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A SINGLE STAGE PRODUCTION SYSTEM WITH UNCERTAIN DEMAND INCOPORATING SHORTAGES

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ABSTRACT

We examine the problem of determining production quantities for a single-product, single stage production system with uncertain demand incorporating shortages. We propose a simple and easy procedure for computing the optimum quantity, the number of replenishments and the total relevant inventory cost for the system We show that the inventory total cost function and the quantity for the system is convex. Also, numerical problems are used to compare the proposed policy to the typical method, which solves the production and inventory problem separately.

KEYWORDS: Production/Inventory model, Optimum quantity, Cost function

1.0 INTRODUCTION

Inventory can be used to protect manufacturer against the randomness in production, respond to variable customer demand, and keep higher availability of goods to maintain high quality customer service (Lee 2005) The amount of inventory needed should depend on the safety stock as to protect against the demand uncertainty, and to achieve a high service level for satisfying customers' demand (Lee 2005). Henig and Gerchak (1990) examines single-stage, periodic review systems with random demand and yield. Yano and Lee (1995) present a detailed review of the inventory system in which yield is a random variable. Wand and Gerchak (1996); Kadir and Rahim (2005) consider systems with uncertain capacity/supply as well. Gurnani *et el.* (2000); Baker and Ehrhardt (1995); Bollapragada and Morton (1999), Donselaar et al. (1996) focus on developing heuristic solution procedure to find order up-to- level in the systems allowing lost sales However, these heuristics methods do not always provide accurate results. Downs *et al* (2001) consider an inventory problem with multiple items; lags in delivery and lost sales Osagiede (2002) considered inventory control model for items with time dependent increasing demand. Osagiede *et al* (2002) presented optimal production policies for items with increasing demand. Osagiede and Omosigho (2005) presented a computer aided solution for inventory problem with linear increasing demand

In this paper, our emphasis is to determine the production quantities for a single-product production system with uncertain demand incorporating shortages. After showing the convexity of the inventory and shortage costs of the optimal policy, which we believe to be necessary for the validation of our results, numerical problems are used to illustrate the proposed policy.

The rest of the paper is organised as follows: In the next section notation and assumptions are presented. Next we present the mathematical model and show the convexity of the optimal policy. We then present some results of the numerical problems. The final section presents conclusions and discusses direction of future research.

2.0 NOTATIONS AND ASSUMPTIONS

T = Cycle time

x = Total demand during the cycle time T (where x is random).

$f(x)$ = Probability that the total demand is x during the cycle time T

k_1 = Storage cost per unit per unit time

k_2 = Shortage cost per unit per unit time.

q = Total inventory level

$W(q)$ = Total inventory cost

λ = Shortage or scarcity rate defined as

$$\lambda = \frac{k_2}{k_1 + k_2}$$

q_1^* = Optimal total inventory level.

T_1 = Time when there is no shortage

T_2 = Time when there is shortage, where $T_1 \cup T_2 = T$

n = Number of replenishment

Assumptions

The following assumptions are used in the model

- Demand is discontinuous, but in practice, we can assume that the rate is constant
- The demand during a certain time interval T is uncertain
- The materials of inventory do not lose value during the interval T
- Storage cost, ordering cost, shortage cost remain constant over time.
- Shortages in the inventory are allowed

3.0 MODEL FORMULATION

In this inventory situation, two cases shall be considered

Case I If the total demand x is less than the inventory q, $(q \geq x)$ This case is represented in the diagram below

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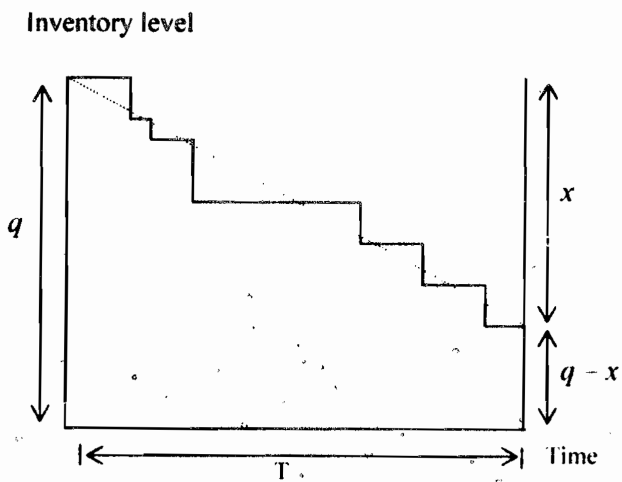

Fig. I

Case II: If the total demand is greater than inventory q, $(q < x)$ we have the following diagram

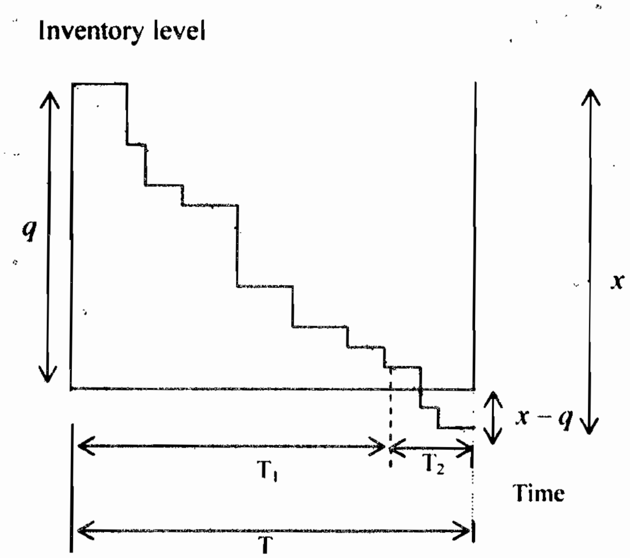

Fig. II

Assume that we can represent the variation of inventory level by a straight line as shown in the diagram for cases I and II, and introducing the concept of average inventory as it is conventionally done, then, we have

In case 1: Average inventory corresponding to situation in case 1 figure I is given by the relations

$$q_a = \frac{1}{2}\left[q + (q - x)\right] = q - \frac{x}{2} \qquad (1)$$

From figure II, it is easy to derive the following relations:

$$\frac{T_1}{T} = \frac{q}{x}, \frac{T_2}{T} = \frac{x - q}{2} \qquad (2)$$

Thus,

In case II: Average inventory corresponding to situation in case II figure II and using equation (2) we have

$$q_b = \frac{1}{2} q \frac{T_1}{T} = \frac{1}{2} \frac{q^2}{x} \qquad (3)$$

From case II, the Average shortage using equation (2) we have (see Kaufman 1963):

$$\mu_b = \frac{1}{2}(x-q)\frac{T_2}{T} = \frac{1}{2}\frac{(x-q)^2}{x} \qquad (4)$$

The mathematical expectation for the total inventory cost will be (see Kaufman 1963):

$$W(q) = k_1 \sum_{x=0}^{q} \left(q - \frac{x}{2}\right) f(x) + k_1 \sum_{x=q+1}^{\infty} \frac{1}{2} \frac{q^2}{x} f(x) + k_2 \sum_{x=q+1}^{\infty} \frac{1}{2} \frac{(x-q)^2}{x} f(x) \tag{5}$$

The problem here is to find the optimal value of q, which is the value $q = q^*$ for which $W(q)$ is minimum. The method that is usually used to find q^* is by calculating the $W(q)$ using its general expression given by (5), for various values of q. The value of q which yield the smallest $W(q)$ is usually taken as the optimal value of q. This method however, involves complex computations.

To avoid these computational complexities, we give the proposition which further describe the structure of the optimal policy. This proposition is ours.

Proposition:

The minimum of $W(q)$ as defined by equation (5) occurs for a value of q^* such that

$$G(q^* - 1) < \lambda < G(q^*), \text{ where}$$

$$\lambda = \frac{k_2}{k_1 + k_2} \quad \textit{and}$$

$$G(q^*) = f(x \le q^*) + (q^* + 1) \sum_{x=q^*+1}^{\infty} \frac{f(x)}{x}. \tag{6}$$

where q^* is the optimal inventory level

Proof:

Given that

$$W(q) = k_1 \sum_{x=0}^{q} \left(q - \frac{x}{2}\right) f(x) + k_1 \sum_{x=q+1}^{\infty} \frac{1}{2} \frac{q^2}{x} f(x) + k_2 \sum_{x=q+1}^{\infty} \frac{1}{2} \frac{(x-q)^2}{x} f(x).$$

then

$$W(q+1) = k_1 \sum_{x=0}^{q+1} \left(q + 1 - \frac{x}{2}\right) f(x) + k_1 \sum_{x=q+2}^{\infty} \frac{1}{2} \frac{(q+1)^2}{x} f(x) + k_2 \sum_{x=q+2}^{\infty} \frac{1}{2} \frac{(x-q-1)^2}{x} f(x) \tag{7}$$

But we can write

$$k_1 \sum_{x=0}^{q+1} \left(q + 1 - \frac{x}{2}\right) f(x) = k_1 \sum_{x=0}^{q} \left(q - \frac{x}{2}\right) f(x) + k_1 \sum_{x=0}^{q} f(x) + k_1 \frac{q+1}{2} f(q+1) \tag{8}$$

$$k_1 \sum_{x=q+2}^{\infty} \frac{1}{2} \frac{(q+1)^2}{x} f(x) = k_1 \sum_{x=q+1}^{\infty} \frac{q^2 f(x)}{2x} + k_1 \sum_{x=q+1}^{\infty} \frac{q f(x)}{x} + \frac{k_1}{2} \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - \frac{k_1}{2} (q+1) f(q+1) \tag{9}$$

and

$$k_2 \sum_{x=q+2}^{\infty} \frac{1}{2} \frac{(x-q-1)^2}{x} f(x) = k_2 \sum_{x=q+1}^{\infty} \left[(x-q)^2 - 2(x-q) + 1\right] \frac{f(x)}{2x}$$

$$- k_2 \left[(q+1-q)^2 - 2(q+1-q) + 1\right] \frac{f(q+1)}{2(q+1)}$$

and so,

$$k_2 \sum_{x=q+2}^{\infty} \frac{1}{2} \frac{(x-q-1)^2}{x} f(x) = k_2 \sum_{x=q+1}^{\infty} \frac{(x-q)^2}{2x} f(x)$$
$$- k_2 \sum_{x=q+1}^{\infty} f(x) + q k_2 \sum_{x=q+1}^{\infty} \frac{f(x)}{x} + \frac{1}{2} k_2 \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \tag{10}$$

Substituting (8), (9) and (10) into equation (5), we obtain

$$W(q) = k_1 \sum_{x=0}^{q+1}\left(q+1-\frac{x}{2}\right)f(x) - k_1 \sum_{x=0}^{q} f(x) - k_1 \frac{(q+1)}{2} f(q+1)$$

$$+ k_1 \sum_{x=q+2}^{\infty} \frac{(q+1)^2}{2x} f(x) - k_1 \sum_{x=q+1}^{\infty} \frac{qf(x)}{x} - \frac{k_1}{2} \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

$$+ \frac{k_1}{2}(q+1)\, f(q+1) + k_2 \sum_{x=q+2}^{\infty} \frac{(x-q-1)^2}{2x} f(x) + k_2 \sum_{x=q+1}^{\infty} f(x)$$

$$- q k_2 \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - \frac{k_2}{2} \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

$$W(q) = \left[k_1 \sum_{x=0}^{\infty}\left(q+1-\frac{x}{2}\right)f(x) + k_1 \sum_{x=q+2}^{\infty} \frac{(q+1)^2}{2x} f(x) + k_2 \sum_{x=q+2}^{\infty} \frac{(x-q-1)^2}{2x} f(x)\right]$$

$$- k_1 \sum_{x=0}^{q} f(x) + k_2 \sum_{x=q+1}^{\infty} f(x) - (qk_1 + qk_2) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - \left(\frac{k_1}{2} + \frac{k_2}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \qquad (11)$$

But recall the properties of a probability mass function (pmf)

(i) $f(x) \geq 0 \quad \textit{for all } x$

(ii) $\sum_{x=-\infty}^{\infty} f(x) = 1$, where $f(x)$ is a (pmf)

Thus $\sum_{x=0}^{\infty} f(x) = 1,$

Hence

$$k_2 \sum_{x=q+1}^{\infty} f(x) = k_2 \sum_{x=0}^{\infty} f(x) - k_2 \sum_{x=0}^{q} f(x)$$

$$= k_2 - k_2 \sum_{x=0}^{q} f(x) \qquad (12)$$

Then, comparing equations (7) and (11) and factorizing the terms in (11) that has $\sum_{x=q+1}^{\infty} f(x)$ in common, we have,

$$W(q) = W(q+1) + k_2 - k_2 \sum_{x=0}^{\infty} f(x) - k_1 \sum_{x=0}^{q} f(x)$$

$$- k_1\left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - k_2\left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

This can be re-written as

$$W(q) = W(q+1) + k_2 - (k_1 + k_2) \sum_{x=0}^{q} f(x) - (k_1 + k_2)\left(q+\frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

Hence, making $W(q+1)$ the subject of the formula, we have

$$W(q+1) = W(q) + (k_1 + k_2) f(x \le q) + (k_1 + k_2)\left(q + \frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - k_2$$

$$= W(q) + (k_1 + k_2)\left[f(x \le q) + \left(q + \frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \right] - k_2 \qquad (13)$$

where $\sum_{x=0}^{q} f(x) = f(x \le q)$

If we set,

$$G(q) = f(x \le q) + \left(q + \frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \qquad (14)$$

Then (13) becomes.

$$W(q+1) = W(q) + (k_1 + k_2) G(q) - k_2 \qquad (15)$$

In a similar manner;

$$W(q-1) = W(q) - (k_1 + k_2)\, G(q-1) + k_2 \qquad (16)$$

Again the following Lemma further strengthens our proposition. The lemma and the proof is ours.

***Lemma*:**

If there exist an inventory level q, where the function $G(q)$ is given by

$$G(q) = f(x \le q) + \left(q + \frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \; for,\ x \ne 0 \text{, then } G(q+1) \ge G(q) \text{ holds}$$

Proof:

$$G(q+1) = f(x \le q+1) + \left(q + 1 + \frac{1}{2}\right) \sum_{x=q+2}^{\infty} \frac{f(x)}{x}$$

$$= f(x \le q) + f(q+1) + \left(q + \frac{1}{2} + 1\right) \sum_{x=q+2}^{\infty} \frac{f(x)}{x}$$

$$= f(x \le q) + f(q+1) + \left(q + \frac{1}{2} + 1\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} - \frac{\left(q - \frac{1}{2}\right)(f(q+1))}{q+1} + \sum_{x=q+1}^{\infty} \frac{f(x)}{x} \quad \frac{f(q+1)}{q+1}$$

$$= G(q) - \frac{1}{2} \frac{f(q+1)}{q+1} + \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$

That is

$$G(q+1) = G(q) + \sum_{x=q+1}^{\infty} \frac{f(x)}{x} + \frac{1}{2}\frac{f(q+1)}{q+1}$$

But since

$$\sum_{x=q+1}^{\infty} \frac{f(x)}{x} + \frac{1}{2}\frac{f(q+1)}{q+1} \geq 0$$

We have

$$G(q+1) - G(q) \geq 0$$

Hence the result.

$$G(q+1) \geq G(q) \qquad (17)$$

Recall that the curve of the inventory total cost function is convex, and so the value q^* which minimizes this function must satisfy these inequalities.

$$W(q_1) > W(q^*) \quad if \; q_1 < q^* \qquad (18)$$

and

$$W(q_2) > W(q^*) \quad if \; q_2 > q^* \qquad (19)$$

Having shown in the lemma that $G(q+1) \geq G(q)$, we shall consider the q^* such that;

$$(k_1 + k_2)\; G(q^*) - k_2 > 0 \qquad (20)$$

and

$$-(k_1 + k_2)\, G(q^* - 1) + k_2 > 0 \qquad \text{for all}$$

$$q_2 > q^* \; and \; q_1 < q^* \qquad (21)$$

Hence equation (18) and (19) are satisfied.
Therefore, the value of q that yields a minimum for W is the value q^*, which satisfies the inequalities in (20) and (21); which together forms the inequality

$$G(q^* - 1) < \lambda < G(q^*) \qquad (22)$$

where

$$\lambda = \frac{k_2}{k_1 + k_2} \qquad (23)$$

therefore;

$$G(q^*) = f(x \leq q^*) + \left(q^* + \frac{1}{2}\right) \sum_{x=q^*+1}^{\infty} \frac{f(x)}{x} \qquad (24)$$

This completes the proof of our proposed model.

4.0 NUMERICAL ILLUSTRATIONS

In this section, we present numerical results to illustrate the usefulness of the model developed in this paper It will be seen that this model avoids complex computational complexity when compared with the direct numerical calculation of the values of $W(q)$ using the expression given in (5) , for various values of q (see Kaufman 1963).

Example I: Let the storage cost per unit per unit time $k_1 =$ ₦5, 000:00 and the shortage cost per unit time $k_2 =$ ₦20, 000:00. The parameters of q, x and the probability mass function pmf, $f(x)$ are as shown in Table I below.

Table I: Parameters for the Problem.

q	1	2	3	4
x	3	1	2	4
$f(x)$	0.2	0.3	0.25	0.25

With $f(x) = 0 \quad \forall \; x \neq \{1, 2, 3, 4\}$

The problem is, what is the inventory level q^*, that minimizes the total inventory cost $W(q)$

NOTE $f(x)$ has been taken to satisfy the properties of a probability mass function (PMF)

(1) $\quad f(x) \geq 0 \; \forall \; x$

(2) $\quad \sum_{-\infty}^{\infty} f(x) = 1$.

The parameters in Table I was constructed for the purpose of Example I. It has not been used before.

We shall approach the problem using the two methods outlined earlier.

Method I: Solution by Numerical Calculation.
We shall calculate the general expression of the cost function $W(q) \; for \; q = 0, 1, 2, 3, 4.$

$$W(q) = k_1 \sum_{x=0}^{q} \left(q - \frac{x}{2}\right) f(x) + k_1 \sum_{x=q+1}^{\infty} \frac{1}{2}\frac{q^2}{x} f(x) + k_2 \sum_{x=q+1}^{\infty} \frac{1}{2}\frac{(x-q)^2}{x} f(x)$$

$$W(1) = k_1 \sum_{x=0}^{q} \left(1 - \frac{x}{2}\right) f(x) + k_1 \sum_{x=2}^{\infty} \frac{1}{2}\frac{1}{x} f(x) + k_2 \sum_{x=2}^{\infty} \frac{1}{2} \frac{(x-1)^2}{x} f(x)$$

$$= 5[0 + 0.5(0.3)] + 5[0.5(0.25) + 0.167(0.2) + 0.125(0.25)] + 20[0.25(0.25) + 0.67(0.2) + 1.125(0.25)]$$

$= ₦\ 10.94$ Thousand.

$$W(2) = k_1 \sum_{x=0}^{2} \left(2 - \frac{x}{2}\right) f(x) + k_1 \sum_{x=3}^{\infty} \frac{2}{x} f(x) + k_2 \sum_{x=3}^{\infty} \frac{1}{2} \frac{(x-2)^2}{x} f(x)$$

$$= 5\{2(0) + 1.5(0.3) + 2(0.25)\}$$

$$+ 5\{0 \cdot 67(0 \cdot 2) + 0.5(0 \cdot 0.25)\} + 20[0.167(0 \cdot 2) + 0 \cdot 5(0 \cdot 25)]$$

= ₦9.213 Thousand

Continuing in this manner, we obtain

$W(3)$ = ₦10.41 Thousand. Out of curiosity we calculate $W(4)$ = ₦14 Thousand

We find, of course that the optimum value of inventory level is $q^* = 2$, since it gives the minimum inventory total cost. It can be noticed that for large values of x, the above calculations will be complex and cumbersome.

Table II: Parameters for the Problem and the Corresponding Values of $W(q)$ *Indicates Minimum Cost.

q	1	2	3	4
x	3	1	2	4
$f(x)$	0.2	0.3	0.25	0.25
$W(q)$	10.94	9.213*	10.41	14

Notice that the optimum occur at $q^* = 2$

Method II: Solution by the New Model:

In our proposed model which states that the minimum $W(q)$ occurs for a value q^* such that

$$G(q^* - 1) < \lambda < G(q^*)$$

where $$\lambda = \frac{k_2}{k_1 + k_2}$$

and $$G(q^*) = f(x \le q^*) + \left(q^* + \frac{1}{2}\right) \sum_{x=q^*+1}^{\infty} \frac{f(x)}{x}$$

we determine the optimum q^* using the above example to see if it corresponds to the values obtained in method I above.

Here we shall calculate $G(q)$ using its expression in (14) given by

$$G(q) = f(x \le q) + \left(q + \frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x}$$ for different values of q

Table III: Various Values for the Proposed Model.*Indicates Minimum Cost.

q	x	$f(x)$	$\frac{f(x)}{x}$	$\sum_{x=q+1}^{\infty} \frac{f(x)}{x}$	$f(x \le q) = \sum_{x=0}^{q} f(x)$	$\left(q+\frac{1}{2}\right)\sum_{x=q+1}^{\infty}\frac{f(x)}{x}$	$G(q) = f(x \le q) + \left(q+\frac{1}{2}\right)\sum_{x=q+1}^{\infty}\frac{f(x)}{x}$
1	3	0.20	0.067	0.2545	0.30	0.3818	0.6818
2	1	0.30	0.30	0.1295	0.55	0.3238	0.8738
3	2	0.25	0.125	0.0625	0.75	0.2188	0.9688
4	4	0.25	0.0625	0.0000	1.00	0.0000	1.0000

Note that in column 6 of Table III, $f(x \le q) = \sum_{x=1}^{q} f(x)$

We shall briefly give the procedure for obtaining our various values in the columns

The entries in Table III were obtained as follows when $q = 1$, $x = 3$, $f(x) = 0.2$, $\frac{f(x)}{x} = 0.067$

$$\sum_{\infty=q+1}^{\infty} \frac{f(x)}{x} = \sum_{x=2}^{\infty} \frac{f(x)}{x} = \frac{f(2)}{2} + \frac{f(3)}{3} + \frac{f(4)}{4}$$

$$= \frac{0.25}{2} + \frac{0.20}{3} + \frac{0.25}{2} = 0.2524$$

$$\left(q + \frac{1}{2}\right) \sum_{x=q+1}^{\infty} \frac{f(x)}{x} = \left(1 + \frac{1}{2}\right) \sum_{x=2}^{\infty} \frac{f(x)}{2}$$

$$= 1.5\left\{\frac{0.25}{2} + \frac{0.20}{3} + \frac{0.25}{4}\right\} = 1.5[0.2545] + 0.3818$$

$$f(x \le q) = f(x \le q) = \sum_{x=1}^{1} f(x) = f(1) = 0.30$$

$$G(q) = G(1) = f(x \le 1) + \left(1 + \frac{1}{2}\right) \sum_{x=2}^{\infty} \frac{f(x)}{x}$$

$$= 0 \cdot 30 + 1.5(0.2545) = 0 \cdot 6818$$

Continuing in this manner, we obtain other values in the Table III.

Notice that in Table III, for $q = 2$, we have

$$[G(1) = 0 \cdot 6818] < \left(\lambda = \frac{4}{5} = 0 \cdot 8\right) < [G(2) = 0 \cdot 8732]$$

This means

$$[G(2-1) < \lambda < \left|G(2) = G(q^{*}-1) < \lambda < G(g^{*})\right|$$

Where $$\lambda = \frac{k_2}{k_1 + k_2} = \frac{20{,}000}{25{,}000} = \frac{4}{5}$$

Therefore the optimum inventory level g^{*} is equal to 2 units, which corresponds to what we obtained by using method I. Clearly, it is easier to calculate entries on Table III in order to find g^{*} than using method I, which involves tedious calculations usually prone to mistakes/errors.

Example 2: Let the storage cost per unit per unit time and the shortage cost per unit time as k_1 = ₦100, 000 00 k_2 = ₦20 000:00 respectively with the parameters q, x and $f(x)$ shown on Table iv below

Table IV. Parameters for the Problem.

q	0	1	2	3	4	5	> 5
x	0	1	2	3	4	5	> 5
$f(x)$	0.1	0.2	0.2	0.3	0.1	0	0

The Problem is: Find the optimal inventory level.

Method I: Solution by Numerical Calculation.

Using the general expression $W(q)$ by (5). Following the pattern of example I, we calculate $W(q)$ for various values of q

$$W(0) = 0.1(0) + 0.1(0) + 2\sum_{x=1}^{\infty} \frac{1}{2} xf(x)$$

$$= 2[(0\cdot5)(0\cdot2) + 1(0\cdot2) + (1\cdot5)(0\cdot3) + 2(0\cdot1) + 2\cdot5(0\cdot1)] = \text{N2.4 million}$$

$$W(1) = k_1\sum_{x=0}^{1}\left(1-\frac{x}{2}\right)f(x) + k_1\sum_{x=2}^{\infty}\frac{1}{2}\frac{f(x)}{x} + k_2\sum_{x=2}^{\infty}\frac{1}{2}\frac{(x-1)^2}{x}f(x) = \text{₦}1.07\text{million}$$

$$W(2) = k_1\sum_{x=0}^{2}\left(2-\frac{x}{2}\right)f(x) + k_1\sum_{x=3}^{\infty}\frac{1}{2}\cdot\frac{4}{x}f(x) + k_2\sum_{x=3}^{\infty}\frac{1}{2}\frac{(x-2)^2}{x}f(x) = \text{₦}0.48\text{million}$$

Continuing in this manner, we obtain, $W(3)$= ₦(0.29) million; $W(4)$ = ₦0.30 million $W(5)$ = ₦0.38 million

Table V. Parameters for the Problem and the Corresponding Values of $W(q)$

*Indicates Minimum Cost.

q	0	1	2	3*	4	5	> 5
x	0	1	2	3	4	5	> 5
$f(x)$	0.1	0.2	0.2	0.3	0.1	0.1	0
$W(q)$	₦2.4	₦1.07	₦ 0.48	₦ 0.29*	₦0 30	₦0 38	-

Notice that g^{*} (optimal inventory level) = 3

Method II: Solution by the Propose Model:

$$G(q^* - 1) < \lambda < G(q^*) \text{ where } \lambda = \frac{k_2}{k_1 + k_2}, \quad G(q^*) = f(x \le q^*) + (q^* + \frac{1}{2}) \sum_{x=q^*+1}^{\infty} \frac{f(x)}{x}$$

where g^* is the optimal inventory level. The values are obtained are represented in Table **vi** below.

Table VI: Various Values for the Proposed Model.*Indicates Minimum Cost.

q	x	$f(x)$	$\frac{f(x)}{x}$	$\sum_{x=q+1}^{\infty} \frac{f(x)}{x}$	$\left(q+\frac{1}{2}\right)\sum_{x=q+1}^{\infty} \frac{f(x)}{x}$	$f(x \le q) = \sum_{x=0}^{q} f(x)$	$G(q) = f(x \le q) + \left(q+\frac{1}{2}\right)\sum_{x=q+1}^{\infty} \frac{f(x)}{x}$
0	0	0.1	∞	0.445	0.2225	0.1	0.3225
1	1	0.2	0.2	0.245	0.3675	0.3	0.6675
2	2	0.2	0.1	0.145	0.3625	0.5	0.8625
3 *	3	0.3	0.1	0.045	0.1575	0.8	0.9575
4	4	0.1	0.025	0.020	0.0900	0.9	0.9900
5	5	0.1	0.020	0.000	0.0000	1.0	1.0000
>5	>5	0	0.000	0.000	0.0000	1.0	1.0000

Notice that in Table VI, for $q = 3$

$$[G(2) = 0.8625] < \left[\lambda = \frac{20}{21} = 0.9524\right] < [G(3) = 0.9575]$$

$$\text{That is } G(q^* - 1) < \lambda < G(q^*) \text{ Where } \lambda = \frac{k_2}{k_1 + k_2} = \frac{2{,}000{,}000}{2{,}100{,}000} = \frac{20}{21}$$

Therefore, from our proposed model the optimal value of the inventory level is 3 units (g^*), which corresponds to what was obtained in method I of example II.

5.0 CONCLUSION

An efficient solution method for inventory problem with uncertain demand, incorporating shortages has been proposed. The model has been derived using the convexity property of the inventory total cost function. The numerical examples and results show that the model is not only consistent with the conventional numerical calculations of the expression of the inventory total cost function, $W(q)$, for various values of q. From the analysis so far, it clearly shows that our proposed model takes less computational efforts. Work is currently going on the computer program for solving more problems and secondly to extend the model to problems for equipment that deteriorates taking demand as continuous random variable.

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