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ANALYTICAL PREDICTION OF HYBRID STEPPING MOTOR STATIC CHARACTERISTICS

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ABSTRACT

This paper is an improvement on one of the methods of predicting static characteristics of the hybrid stepping motor using measured flux-linkage data. The total flux-linkage data is approximated using analytical functions. Instead of separating the data into the permanent magnet and current-dependent components, the present method fits it compositely. This composite method of approximation fits the flux-linkage data and predicts the static torque/rotor angular position characteristics more accurately than the separate fitting method.

KEYWORDS: Stepping motors, Hybrid stepping motors, Static torque prediction, Stepping motor characteristics, Static characteristics simulation.

INTRODUCTION

Stepping motors are electromechanical incremental actuators which convert digital pulses into discrete linear or angular motion of essentially uniform magnitude, instead of continuous motion as in conventional motors. Each pulse moves the rotor shaft and latches it magnetically to a precise position to which it is stepped.

The existence of stepping motors dates back to the 1920's, but interest in their use and prediction of characteristics began in the 1960's, when computer manufacturers discovered their usefulness in terminal devices. Since then many papers have been published in this area. While some are for design purposes, others are for calculating the static torque/rotor angular position characteristics for this class of motor. These are based on permeance (Harris et al, 1975; Harris and Finch, 1979; Chai and Konecny, 1982) finite element (Brauer, 1982; Ishikawa et al, 1998, Ishikawa et al, 2000), or measured flux-linkage data (Bryne and O'Dwyer, 1976; Pickup and Russell, 1979; Haller et al, 1981; Agber, 1985; Agber, 2004) methods.

The flux-linkage method fits the measured flux-linkage data numerically (Agber, 1985; Agber, 2004), or analytically using either exponential functions of current, *i*, at constant rotor angular position, θ (Bryne and O'Dwyer, 1976) or polynomial functions in *i*, and trigonometrical functions in θ (Pickup and Russell, 1979; Haller et al, 1981; Agber, 1985).

This paper examines a method of predicting the static characteristics of the hybrid permanent magnet (PM) stepping motor, where the total flux-linkage data is separated into PM and current-dependent components and fitted separately. It shows that the accuracy of fitting and prediction of characteristics is improved when the flux-linkage data is fitted without separation.

Formulation of the Method

The electromagnetic torque $T(i,\theta)$ of a system can be calculated from its magnetic coenergy $W'(i,\theta)$ at constant current as follows:

$$T(i,\theta) = \frac{\partial W'(i,\theta)}{\partial \theta} \qquad (1)$$

where *i* and θ denote the stator current and the rotor angular position.

The $W'(i,\theta)$ of such a system can be calculated if the flux-linkage data is known as illustrated by Fitzgerald et al, (1971).

$$W(i,\theta) = \int_0^{m} \varphi_T(i,\theta) di' \qquad (2)$$

where *i'* is a dummy variable for current and $\varphi_T(i,\theta)$ is the total flux-linkage.

The total flux-linkage of the hybrid PM stepping motor has two components (Pickup and Tipping, 1976), i.e. the PM $\varphi_{pm}(\theta)$ and the current-dependent $\varphi(i,\theta)$, that is:

$$\varphi_T(i,\theta) = \varphi_{pm}(\theta) + \varphi(i,\theta) \qquad (3)$$

By substituting Equation (3) into Equation (2) and simplifying, the following expression is obtained.

$$W(i,\theta) = i\varphi_{pm}(\theta) + \int_i^{m} \varphi(i,\theta) di' \qquad (4)$$

Equation (4) is then substituted into Equation (1) and a general expression for the torque of the PM hybrid motor is obtained.

$$T(i,\theta) = i\frac{d\varphi_{pm}(\theta)}{d\theta} + \frac{\partial\{\int_i^{m} \varphi(i,\theta) di'\}}{\partial\theta} \qquad (5)$$

As can be seen in Equation (5), the torque developed by a PM hybrid stepping motor consists of two components. The first is due to the interaction of the PM flux-linkage with the stator current, while the second is as a result of the interaction between the current-dependent flux-linkage and the current that produces it. It should be noted that the breakdown of the total flux-linkage into two components is a theoretical assumption. In practice, the current-dependent flux component does not exist independently in the same form.

The Separate Fitting Method.

Representation of the permanent magnet's flux-linkage

The motor under investigation is a 1.8^0 bifilar wound GEC hybrid stepping motor Model VM156 – 270BK. It has an axially magnetized rotor with 50 teeth (N_r = 50). In the usual hybrid arrangement, this excitation is provided by a permanent magnet.

A static rig, comprising a torque and displacement transducers, torque and flux meters, a stabilized DC supply and an X-Y plotter, was constructed and used to obtain the static data.

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The choice of a function $f_{pm}(\theta)$ to represent the PM flux-linkage data has largely been influenced by the form of the measured curve and the static and dynamic requirements it has to satisfy. These include:

i) The function must be symmetrical about the stable ($N_r\theta = 0$) and unstable ($N_r\theta = \pi$) torque zeros (where N_r denotes the number of rotor teeth).

ii) It must be equal and opposite about $N_r\theta = \pi/2$.

iii) It must be continuous and capable of specifying continuous derivatives.

The use of periodic function to represent the PM flux-linkage data is reasonable, since the variation of the flux-linkage with θ is periodic. These requirements are satisfied by a cosine series with odd harmonics in θ as shown below.

$$f_{pm}(\theta)=\sum_{n=1}^{2l-1} A_n Cos(nN\theta) \qquad (6)$$

where $n = 1, 3, \ldots\ldots, 2l-1$ – an odd integer.

To fit $f_{pm}(\theta)$ to the PM flux-linkage data requires a coefficient A_{00}, which accounts for the fact that the reference (zero) level of flux-linkage has been taken at the value of $N_r\theta$, where the absolute flux-linkage of the reference phase is at a maximum value. Equation (6) will then become:

$$f_{pm}(\theta) = A_{00} + \sum_{n=1}^{2l-1} A_{0n} Cos(nN_r\theta) \qquad (7)$$

Equation (7) is fitted to the PM flux-linkage data and with $n = 3$ (or $l = 2$), the discrepancy per experimental point is 0.1mWb-t (or 0.11% of the average flux-linkage at 15.0A), which is within the experimental accuracy of the measurements. The measured and fitted curves are shown in Figure 1.

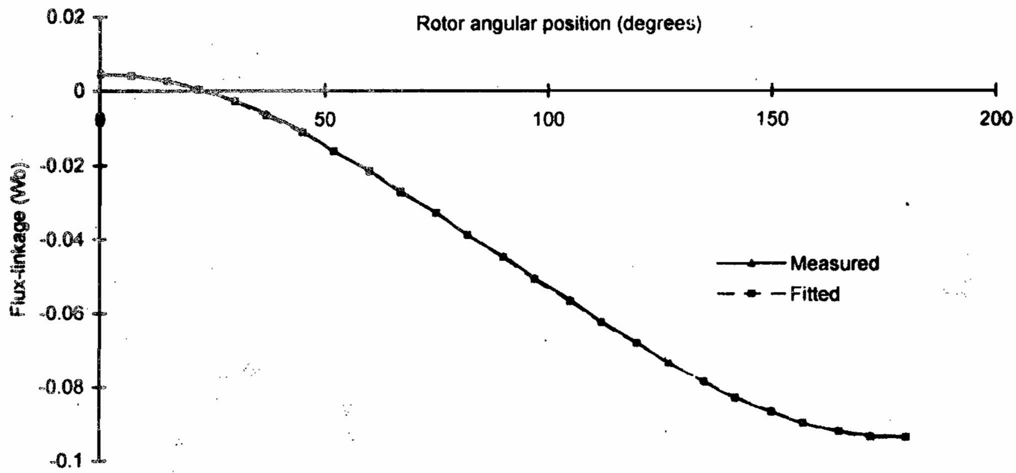

Fig. 1: Permanent magnet flux-linkage/rotor angular position characteristics

Representation of the current-dependent flux-linkage

The current-dependent flux-linkage is obtained by subtracting the PM flux-linkage from the total flux-linkage data, thus:

$$\phi(i,\theta)=\varphi(i,\theta)-\varphi_{pm}(\theta) \qquad (8)$$

The mathematical function representing the flux-linkage data must satisfy the following conditions:

i) It must be symmetrical about the stable ($N_r\theta = 0$) and unstable ($N_r\theta = \pi$) torque zero positions.

ii) It must be continuous and capable of specifying continuous derivatives $\partial\varphi(i,\theta)/\partial i$ and $\partial\varphi(i,\theta)/\partial\theta$ with accuracy.

iii) It must be sign sensitive in i.

These requirements are met with a function containing a cosinusoidal series in θ and a polynomial series of odd powers of i (Pickup and Russell, 1979; Agber, 1985). A similar representation involving a cosine series in θ and a polynomial for i, has been used elsewhere (Haller et al, 1981). The variant used here differs only in the exclusion of the even powers of i, which would violet the conditions set out above. The implementation chosen here fits the current-dependent flux-linkage data with the function suggested above.

$$f(i,\theta)=\sum_{k=0}^{m}\sum_{j=1}^{2l-1} A_{jk} i^j Cos(kN_r\theta) \qquad (9)$$

where $k = 0, 1, 2, \ldots\ldots, m$

$j = 1, 3, 5, \ldots\ldots, 2l-1$ – an odd integer.

The order of approximation (j and k) used depends upon the number of data points and the accuracy of representation needed. Taking into account the economy of computing, it was found that a fifth-order Fourier series ($k = 5$) and a ninth-order polynomial ($j = 9$ or $l = 5$) gives a reasonable compromise between measured and approximated curves. The measured and fitted flux-linkage curves for i = 5.0, 10.0, 15.0, 20.0, and 25.0A are shown in Figure 2. The average discrepancy between measured and fitted flux-linkage data using least square error routine is 0.42mWb-t or 0.26% of that measured at the motor rated current of 15.0A.

Prediction of Static Torque

Once a suitable representation for the flux-linkage data is obtained, the prediction of $T(i,\theta)$ characteristics can be performed using Equations (1 – 5). The total flux-linkage expression ($f_T(i,\theta)$) is given as the sum of Equations (7) and (9) as follows:

$$f_T(i,\theta) = A_{00} + \sum_{n=1}^{3} A_{0n} Cos(nN_r\theta) + \sum_{k=0}^{5}\sum_{j=1}^{9} A_{jk} i^j Cos(kN_r\theta) \qquad (10)$$

where j = 1, 3,.., 9 an odd integer

k = 0, 1, 2,.., 5

n = 1, 3 an odd integer.

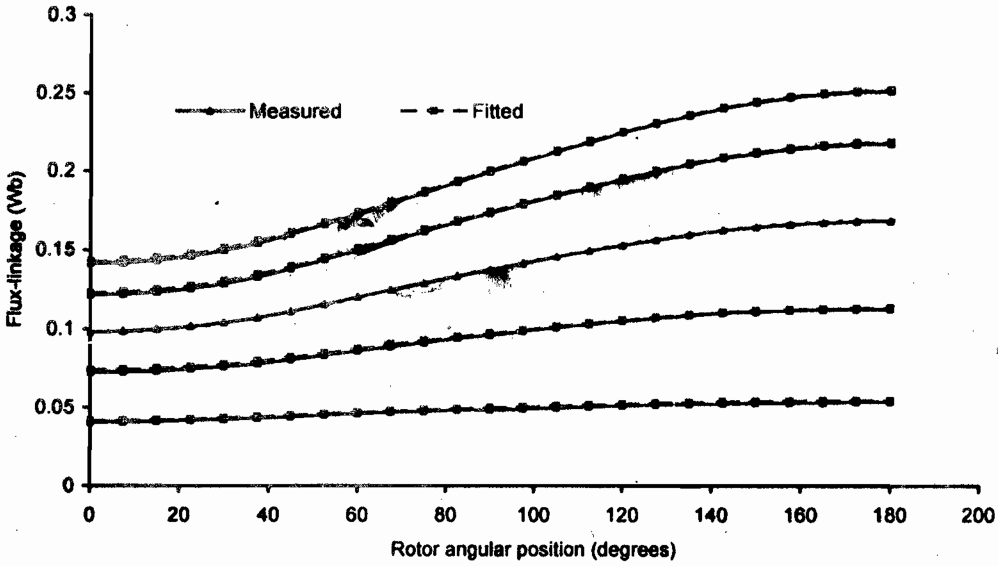

Fig. 2: Current-dependent flux-linkage/rotor angular position characteristics for I = 5.0, 10.0, 15.0, 20.0 and 25.0Amps

Equation (10) can be integrated with respect to i at constant θ as shown in Equation (4) to obtain $W^{'}(i,\theta)$. The coenergy is then differentiated with respect to θ at constant i to yield $T(i,\theta)$ as in equation (5). The static torque expression can, therefore, be written as

$$T(i,\theta) = -N_r\left[i\sum_{n=1}^{3} n A_{0n} Sin(nN_r\theta) + \sum_{k=1}^{5}\sum_{j=2}^{10} k j^{-1} A_{jk} i^{j} Sin(kN_r\theta)\right] \qquad (11)$$

Equation (11) predicts the $T(i,\theta)$ characteristics for the PM hybrid stepping motor with the discrepancy per experimental point of 0.95Nm or (3.35% of average holding torque at the motor rated current of 15.0A). The measured and predicted $T(i,\theta)$ characteristics for selected currents are displayed in Figure 3.

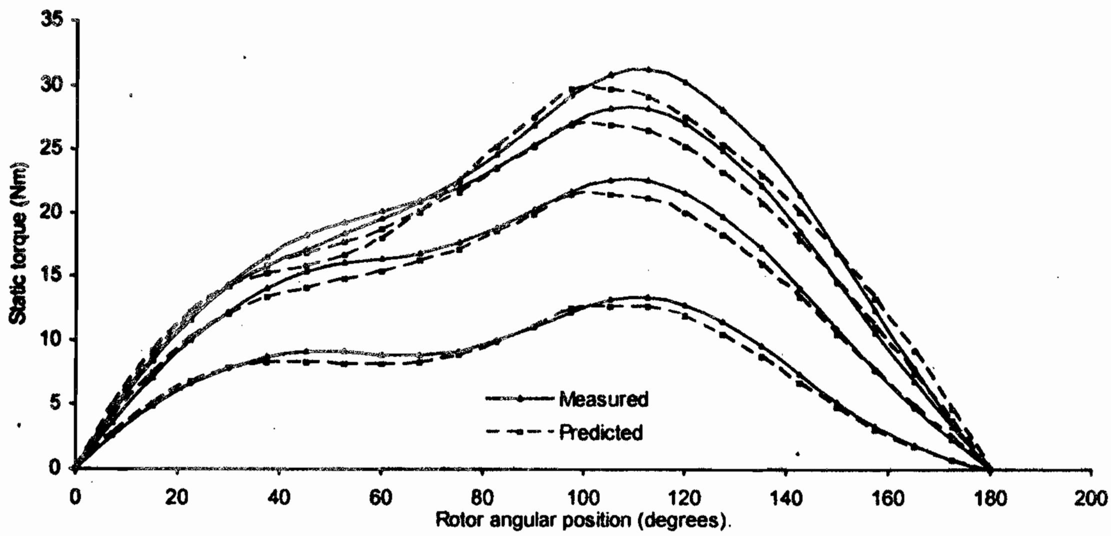

Fig. 3: Static torque/rotor angular position characteristics using the separate fitting method for I = 5.0, 10.0, 15.0, 23.0Amps.

The Composite Fitting Method

This new method is prompted by the large discrepancy that existed between the measured and approximated flux-linkage curves at lower and higher values of current in the previous separate fitting method. Although the discrepancy in the approximation does not introduce very significant average error in $T(i,\theta)$ characteristics, the point-by-point error at individual levels of stator excitation is considerable.

This method uses Equation (10) to represent the total flux-linkage data and Equation (11) to predict $T(i,\theta)$ characteristics. It does not separate the two flux components but fits them compositely. This method improves the approximation and considerably reduces the error in torque as shown in Figures 4 and 5 respectively. The average discrepancy per experimental point in the approximation of the flux-linkage data is 0.4mWb-t (0.29% of the average flux-linkage measured at 15.0A) and in the prediction of $T(i,\theta)$ is 0.64Nm (2.27% of the measured holding torque at 15.0A).

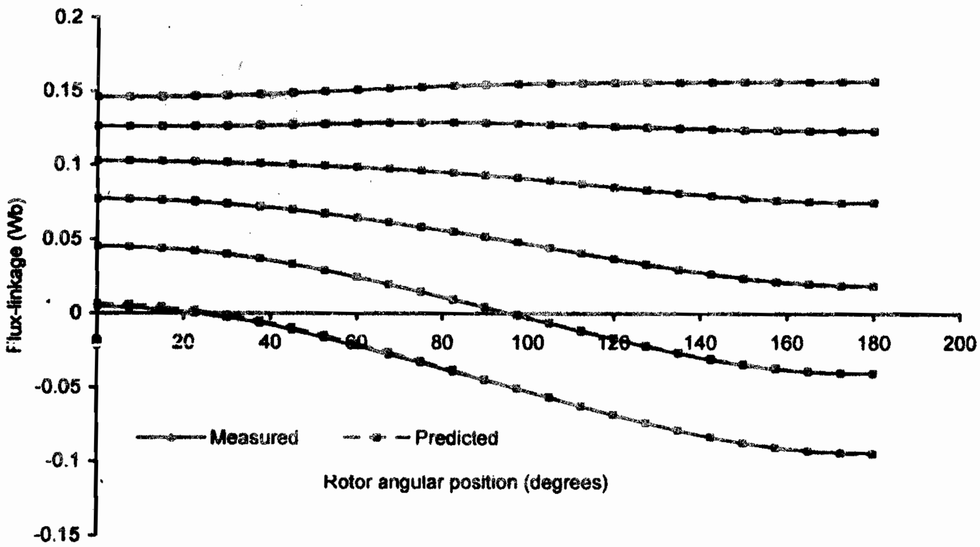

Figure 4. Measured and predicted flux-linkage/rotor angular position characteristics using the composite fitting method for I = 0.0, 5.5, 10.0, 15.0, 20.0 and 25.0Amps.

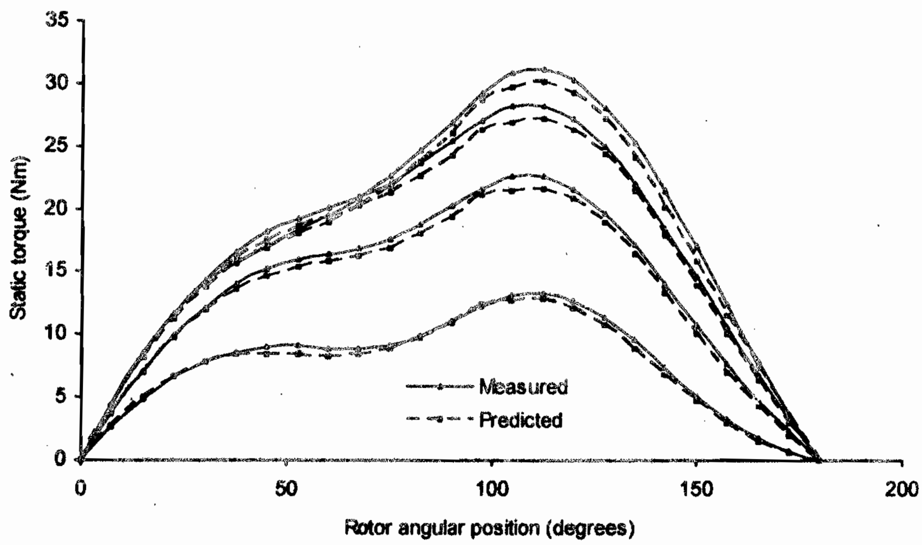

Fig. 5: Static torque/rotor angular position characteristics using the composite fitting method for I = 5.0, 10.0, 15.0 and 23.0Amps.

DISCUSSIONS

Two methods of representing the flux-linkage data of a hybrid stepping motor have been presented. The first, called the separate fitting method, is based on an already developed method (Pickup and Russell, 1979), in which the total flux-linkage data of a PM hybrid stepping motor is separated into the PM and current-dependent components and analytical function derived for each of them. The two flux-linkage components are fitted to these functions individually to generate coefficients that are used in conjunction with the functions to form the total flux-linkage equation, which is used to represent the total flux-linkage data. Equation (10) is integrated with respect to current (i) to produce coenergy $W'(i,\theta)$, which when differentiated with respect to rotor angular position (θ), yields static torque.

The second method, called the composite fitting method, uses the same principles as the first, but does not separate the total