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MATHEMATICAL MODELING OF CARBON MONOXIDE POLLUTANT IN BAUCHI METROPOLIS

S. ABDULSALAM., A. AYUB. and F. M. SIRAJUDEEN

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ABSTRACT

Mathematical model for the prediction of carbon monoxide (CO) level in Bauchi metropolis was developed from basic principles The model was simplified and evaluated using Crank Nicholson implicit method and matrix inversion method (EXCEL) respectively An experimental work was also carried out to determine the pollutant levels at four locations or areas with high traffic congestion in Bauchi metropolis. These areas were Yelwa, Wunti, Central Market and Gidan Mai. Results obtained from the experimental work indicate that the CO level at Yelwa, Wunti and Central Market were above the acceptable limit by the Nigerian Ambient Air Quality Standard (NAAQS). Whereas, only the CO level at Gidan Mai was within the acceptable limit. Also, the experimental CO levels obtained were compared with the predicted CO values obtained from the model and it was observed that the model represents the real situation at 95% confidence level using chi-squares statistical method.

KEYWORDS: Carbon monoxide; mathematical model; pollutant level; high traffic congestion

INTRODUCTION

Carbon monoxide (CO) is a colorless, odorless, poisonous gas formed when carbon in fuels is not burned completely (Http 1, 2006; Http.2, 2005). It is a byproduct of highway vehicle exhaust. In cities, automobile exhaust can cause as much as 95 percent of *all* CO emissions (*Http* 3, 2005) These emissions can result in high concentrations of CO, particularly in local areas with high traffic congestion Other sources of CO emissions include industrial processes and fuels combustion in sources such as boilers and incinerators (Http 1, 2006) The accumulation of CO in air at high level is very hazardous to human health (Http 2, 2005; Http 4, 2006)

CO enters the blood stream and reduces oxygen delivery to the body's organs and tissues. The health threat from exposure to CO is most serious for those who suffer from cardiovascular disease (Allred, *et al.*, 1989; Http.1, 2006; Http.2, 2005). Healthy individuals are also affected, but only at higher levels of exposure.

Exposure to elevated CO levels is associated with visual impairment, reduced work capacity, and reduced manual dexterity, poor learning ability, and difficulty in performing complex tasks (Http 3, 2005; Http.5, 2006).

Due to the health and environmental effects of CO, it is important to evaluate the environment periodically. In order to keep a pollutant free air

The aim of this research work was to develop a mathematical model from the basic principle for the prediction of CO level and to validate this model with experimental data obtained from areas with heavy traffic congestion in Bauchi metropolis

MODEL FORMULATION FOR THE PREDICTION OF CO LEVEL

The atmosphere over the modeling areas is perceived as a well-mixed box, the evolution of pollutant in the box was based on the principle of conservation of mass (Thompson and Ceckler, 1977).

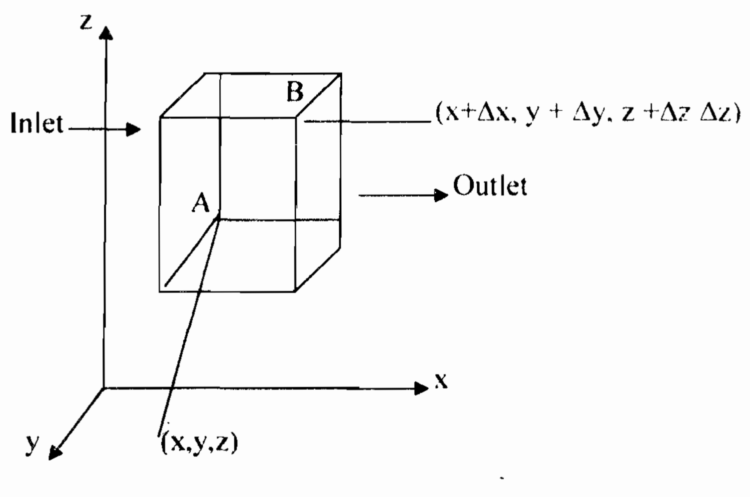


Figure 1: An Elemental Volume of CO Level in the Atmosphere

Consider an elemental volume in which CO is flowing as shown in Figure 1.0. The material balance over the volume element can be written as (Levenspiel, 1988):

S. Abdulsalam , Dept of Chemical Engineering Programme, Abubakar Tafawa Balewa University, Bauchi, P M. B 0248, Bauchi, Nigeria
A. Ayub, Department of Chemical Engineering, University of Lagos, Lagos, Nigeria.
F. M. Sirajudeen, Department of Business Administration and Management, Federal Polytechnic, Bauchi - Nigeria.

$$\begin{bmatrix} \text{Rate of flow of CO} \\ \text{out of volume element} \\ \Delta x\Delta y\Delta z \\ I \end{bmatrix} - \begin{bmatrix} \text{Rate of flow of CO} \\ \text{into the volume element} \\ \Delta x\Delta y\Delta z \\ II \end{bmatrix} + \begin{bmatrix} \text{Rate of accumulation of CO} \\ \text{in the volume element} \\ \Delta x\Delta y\Delta z \\ III \end{bmatrix}$$

$$\begin{bmatrix} \text{Rate of generation of CO} \\ \text{in the volume element} \\ \Delta x\Delta y\Delta z \\ IV \end{bmatrix} = 0 \quad (1)$$

If M_{CO} and N_{CO} are molecular weight and molar flux of CO respectively, then the mass rate of CO entering volume element at common corner A with coordinate x, y and z (Treybal, 1980) is given as:

$M_{CO}\,[(N_{COx})_x\,\Delta y\Delta z + (N_{COy})_y\,\Delta x\Delta z + (N_{COz})_z\,\Delta x\Delta y]$ (2)

Similarly, the mass rate of CO out of volume element ΔxΔyΔz at common corner B is given by; (Levenspiel, 1988)

$M_{CO}\,[(N_{COx})_{x+\Delta x}\,\Delta y\Delta z + (N_{COy})_{y+\Delta y}\,\Delta x\Delta z + (N_{COz})_{z+\Delta z}\,\Delta x\Delta y]$ (3)

The total CO in the volume element ΔxΔyΔz and its rate of accumulation is given by: (Treybal, 1988)

$$\Delta x\Delta y\Delta z\frac{\partial \rho_{CO}}{\partial t} \quad (4)$$

where ρ_{CO} of the density of CO.

The rate of generation of CO in the volume element is zero since there is no chemical reaction taken place. Therefore, the fourth term of equation (1) becomes zero. Substituting Eqns. (2), (3) and (4) in (1) and dividing by ΔxΔyΔz and taking limit as Δx→0, Δy→0, and Δz→0, we have;

$$M_{CO}\left(\frac{\partial N_{COx}}{\partial x} + \frac{\partial N_{COy}}{\partial y} + \frac{\partial N_{COz}}{\partial z}\right) + \frac{\partial \rho_{CO}}{\partial t} = 0 \quad (5)$$

Let us represent flux due to molecular diffusion of CO as J_{CO}, N_{CO} as molar flux of CO, C_{CO} concentration of CO and the wind velocity in the x-direction as U_x. Therefore, from Fogler (2001), the flux N_{CO} is given by,

$N_{CO} = U_x\,.C_{CO} + J_{CO}$ (6)

In addition, the flux due to molecular (Fogler, 2001) or turbulent diffusion (J_{CO}) is given by:

$$J_{CO} = -\frac{K\partial C_{CO}}{\partial x} \quad (7)$$

where K is the dispersion coefficient.
Also,

$$C_{CO} = \frac{\rho_{CO}}{M_{CO}} \quad (8)$$

Substituting equations (7) and (8) in (6) and differentiating gives;

$$\frac{\partial(M_{CO}N_{COx})}{\partial x} = \frac{U_x\partial \rho_{CO}}{\partial x} - kM_{CO}\frac{\partial^2 C_{CO}}{\partial x^2} + \rho_{CO}\frac{\partial Ux}{\partial x} \quad (9)$$

Substituting equation (9) and its analogous in the y and z-directions in Eqn (5), gives:

$$\underset{I}{\frac{\partial \rho_{CO}}{\partial t}} = \underset{II}{-M_{CO}K\left(\frac{\partial^2 C_{CO}}{\partial x^2} + \frac{\partial^2 C_{CO}}{\partial y^2} + \frac{\partial^2 C_{CO}}{\partial z^2}\right)} + \underset{III}{\left(U_x\frac{\partial \rho_{CO}}{\partial x} + U_y\frac{\partial \rho_{CO}}{\partial y} + U_z\frac{\partial \rho_{CO}}{\partial z}\right)}$$
$$\underset{IV}{-\rho_{CO}\left(\frac{\partial Ux}{\partial x} + \frac{\partial Uy}{\partial y} + \frac{\partial Uz}{\partial z}\right)} \quad (10)$$

The fourth term (IV) of equation (10) is the continuity equation; therefore, it is equal to zero (Treybal, 1980). Also dividing equation (10) by molecular weight, M_{CO}, we have;

$$\frac{\partial C_{CO}}{\partial t} = K\left(\frac{\partial^2 C_{CO}}{\partial x^2} + \frac{\partial^2 C_{CO}}{\partial y^2} + \frac{\partial^2 C_{CO}}{\partial z^2}\right) - \left(Ux\frac{\partial C_{CO}}{\partial x} + Uy\frac{\partial C_{CO}}{\partial y} + Uz\frac{\partial C_{CO}}{\partial z}\right) \quad (11)$$

Equation (11) is the desired model equation for the prediction of CO level in the atmosphere.

MODEL SIMPLIFICATION

Let us assume that CO flow in the atmosphere to be in the wind direction, which corresponds to the x-axis. Therefore, equation (11) becomes:

$$\frac{\partial C_{CO}}{\partial t} = K\frac{\partial^2 C_{CO}}{\partial x^2} - Ux\frac{\partial C_{CO}}{\partial x} \quad (12)$$

Using Crank Nicholson implicit method (Stroud and Booth, 2003), equation (12) becomes:

$$-C^{n+1}_{CO,j+1} + \alpha C^{n+1}_{CO,j} - C^{n+1}_{CO,j-1} = C^{n}_{CO,j+1} + \beta C^{n}_{CO,j} + C^{n}_{CO,j-1} \quad (13)$$

where:

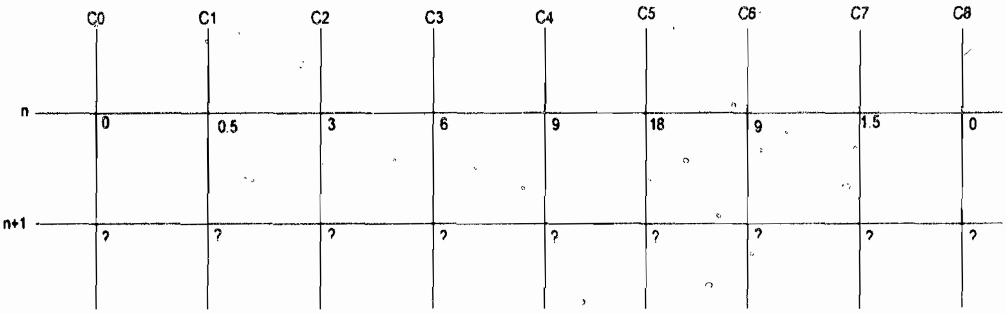
$$\alpha = (1+2R1+S1)/R1 \quad and \quad \beta = (1-2R1+S1)/R1$$

$$R1 = \frac{\Delta t K}{2\Delta x^2} \quad and \quad S1 = \frac{Ux\Delta t}{\Delta x}$$

Substituting values of j from 1 to 7 in equation (13) we have;
Ax = b
where:

$$A = \begin{bmatrix} \alpha & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & \alpha & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & \alpha & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \alpha & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & \alpha & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & \alpha & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & \alpha \end{bmatrix}, \quad x = \begin{bmatrix} C_{CO1} \\ C_{CO2} \\ C_{CO3} \\ C_{CO4} \\ C_{CO5} \\ C_{CO6} \\ C_{CO7} \end{bmatrix} \quad and \quad b = \begin{bmatrix} C_{co_{2,j}} + \beta C_{co_{1,j}} + C_{co_{0,j}} + C_{co_{0,j+1}} \\ C_{co_{3,j}} + \beta C_{co_{2,j}} + C_{co_{1,j}} \\ C_{co_{4,j}} + \beta C_{co_{3,j}} + C_{co_{2,j}} \\ C_{co_{5,j}} + \beta C_{co_{4,j}} + C_{co_{3,j}} \\ C_{co_{6,j}} + \beta C_{co_{5,j}} + C_{co_{4,j}} \\ C_{co_{7,j}} + \beta C_{co_{6,j}} + C_{co_{5,j}} \\ C_{co_{8,j}} + \beta C_{co_{7,j}} + C_{co_{6,j}} + C_{co_{8,j+1}} \end{bmatrix}$$

Equation (13) is an expression in which the unknown pivotal value (n+1) is expressed in terms of the known pivotal value nth. The (n+1) values were obtained by assuming some values for CO concentrations (nth values) at different grid points after several iterations (Stroud and Booth, 2003). For instance, the grid representation for Yelwa is given as follows:



To solve equation (13), boundary conditions are needed. Therefore, the following boundary conditions were used:

$$0 < C_{CO} < 25\,ppm$$

$$5 < t < 20hr$$

Since there were no previous data for CO concentration in Bauchi metropolis, the above boundary condition was based on the minimum and maximum CO concentration obtained in the experimental work as will be presented subsequently. Also, the time boundary condition was based on time boundaries for the experimental work.

Other parameters required in solving the matrix are; the average wind speed which was obtained from FMEnv/ZERI(2005) to be 0.9m/s for Bauchi metropolis between May and June, the dispersion coefficients, K was obtained to be $0.66m^2/s$ by a method well detailed in (Http.6, 2005) and Δt and Δx were taken to be 2 and 1(because the measurements were taken 1m above the grounnd) respectively.

The values of α and β were calculated from equation (13) to be 6.24 and 2.24 respectively and used to solve the resulting matrix using the Microsoft Excel spreadsheet having matrix function because it is an efficient method in terms of time and accuracy (Stroud and Booth, 2003).

EXPERIMENTAL PROCEDURE

Experimental work was carried out to measure the concentrations of CO along four areas with high traffic congestion in Bauchi metropolis. These areas were: Yelwa, Wunti, Central Market and Gidan Mai.

Air sample at Yelwa was measured using the automatic air pollumeter dectector. These measurements were taken one meter from the ground level. Each measurement was repeated three times and the average was recorded as presented in Table 1. These measurements were taken at two (2) -hourly intervals from 7.00(7.30) am to 7.00(7.30) pm for seven days.

The above procedure was repeated for Wunti, Central Market and Gidan Mai, their results are as presented in Tables 2 to 4

RESULTS AND DISCUSSION

Table 1: Observed CO Concentration in the Atmosphere over a period of seven Days for Yelwa

Date	Concentrations of Carbon monoxide (ppm)						
	7.00am	9.00am	11.00am	1.00pm	3.00pm	5.00pm	7.00pm
23-05-05	0.6	3.3	9.3	10.0	11.7	7.3	2.3
24-05-05	0.6	4.7	9.3	10.7	13.5	7.2	3.6
25-05-05	1.0	3.2	6.7	11.0	12.7	8.2	0.5
26-05-05	0.6	3.7	7.3	8.0	12.3	7.0	2.3
27-05-05	0.6	5.3	8.7	11.7	13.4	7.3	2.0
28-05-05	0.6	4.0	4.0	11.0	12.3	6.0	2.0
30-05-05	1.0	5.7	7.3	10.0	10.0	4.7	1.7
Mean	0.714	4.27	7.51	10.34	12.27	6.83	2.06

Table 2: Observed CO Concentration in the Atmosphere over a period of seven Days for Wunti

Date	Concentrations of Carbon monoxide (ppm)						
	7.30am	9.30am	11.30am	1.30pm	3.30pm	5.30pm	7.30pm
23-05-05	1.0	4.2	8.2	11.7	15.6	9.3	2.7
24-05-05	1.0	5.0	9.2	11.3	14.0	8.3	2.7
25-05-05	0.6	5.4	8.2	12.5	16.0	10.0	4.0
26-05-05	0.6	5.3	9.8	12.3	16.2	10.3	4.0
27-05-05	1.0	4.7	10.3	13.4	15.3	10.7	3.3
28-05-05	1.3	4.0	10.7	12.4	16.1	9.8	2.4
30-05-05	1.0	9.3	11.2	14.3	17.3	13.7	2.0
Mean	0.93	5.40	9.66	12.56	15.80	10.30	3.01

Table 3: Observed CO Concentration in the Atmosphere over a period of seven Days for Central Market

Date	Concentrations of Carbon monoxide (ppm)						
	7.00am	9.00am	11.00am	1.00pm	3.00pm	5.00pm	7.00pm
02-06-05	0.6	2.7	8.7	12.0	16.0	13.5	3.3
03-06-05	1.0	3.4	7.2	10.7	13.0	8.7	4.3
04-06-05	0.6	4.7	9.3	12.0	15.3	8.0	3.3
06-06-05	0.3	6.3	8.0	13.3	15.4	10.0	5.0
07-06-05	1.0	5.7	9.0	13.2	15.0	12.6	3.8
08-06-05	1.0	5.2	8.7	11.3	15.3	10.0	4.0
09-06-05	0.6	5.2	8.7	10.8	13.3	8.0	4.2
Mean	0.73	4.74	8.51	11.90	14.76	10.11	3.99

Table 4: Observed CO Concentration in the Atmosphere over a period of seven Days for Gidan Mai

Date	Concentrations of Carbon monoxide (ppm)						
	7.30am	9.30am	11.30am	1.30pm	3.30pm	5.30pm	7.30pm
02-06-05	0.0	2.0	2.7	3.3	4.0	3.7	0.6
03-06-05	0.3	3.3	4.0	6.0	6.0	4.0	1.0
04-06-05	0.3	3.0	6.0	6.3	8.0	2.7	0.6
06-06-05	0.0	2.0	3.3	4.7	6.0	4.0	0.3
07-06-05	0.6	2.0	3.7	5.0	6.3	3.0	0.5
08-06-05	0.3	2.3	3.3	4.0	6.0	4.0	0.3
09-06-05	0.0	2.3	3.7	4.6	5.4	4.0	0.0
Mean	0.21	2.21	3.81	4.84	5.96	3.63	0.56

Table 5: Predicted CO Concentration from Model Formulated for four locations around Bauchi Metropolis

Time	CO Concentration (ppm)			
	Yelwa	Wunti	Central Mkt	Gidan Mai
7.00/7.30am	1.43	1.76	1.74	0.71
9.00/9.30am	3.42	4.48	4.61	1.99
11.00/11.30am	6.23	8.27	9.06	3.50
1.00/1.30pm	10.06	13.25	14.05	5.05
3.00/3.30pm	12.45	15.63	14.81	5.30
5.00/5.30pm	9.22	10.56	9.62	3.16
7.00/7.30pm	3.99	4.41	4.26	1.43

Mkt: *market*

From the results obtained, the predicted and observed (mean) carbon monoxide (CO) concentrations were plotted for Yelwa, Wunti, Central market and Gidan Mai and the graphical representations are presented in Figure 2-5. From these plots, it can be seen that CO concentrations were low during the morning hours.

The CO concentration increased got to the maximum value at 3.00pm. From 3.00pm down the concentration of CO emitted into the atmosphere decreased.

The CO level observed and predicted between 11.00am (11.30am) and 5.00pm (5.30pm) exceeded the standard of 10ppm stipulated by the Nigerian Ambient Air Quality Standard (NAAQS, 2001) for Yelwa, Wunti and Central Market. These concentrations of CO pose serious health hazard to the public, especially to the people leaving around these areas.

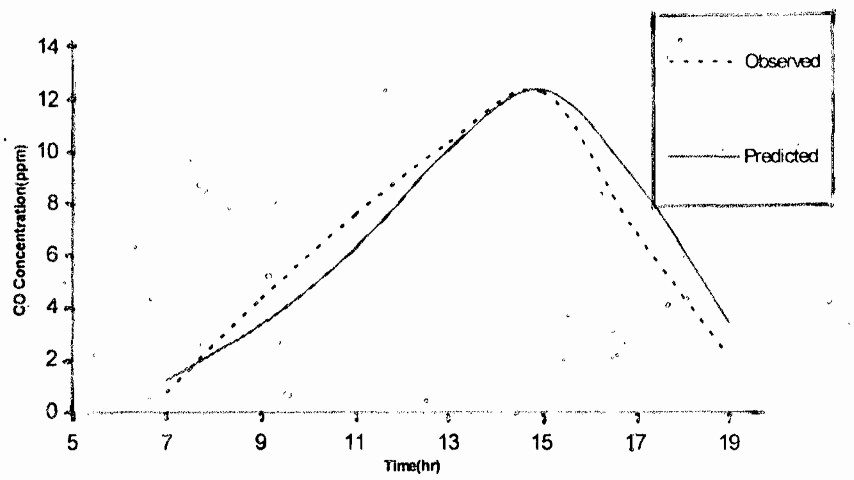


Figure 2: Observed and Predicted CO Concentration with Time for Yelwa

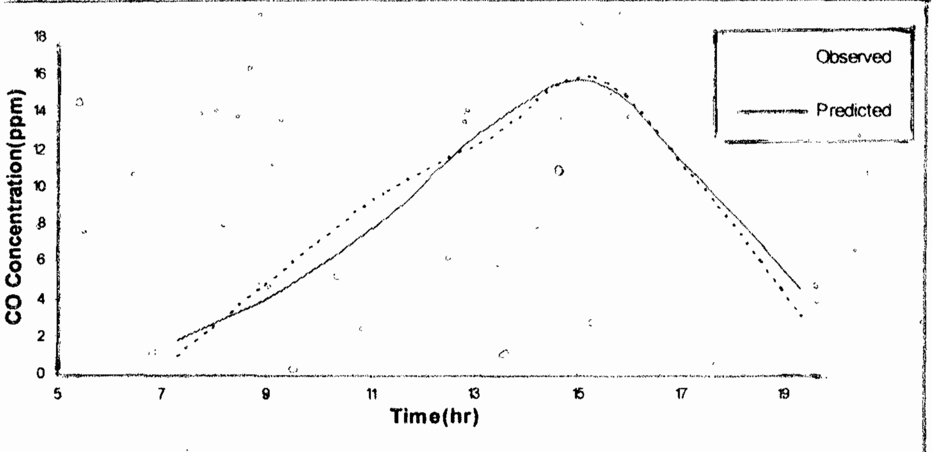


Figure 3: Observed and Predicted CO Concentration with Time for Wunti

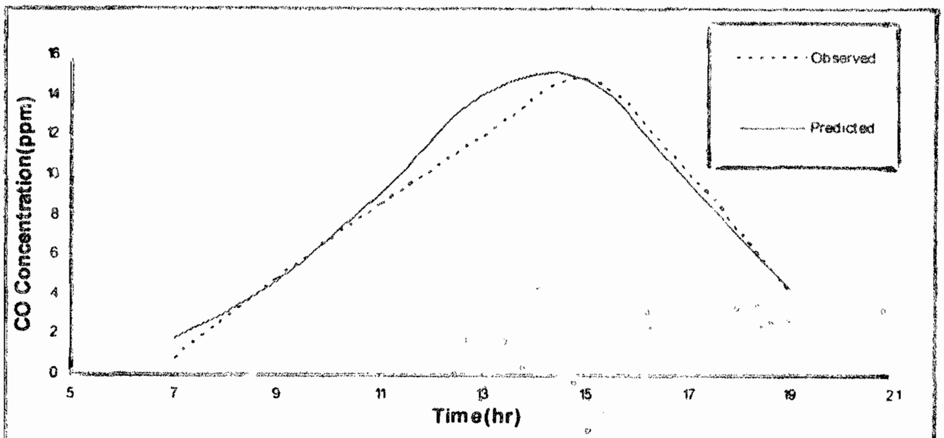


Figure 4.0. Observed and Predicted CO Concentration with Time for Central Market

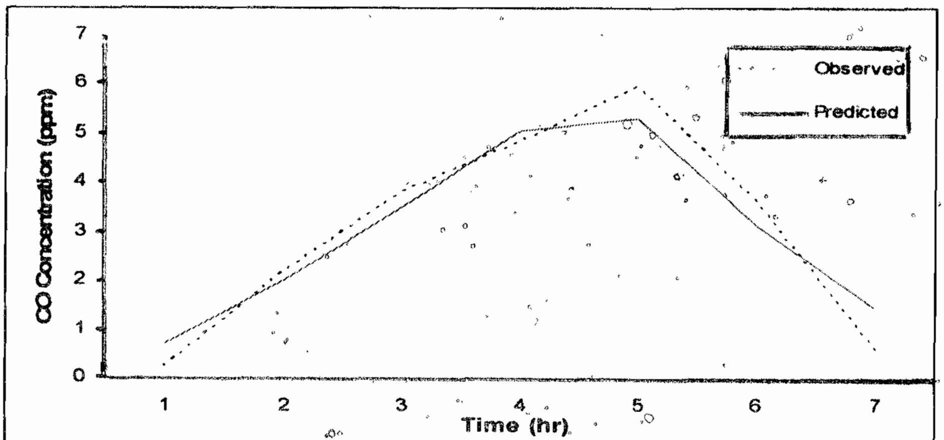


Figure 5: Observed and Predicted CO Concentrations with Time for Gidan Mai

The level of CO for Gidan Mai did not exceed the stipulated standard during the period of investigation. This indicates that the people in this area were not exposed to danger of CO pollutant.

Because of the above, serious measures have to be taken to control the air pollution problem in Yelwa, Wunti and Central market areas. This can be achieved through the formulation of policies aimed at minimizing the concentrations of CO emitted. It can also, be seen in Figure 2-5 that the observed and predicted CO concentrations show high level of correlation, although, there were some discrepancies between them, which can be attributed to some uncertainties inherent in the model formulation, atmospheric conditions (such as wind speed and dispersion coefficient). It was observed at 95% confidence level using chi-squares statistical method that there is a good fit between the observed and predicted CO values for the four areas sampled. Summary of chi-squares statistical analysis is as shown in Table 6.

Table 6: Summary of Chi-squares at 95% Confident Level between Observed and Predicted CO Concentrations in Bauchi Metropolis

Location	Degree of freedom	Evaluated Chi-square value	Chi-square value at 95% Confident Level	Significant Difference
Yelwa	6	1.4789	1.635	No
Wunti	6	1.3089	1.635	No
Central Mkt	6	0.9962	1.635	No
Gidan Mai	6	1.0925	1.635	No

Mkt: market ; evaluated chi-square values were computed from the observed (mean) and predicted CO values in Tables 1-5.

CONCLUSION

From the results obtained, the following can be concluded:

Since there were no significant difference between the observed and predicted CO concentrations at 95% confidence level using chi-squares statistical method, the developed model can be use to predict CO level in Bauchi metropolis.

It was observed that the CO level between the hours of 11.00(11.30) am and 5.00(5.30) pm for Yelwa, Wunti and

Central market exceeded the standard acceptable level of 10ppm.
The level of CO observed at Gidan Mai was within the acceptable limit of 10ppm

RECOMMENDATIONS

Government should find ways of reducing the CO level in areas where it exceeded the stipulated standard.
Generally, the mathematical model developed can be used for the prediction of CO discharged in to the atmosphere in any environment using the required atmospheric data such as wind speed and dispersion coefficient.

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