalies, also called points of inflections of gravity anomaly profile, were used to define the boundary enclosing the underground mineral deposit. Also, based on the mathematical formulations derived from Gauss's theorem in Kearey and Brooks (1988), Telford et al (1990) and Reynolds (1998), the residual gravity anomalies were used in the determination of the quantity of mineral deposit. From the review of previous works, it is observed that the procedure being used for the location of the underground mineral body defines the region within which the mineral could be found but does not define the location of the centre of mass of the mineral body. Also, the approach, currently in use, for the determination of the quantity of mineral deposit is the direct application of Gauss's theorem and this leaves rooms for further investigations into the application this theorem. Therefore, a better approach, which leads to the use of more acceptable functional models and procedures for the determination of the most probable location of the centre of mass and quantity of mineral deposit, is highly desirable. This could lead to the solution of the inverse problem in gravity method of geophysical exploration and hence leads to cheaper, easier and less risky mineral exploration. Therefore, it is the objective of this study to use the concepts of Newton's law of gravitation and Fourier analysis in determining the location and quantity of the sub-surface mineral body that is responsible for the residual gravity anomalies distribution on the earth's surface

METHODOLOGY

The method adopted is to utilize the optimum residual gravity anomalies of a given area to determine the location of centre of mass and quantity of the mineral deposits using the applications of Newton's law of gravitation and Fourier analysis.

Data Acquisition

The data used for the study as extracted from Idowu (2005), is shown in Table 1. It is an actual field data obtained during gravity survey of Gongola basin by Shell Nigeria Exploration and Production Company (SNEPCO) in 1995. The gravity survey covers parts of Bauchi, Gombe and Plateau States of Nigeria as shown in figure 1. The section used in this study covers one of the areas suspected for mineral accumulation in Bauchi State. Columns 1, 2, 3, 4, 5, 6 and 7 of the table show the gravity station numbers, x-coordinates, y-coordinates, z-coordinates, observed gravity anomalies, residual gravity anomalies and the horizontal distances of station intervals (STI) respectively. Columns 1 – 5 were collected from SNEPCO (1995) while columns 6 and 7 were computed by Idowu (2005). Also, the density of the mineral deposit and the density of the rock hosting the mineral body were given as $0.75 g/cm^3$ and $2.00 g/cm^3$ respectively.

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Table 1: Residual gravity anomalies and station intervals at the survey area

St No.	x (m)	y (m)	z (m)	Δg (mGal	$\bar{R}$ (mGal)	STI (m)
V0020	687925.100	1114812.000	308.400	-15.175800	-0.063056	-
V0040	688142.400	1115256.300	305.900	-15.058200	-0.423328	494.593
V0060	688444.700	1115631.900	303.400	-15.001400	-0.686695	482.142
V0080	688621.900	1116031.300	306.000	-14.902900	-0.569578	436.944
V0100	688519.300	1116510.100	296.700	-14.705000	-1.315941	489.669
V0120	688373.600	1116978.100	286.000	-14.625600	-1.040723	470.226
V0140	688422.800	1117457.400	287.300	-14.237200	-1.542023	502.463
V0160	688269.400	1117921.300	296.200	-13.687200	-2.343619	466.963
V0180	688156.900	1118398.800	302.600	-13.383500	-2.362164	490.574
V0200	688051.000	1118875.800	306.100	-13.235300	-2.228070	488.614
V0220	687876.500	1119331.600	315.500	-12.876100	-1.525532	488.061
V0240	687753.700	1119810.900	317.000	-12.483600	-1.721616	494.781
V0260	687648.600	1120288.300	324.600	-12.219700	-0.951945	488.832
V0280	687523.500	1120756.400	328.300	-12.054900	-0.622040	484.528
V0300	687442.100	1121242.800	326.700	-12.019400	-0.868855	493.164
V0320	687354.900	1121729.800	324.300	-11.881300	-1.331219	494.745
V0340	687364.500	1122224.400	323.700	-11.604800	-1.689661	494.693
V0360	687258.900	1122705.800	323.200	-11.492300	-1.870601	492.846
V0380	687192.600	1123190.100	328.300	-11.118800	-1.558140	488.817
V0400	687281.400	1123673.800	333.500	-11.347000	-0.689287	491.784
V0420	687356.400	1124151.300	337.900	-11.289700	-0.287607	483.354
V0440	687703.200	1124488.300	340.500	-11.194400	-0.160651	483.569

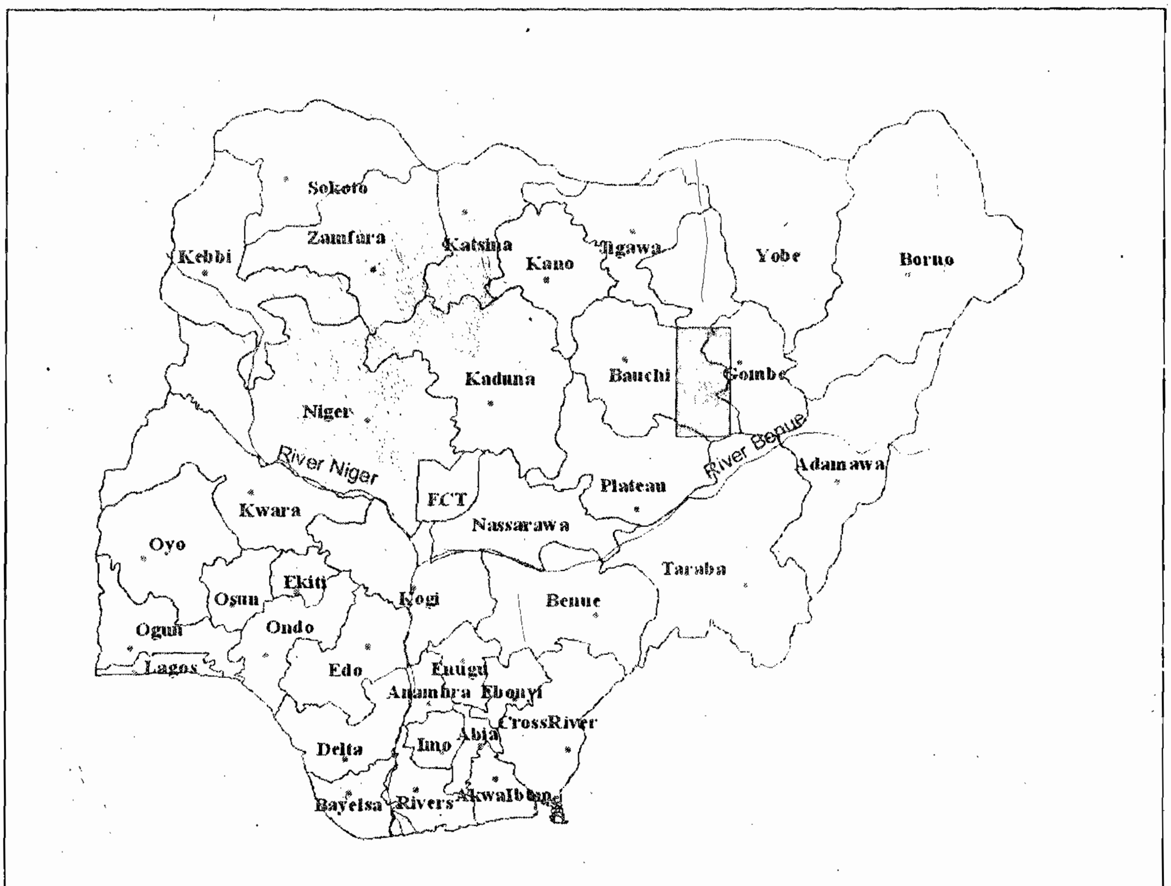

Figure 1: Map of Nigeria showing the study area
Source: **SNEPCO, 1995**

LEGEND

The Study Area

River

State

Data Processing

This includes the mathematical formulation of the computational procedure for the solution of inverse problem in the gravity method of geophysical exploration. That is, the mathematical formulation for the determination of the location and quantity of the underground mineral deposit that caused the optimum residual gravity anomalies. The application of Newton's law of gravitation leads to the simultaneous determination of x, y, z, and M while the concept of Fourier analysis leads to the determination of M and the depth (D), that is, the height from the centre of mass of the mineral body to the surface of the earth.

Determination of the Location and Quantity of Mineral Deposit by Newton's law of Gravitation.

In Figure 2, let $\underline{G}$ be the position of underground anomalous body of mass M that caused the residual gravity anomaly at position P_i on earth's surface (S_f).

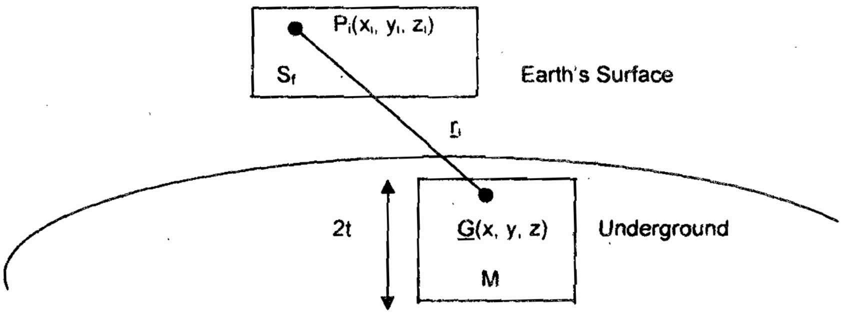

Figure 2: Underground location of anomalous body

The residual gravity anomaly ($\bar{R}_i$) measured at P_i for the anomalous body, whose mass and distance from P_i are M and r_i respectively, is given by (1) as in Ezeigbo (2004).

$$\bar{R}_i = KM(z - z_i)/r_i^3 \qquad (1)$$

It can be observed that the measured quantity ($\bar{R}_i^b$) of $\bar{R}_i$ in equation (1) can be expressed as non-linear function of x, y, z and M. The function may be linearised, using Taylor's series expansion and the result, if the expansion is truncated at first order term, is:

$$\delta \bar{R}_i = -(3KM^0/r_i^5)(z^0 - z_i)(x^0 - x_i)\,dx - (3KM^0/r_i^5)(z^0 - z_i)(y^0 - y_i)\,dy$$
$$+ (KM^0/r_i^5)(r_i^2 - 3(z^0 - z_i)^2)\,dz + (K/r_i^3)(z^0 - z_i)\,dM + (\bar{R}_i^b - \bar{R}_i^c) \qquad (2)$$

where x^0, y^0, z^0 and M^0 are the approximate values of x, y, z and M, while dx, dy, dz and dM are the respective corrections to the approximate values to obtain the adjusted values (x^a, y^a, z^a, M^a) of the parameters (x, y, z, M). Also, $\bar{R}_i^c$ is the computed value of $\bar{R}_i$ obtained using the approximate values (x^0, y^0, z^0 and M^0) in equation (2) which can be combined and written in matrix form as

$$\bar{V} = \bar{A}\underline{x} + \underline{l} \qquad (3)$$

where $\bar{V} = (\delta \bar{R}_i)^T$, $\underline{x} = (dx, dy, dz, dM)^T$ and

$\underline{l} = (\bar{R}_i^c - \bar{R}_i^b)^T$

$\bar{A}$ is a design matrix of the coefficients of the parameters (dx, dy, dz and dM) while superscript T indicates the transpose of a vector and/or matrix. Applying least squares technique to solving equation (3), the estimates ($\hat{x}$) of $\underline{x}$ can be obtained as:

$$\hat{x} = -(\bar{A}^T \underline{P} \bar{A})^{-1}(\bar{A}^T \underline{P}\underline{l}) \qquad (4)$$

where: $\underline{P}$ = weight matrix of the observations.

The error covariance matrix ($\Sigma \bar{x}$) of the parameters is given by

$$\Sigma \bar{x} = \hat{\sigma}^2 (\bar{A}^T \underline{P} \bar{A})^{-1} \qquad (5)$$

Where: $\hat{\sigma}^2 = \bar{V}^T \underline{P} \bar{V}/(n-m)$ (A-posterior variance)

The approximate values can be computed as

$$x^0 = \Sigma x_i/n \qquad (6)$$
$$y^0 = \Sigma y_i/n \qquad (7)$$
$$z^0 = \Sigma z_i/n - D \qquad (8)$$

The geometric form of the mineral body is generally taken to be three-dimensional. This, in some cases for simplicity reason, is usually represented by a sphere (Telford et al., 1990 and Reynolds, 1998). Therefore, in this method, the shape of the mineral body is assumed to be a sphere. D, for a sphere, is given by equation (9) as in the works of Telford et al. (1990) and Reynolds (1998).

$$D = 1.305\,\bar{d}_{1/2} \qquad (9)$$

Where $\bar{d}_{1/2}$ is the horizontal distance from the point of maximum value of residual gravity anomaly to the point at which the value of the residual gravity anomaly reduces to half of its maximum value. Equation (10), which is generally used to compute the quantity of the mineral deposit, as in Telford et al. (1990) and Reynolds (1998), is also used here to compute the approximate value of M.

$$M^0 = \rho_1(1/2\pi K\Delta\rho)\Sigma_{i=1}^{n-1}\bar{R}_i\hat{T}_i \quad (10)$$

where: $\hat{T}_i$ = distance interval between two gravity stations
i = number of intervals
$\bar{R}_i$ = mean residual gravity anomaly for each interval
ρ_1 = density of the mineral deposit
$\Delta\rho = \rho_1 - \rho_2$ (density contrast)
ρ_2 = density of the host rock
K = Newton's gravitational constant

The estimates (x^a, y^a, z^a, M^a) of the parameters (x, y, z, M) can therefore be obtained and written in matrix form as:

$$(x^a, y^a, z^a, M^a)^T = (x^0+dx,\ y^0+dy,\ z^0+dz,\ M^0+dM)^T \quad (11)$$

Other information required for the location of underground mineral resources is the limiting depth (D_L). This refers to the maximum depth, from the earth's surface, at which the top of a mineral body could lie and still produce the measured residual gravity anomaly. For a sphere, as in the works of Kearey and Brooks (1988), Telford et al. (1990) and Reynolds (1998):

$$D_L = D - t \quad (12)$$

$$t = (\bar{R}_{max}D^2/0.028\Delta\rho)^{1/3} \quad (13)$$

Where: $\bar{R}_{max}$ = maximum value of residual gravity anomaly
t = half of the thickness of the mineral body = radius of the sphere

Determination of the Quantity and Depth of the Mineral Deposit by Fourier Analysis.

Fourier analysis was applied in Tsuboi (1983) in discussing the determination of underground distribution that is responsible for the residual gravity anomaly distribution on the earth's surface. This was based on the assumption that the residual gravity anomaly values and the underground mass at depth (d) vary along the same horizontal direction (s) as shown in Figure 3.

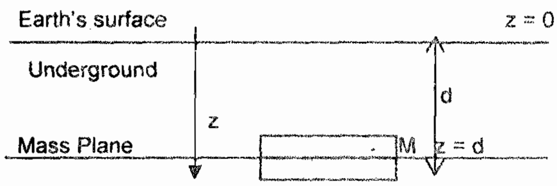

Figure 3: An underground mass plane at depth (d)

The residual gravity anomalies and the underground mass distribution at depth (d) can functionally be expressed by equation (14) as in Tsuboi (1983).

$$\bar{R}(s) = \sum_m mA_m \begin{matrix}Cos\\Sin\end{matrix} ms\, e^{-md} \quad (14)$$

$$M(s) = \sum_m C_m \begin{matrix}Cos\\Sin\end{matrix} ms \quad (15)$$

By Gauss's theorem (Tsuboi, 1983),

$$2\pi K\, M(s) = \sum_m mA_m \begin{matrix}Cos\\Sin\end{matrix} ms$$

Therefore, $mA_m = 2\pi KC_m$

Let $B_m = mA_m e^{-md}$,

Then, $$\bar{R}(s) = \sum_m B_m \begin{matrix}Cos\\Sin\end{matrix} ms \quad (16)$$

From the above relations, it can be seen that:

$$A_m = B_m e^{md}/m = 2\pi KC_m/m$$

$$B_m = mA_m e^{-md} = 2\pi KC_m e^{-md}$$

$$C_m = mA_m/2\pi K = B_m e^{md}/2\pi K$$

Therefore, the mass M(s) o
f the mineral deposit at the depth (d) can be given as:

$$M(\underline{s}) = (1/2\pi K)\sum_m B_m \begin{matrix}Cos\\ Sin\end{matrix} m\,s\,e^{md} \quad (17)$$

A_m, B_m and C_m are constant coefficients while 'm' is a positive integer (0, 1, 2,...,N), which is the half of the number of points where residual gravity anomalies are obtained. In other words, the number of stations where residual gravity anomalies are obtained = 2m.

It can be seen that equation (17) makes it possible to determine the underground mass distribution directly from the residual gravity anomaly distribution obtained on the earth's surface. The expression for the determination of underground mass distribution by Fourier analysis, as shown in equation (15) can be rewritten in the following form:

$$y = f(\underline{s}) = \sum C_m \begin{matrix}Cos\\ Sin\end{matrix} m\,\underline{s}$$

$$= C_0 + C_1 Cos\,\underline{s} + C_2 Cos2\,\underline{s} + \ldots + C_m Cosm\,\underline{s} + C_{m+1} Sin(m+1)\,\underline{s} +$$

$$C_{m+2} Sin(m+2)\,\underline{s} + \ldots + C_{2m} Sin(2m)\,\underline{s}$$

Let the coefficients of 'Cos' and 'Sin' be represented by a_m and b_m respectively. Then,

$$y = f(\underline{s}) = a_0 + a_1 Cos\,\underline{s} + a_2 Cos2\,\underline{s} + \ldots + a_m Cos(m\,\underline{s}) + b_1 Sin\,\underline{s} +$$

$$b_2 Sin2\,\underline{s} + \ldots + b_m Sin(m\,\underline{s}) \quad (18)$$

Equation (18) is known as Fourier series and the coefficients (a_m and b_m) are called Fourier coefficients. J. B. J. Fourier has suggested that an ordinary continuous one-valued function $f(\underline{s})$ given within a domain $0 \le \underline{s} \le 2\pi$ can be expressed by the sum of trigonometric functions having various amplitudes a_m and b_m with $m\,\underline{s}$ as arguments for m = 1, 2, 3, ..., N. If the values of $f(\underline{s})$ are known continuously throughout $0 \le \underline{s} \le 2\pi$, then the Fourier coefficients a_m and b_m are given by equations (19) to (21) as in Tsuboi (1983).

$$a_0 = (1/2\pi)\int_0^{2\pi} f(\underline{s})d\underline{s} \quad (19)$$

$$a_{m'} = (1/\pi)\int_0^{2\pi} f(\underline{s})Cos(m\,\underline{s})d\underline{s} \quad (20)$$

$$b_{m'} = (1/\pi)\int_0^{2\pi} f(\underline{s})Sin(m\,\underline{s})d\underline{s} \quad (21)$$

Where: $m' = 1, 2, 3, \ldots, N$

Also, it was stated that if the values of $y = f(\underline{s})$ are known only at 2m equidistance points within $0 \le \underline{s} \le 2\pi$, the values of the Fourier coefficients equal to the 2m values of y can be obtained. Therefore, the process of gravity anomaly analysis for the determination of the quantity and depth of the underground mineral deposit by Fourier series can be summarized as:

(a) Analysis of the residual gravity anomaly ($R(\underline{s})$) into Fourier series. That is to find the coefficients (a_m and b_m) such that the values of the series will be equal to 2m observed values of $R(\underline{s})$ within $0 \le \underline{s} \le 2\pi$ where $R(\underline{s})$ is given by.

$$\bar{R}(\underline{s}) = \sum_m a_m CoS(m\,\underline{s}) + b_m Sin(m\,\underline{s}) \quad (22)$$

(b) Calculation of $a_m e^{md}$ and $b_m e^{md}$ with assumed 'd'.
The estimate of 'd' is generally based on the consideration of the behaviour of the values of $B_m e^{md}$ according to the values of 'm'. In Tsuboi (1983), it was shown that if $B_m e^{md}$ becomes larger as 'm' increases, the series $M(\underline{s})$ will not converge smoothly. Therefore, the series $M(\underline{s})$ will converge smoothly and hence the most probable value of the underground mass will be obtained when the value of $B_m e^{md}$ tends to decrease with increase in 'm'.

(c) Synthesizing the series:

$$M(\underline{s}) = (1/2\pi k)\sum_m (a_m e^{md} Cos(m\,\underline{s}) + b_m e^{md} Sin(m\,\underline{s})) \quad (23)$$

(d)' The value of 'd' in the above formula is expressed in radian when the domain of interest is taken to be 2π. If the actual domain d in length is known, the actual depth d' corresponding to 'd' is given by equation (24) as in Tsuboi (1983)

$$d' = d\,d/2\pi \quad (24)$$

So far, the method of Fourier analysis which uses the single Fourier series has been discussed. This is applicable to the analysis of gravity anomalies in a situation where the residual gravity anomalies vary along a profile (a single horizontal direction). This method can be extended to be applicable to two-dimensional case. In this case, the residual gravity anomalies have to vary along two horizontal directions. In other words, the residual gravity anomalies should be obtained at the various grid points designed for the study area. Therefore, a double Fourier series can be used instead of a single Fourier series. The mathematical formulation for this is fully discussed in Tsuboi (1983).

Numerical Investigations

The mathematical formulations for the solution of inverse problem in the gravity method of mineral exploration have been discussed. Here, numerical tests are carried out in order to determine the adequacy or otherwise of the mathematical formulations in the determination of location and quantity of the underground mineral resources. The investigations include the following:

(a) The application of Newton's law of gravitation to determine the location and quantity of mineral deposit.

(b) The use of Fourier analysis to determine the quantity and depth of the mineral body.

Determination of location and quantity of the mineral body by Newton's gravitational method

Here, the required four parameters, x, y, z and M, which specify the location and quantity of the mineral deposits, are computed using equations (1), (2), (3), (4), (5), (6), (7), (8), (9), (10) and (11). Also, equations (12) and (13) are used to compute the limiting depth and the thickness of the mineral deposit respectively.

Determination of quantity and depth of mineral body by Fourier analysis

In this study, the application of the Fourier analysis is limited to the use of Fourier series in one-dimensional case. This is because the data were observed along one horizontal direction in the survey area. The computations involved is carried out using the following steps.

(a) Evaluation of Fourier series to obtain the Fourier coefficients (a_m and b_m).

The number of the Fourier coefficients = number of points where residual gravity anomalies are obtained = 2m within $0 \le s \le 2\pi$.

The requirement for a satisfactory result, in this method, is that the gravity station intervals must be equal. That is, the distances between gravity stations i and i+1 for all i's must be equal. In order words, the distance between gravity stations 1 and 2 = the distance between gravity stations 2 and 3 = the distance between gravity stations 3 and 4 and so on. However, as shown in column 7 of table (1), the station intervals are not equal. Therefore, for the purpose of this investigation, the average of the different twenty-one station intervals is used to represent the constant gravity station interval. Also, 2m = 22. Therefore, the Fourier coefficients for the residual gravity anomalies are obtained as:

$$a_0 = (1/2m)\sum_{i=1}^{2m} R_i \quad (25)$$

$$a_j = (1/m)\sum_{i=1}^{2m} R_i \, Cos(2\pi(i-1)j/2m) \quad (26)$$

j = 1, 2, 3, ..., m

$$b_j = (1/m)\sum_{i=1}^{2m} R_i \, Sin(2\pi(i-1)j/2m) \quad (27)$$

j = 1, 2, 3, ..., m-1

(b) Calculate $a_m e^{md}$ and $b_m e^{md}$ for m = 0, 1, 2, 3, ..., m; with an assumed initial value of depth (d).

(c) Computation of $M(s)$ using equation (23).

(d) The initial value of 'd' is increased and then used to compute the next values of $a_m e^{md}$, $b_m e^{md}$ and hence $M(s)$. This is an iterative process which continues until the value of $M(s)$ decreases smoothly and converges to the minimum value.

(e) $\hat{d}$ is computed as:

$$\hat{d} = \sum_{i=1}^{2m-1} [(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2]^{1/2} \quad (28)$$

Therefore, the actual depth (d') of the mineral deposit is obtained using equation (24).

Computer programs written in Fortran 77 language were designed and used to carry out the necessary computations. The results obtained are presented below.

Presentation of Results

The parameters for the location and quantity of the suspected mineral deposit in the study area are shown in Table 2. These include the coordinates of the centre of mass (X, Y, Z), the limiting depth (D_L), surface elevation (h), thickness (2t), depth of the centre of mass (D), the total depth

(D_T) and the quantity (M) of the mineral deposit as computed using the application of Newton's law of gravitation plus the quantity and depth of the mineral deposit using the application of Fourier analysis Also, SNEPCO evaluated their data which led to the determination of the surface coordinates (parameters) of the exploratory well to be drilled for mineral resources. The coordinates, as shown in Table 2, were chosen by SNEPCO based on the easy accessibility to the exploration site Also, the maximum depth drilled (without success of existence of mineral in commercial quantity) was 2785m (SNEPCO 1999).

Table 2: *Location and quantity of mineral deposit.*

Parameters	Computed Parameters by Newton's Method	Computed Parameters by Fourier Method	SNEPCO's Parameters
X(m)	687849.245 ± 0.0002	-	687606.959
Y(m)	1119612.114 ± 0.0007	-	1118822.023
Z(m)	-2727.859 ± 0.0012	-	-
h(m)	314.641	-	313.830
M(tons)	1.794×10^5	1.797×10^5	
2t (m)	3.684	-	
D (m)	3042.500	3036.127	
D_L (m)	3040.658	-	2785.000
D_T (m)	3044.342	-	

The statistical values used for examining the reliability of the procedure used for the computation of parameters used for the determination of the location and quantity of the mineral deposit are shown in Table 3. This includes the level of significance (α), degree of freedom (DF), Upper limit of table statistic (UL), computed statistic (CS) and lower limit of table statistic (LL).

Table 3: Computed and table statistics for location and quantity of mineral deposit.

α	DF	UL	CS	LL
0.05	18	31.53	18.08	8.23

Analysis of Results

It appears, as shown in column 2 of Table 2, that the computed parameters by Newton's method give the possible representations of the location and quantity of the mineral deposit. Statistical investigation was carried out to test the reliability of these results. That is, to show that the procedure used for the computations of the parameters has (or has not) introduced distortions in their values. In order words, $\bar{V}^T P \bar{V}$ was statistically examined to know whether it falls within the specified confidence limits or not. This was achieved by means of Chi-Squares (χ^2) hypothesis test. That is,

Null hypothesis: H_0: $\bar{V}^T P \bar{V} = \sigma_0^2$ ($\bar{V}^T P \bar{V}$ is within the confidence limits)

Alternative hypothesis: H_1: $\bar{V}^T P \bar{V} \neq \sigma_0^2$ ($\bar{V}^T P \bar{V}$ is outside the confidence limits)

Where $\bar{V}^T P \bar{V} / \sigma_0^2$ = Computed statistic.

This is a two-tail test where the Null Hypothesis is rejected if the computed statistic is outside the confidence limits. The confidence limits are the upper limit and the lower limit of the table statistic. They are obtained in the statistical table, in Ayeni (2001), as $\chi^2_{1-\alpha/2,df}$ for the upper limit and $\chi^2_{\alpha/2,df}$ for the lower limit. From Table 3, it appears that the computed statistic falls within the confidence limits. This suggests that the null hypothesis, that $\bar{V}^T P \bar{V}$ is within the confidence limits, should not be rejected. This confirms that the computed parameters satisfactorily represent the location and quantity of the mineral deposit. Also, from rows 5 and 7 of Table 2, the values of the quantity and depth of mineral deposit as determined by the applications of Newton's law of gravitation and Fourier analysis compare favorably well. The differences observed in their respective values are probably due to the fact that the gravity station interval used in the application of Fourier analysis is an estimated interval. This is because Fourier analysis requires gravity observations at equal interval but the observations used in this study were taken at unequal interval as shown in Table 1. On the other hand, the application of Newton's law of gravitation is free from the condition of equal gravity station intervals. Therefore, based on the available data, it can be inferred that the quantity and depth of the mineral body obtained by the application of Newton's law of gravitation is better than those obtained by the application of Fourier analysis. However, if the gravity stations where observations are carried out are of equal intervals, the two applications are most likely to produce the same results. Furthermore, the coordinates of the mineral location as given by SNEPCO (1999) and as determined in this study are not the same. This might not be unconnected with the following reasons:

(a) The condition of easy accessibility to the exploration well imposed by SNEPCO in deciding the location of the well

(b) Unlike the objective of this work, SNEPCO did not target the centre of mass of the Mineral deposit while determining the coordinates of the mineral body.

Therefore, it can be inferred that if SNEPCO had targeted the centre of mass of the mineral they would have drilled further to possibly get mineral in commercial quantity.

CONCLUSIONS

This paper has attempted to solve the inverse problem in the gravity method of geophysical exploration. Specifically, it tried to utilize the residual gravity anomalies in determining the location and quantity of the underground mineral deposit. The application of Newton's law of gravitation was used for the simultaneous determination of the parameters, x y, z and M. The parameters obtained were found to be satisfactory at five percent level of significance. Also, the quantity and depth of the underground mineral resources were determined using the concept of Fourier analysis. The quantities and the depths of the mineral deposit obtained using the applications of Newton's law of gravitation and Fourier analysis are found to be precise. Therefore, it can be inferred that the solution of the inverse problem in the gravity method of geophysical exploration appears to have been satisfactorily achieved.

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