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ANALYSES OF TECHNIQUES ON STRUCTURAL FATIGUE FAILURE DETECTION.

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ABSTRACT

Machines and structures are subjected to variable loading conditions where the stress cycle does not remain the same during the operation of the machine. Fatigue is undoubtedly one of the most serious of all causes of breakdowns of machines and structures which results in sudden failures. The use of the time domain identification, the spectral density and the random decrement signature techniques in the early detection of fatigue failure have been analyzed.

The time domain identification technique identifies the characteristic roots and the number of modes from the free responses of machines and structures under test. The spectral density technique obtains a broad picture of the frequencies and energy of the structural modes of the system. The random decrement signature technique establishes standard signatures for the loading conditions of the machines and structures. These techniques are useful in detecting fatigue failure in machines and structures under irregularly varying service.

KEY WORDS:fatigue, failure detection techniques

INTRODUCTION

The weakness of metals under repeated loading is a very familiar phenomenon to engineers handling the design of machine parts or their operation. This phenomenon of decreased resistance of a material to varying stresses is called fatigue(Seireg, 1969). Fatigue failure occur in structures when subjected to irregularly varying service loads. The load histories of these structures recorded under actual operating conditions are in the form of random vibrations. Such random responses are obtained from tractors, farm implements, automobiles and aircrafts under tests. The detection and prediction of fatigue failure of a machine part subjected to irregularly varying service loads is a major design problem. In communications, a desired signal in some systems could be random or being accompanied by an undesired random waveform, noise. The noise interferes with the message and ultimately limits the performance of the system, unless we are able to determine how such a waveform will alter the desired output of the system.

A random vibration (Fig. 1) contains frequencies in a continuous distribution over a wide range(Thompson, 1972). The curve is so complicated and variable that it cannot be used to detect changes although all of the information is centered within the time history. The characteristic of such waveforms is that its instantaneous value cannot be predicted in a deterministic sense. Hence a random load history precludes the possibility of any order of repetition. However, these random waveforms are used to predict fatigue lives for parts subjected to irregularly varying loads.

In this paper, the analysis of three techniques; the Time Domain Identification, Spectral Density and the Random Decrement Signature in predicting fatigue failure in machines and structures have been presented.

THEORY OF TECHNIQUES

1. The Time Domain Identification Technique

   This technique uses the free responses (free decay) of a structure under test to identify its vibration parameters; namely, frequencies, damping factors and modal vectors in complex form. The experimental identification of structural modes of vibration are carried out by measuring the input(or inputs) to the structure under test and the resulting responses due to the input. From the measured free responses of the *n* stations on the structure, the modal vectors of a matrix [A] containing *n* modes can be obtained.

   Considering a multi-degree freedom, linear system with damping, the time domain method for the free response for the system is(Thompson, 1972);

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$$M\ddot{y} + C\dot{y} + Ky = 0 \quad \text{------------------(1)}$$

Where M is the mass of the structure, kg
C is the viscous damping coefficient, kg/ms^{-1}
K is the spring constant, kg/metre
Y is the displacement, metres.

For a system of *n* lumped masses

$$[M]\{\ddot{y}\} + [C]\{\dot{Y}\} + [K]\{Y\} = 0 \quad \text{---------------(2)}$$

Considering that

$$[M]\{\dot{y}\} - [M]\{\dot{y}\} = \{0\} \quad \text{---------------------(3)}$$

Equations (2) and (3) can be put in matrix form

$$\begin{bmatrix} [M] & [0] \\ [C] & [M] \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} [0] & -[M] \\ [k] & [0] \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad \text{----------(4)}$$

Therefore

$$\begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} [M] & [0] \\ [C] & [M] \end{bmatrix}^{-1} \begin{bmatrix} [0] & -[M] \\ [k] & [0] \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad \text{----------(5)}$$

By introducing a matrix $[a_i]$, such that

$$\begin{bmatrix} [M] & [0] \\ [C] & [M] \end{bmatrix} \begin{bmatrix} [a_1] & [a_2] \\ [a_3] & [a_4] \end{bmatrix} = \begin{bmatrix} [I] & [0] \\ [0] & [I] \end{bmatrix} \quad \text{----------(6)}$$

Where $[1]$ is a unit matrix

Expanding and simplifying

$$[M][a_1] = [I] \qquad \therefore [a_1] = [m]^{-1}$$

$$[M][a_1] + [M][a_3] = [0]$$

$$[M][a_3] = -[C][a_1] = -[C][M]^{-1}$$

$$\therefore [a_3] = -[M]^{-1}[C][M]^{-1}$$

$$[M][a_2] = [0]$$

$$\therefore [a_2] = [0]$$

$$[C][a_2] + [M][a_4] = [I]$$

$$\therefore [a_4] = [M]^{-1} \quad \text{Since } [a_2] = [0]$$

Therefore

$$\begin{bmatrix} [a_1] & [a_2] \\ [a_3] & [a_4] \end{bmatrix} = \begin{bmatrix} [M]^{-1} & [0] \\ -[M]^{-1}[C][M]^{-1} & [M]^{-1} \end{bmatrix} \quad \text{------------(7)}$$

From (6) and (7)

$$\begin{bmatrix} [M]^{-1} & [0] \\ -[M]^{-1}[C][M]^{-1} & [M]^{-1} \end{bmatrix} = \begin{bmatrix} [M] & [0] \\ [C] & [M] \end{bmatrix} \quad \text{------------(8)}$$

Substituting (8) into equation (5)

$$\begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} [M]^{-1} & [0] \\ -[M]^{-1}[C][M]^{-1} & [M]^{-1} \end{bmatrix} \begin{bmatrix} [0] & -[M] \\ [k] & [0] \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad \text{------(9)}$$

$$\begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} [0] & -[I] \\ [M]^{-1}[k] & [M]^{-1}[C] \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad \text{------------(10)}$$

$$\begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} - \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[k] & -[M]^{-1}[C] \end{bmatrix} \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad \text{------------(11)}$$

If

$$\begin{Bmatrix} x \end{Bmatrix} = \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} \quad \text{And} \quad \begin{Bmatrix} \dot{x} \end{Bmatrix} = \begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix}$$

Then equation (11) becomes

$$\begin{Bmatrix} \dot{x} \end{Bmatrix} - [A] \begin{Bmatrix} x \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \quad \text{------------(12)}$$

$$\begin{Bmatrix} \dot{x} \end{Bmatrix} = [A] \begin{Bmatrix} x \end{Bmatrix}$$

$$\therefore [A] = \begin{Bmatrix} x \end{Bmatrix} \begin{Bmatrix} x \end{Bmatrix}^{-1}$$

$$\begin{Bmatrix} x \end{Bmatrix}^{-1} = \begin{bmatrix} x(t_1) & x(t_2) & \text{---------} & x(t_{2n}) \end{bmatrix}$$

Where $[A]$ is a 2n x 2n matrix $\begin{bmatrix} [0] & [I] \\ -[M]^{-1}[k] & -[M]^{-1}[C] \end{bmatrix}$

From the free responses of n stations with n modes on a structure under test, matrix $[A]$ is formed such that

$$[A] = \begin{bmatrix} y \\ z \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}^{-1} \qquad \text{------------ (13)}$$

Where $X_{ij} = X_i(t_j)$
$Y_{ij} = X_i(t_j + dt)$
$Z_{ij} = X_i(t_j + 2dt)$

For $(i = 1, n; j = 1, 2n)$, dt is the change in time.

Assume $$\{x\} = e^{\lambda t}\{\psi\} \qquad \text{------------ (14)}$$

Where $\{\psi\}$ is a modal vector matrix.

Substituting (14) in (12) and simplifying

$$\left[-\lambda[I] + [A]\right]\{\psi\} = \{0\}$$

Which shows that

$$\left|-\lambda[I] + [A]\right| = 0 \qquad \text{------------ (15)}$$

The eigenvalues a_i are related to the characteristic roots λ_i of the solution of equation (15) through the equation

$$a_i = e^{\lambda i dt} \qquad \text{------------ (16)}$$

The eigenvectors of the matrix $[A]$ are modal vectors. The number of mode in the response is determined by adjusting the number of apparent stations and the order of the mathematical model

2. The Spectral Density Technique

The autocorrelation function R_y, between two random responses $y(t_1)$ and $y(t_2)$ is(Peebles Jnr., 1980)

$$R_y(t_1,t_2) = E[y(t_1)\, y(t_2)] \qquad \text{------------ (17)}$$

In the stationary case, the autocorrelation function $R_y(t_1,t_2)$ of a second- order process is a function only of the time difference r, between t_1 and t_2 and not absolute time.

For time assignments

$t_1 = t$ and $t_2 = t + r$ with r a real number,

$$R_y(t, t+r) = E[y(t)\,y(t+r)] \quad \text{-----}(18)$$
$$= R_y(r)$$

In the time ergodic case, the autocorrelation fuvction $R_y(r)$ may also be found by the time averaging on any sample function of the ensemble, as shown below;

$$R_y(r) = \underset{T \to \infty}{\text{Limit}} \frac{1}{T} \int_0^T y(t)\,y(t+r)\,dt \quad \text{-----}(19)$$

The Fourier transforms play an important role in the spectral characteristics of random responses. Hence the Fourier transform of the time average of the autocorrelation function $R_y(r)$ is expressed as follows(Peebles Jnr.,1980)

$$F_y(f) = \int_{-\infty}^{\infty} R_y(r)\, e^{-i2\Pi fr}\, dr \quad \text{-----}(20)$$

$$= \int_{-\infty}^{\infty} \left[1/T \int_{-\infty}^{\infty} y(t)\, y(t+r)\, dt \right] e^{-i2\Pi fr}\, dr$$

$$= 1/T \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} y(t)\, y(t+r)\, e^{i2\Pi ft}\, e^{-2\pi f(t+r)} \right] dt\, dr$$

Let $s = t + r$

$$F_y(f) = 1/T \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \left(y(t)\, e^{i2\Pi ft}\, dt\right)\left(y(s)\, e^{-i2\Pi fs}\, ds\right) \right]$$

$$= 1/T \int_{-\infty}^{\infty} y(t)\, e^{i2\Pi ft}\, dt \int_{-\infty}^{\infty} y(s)\, e^{-i2\Pi fs}\, ds$$

$$= 1/T\, A^*(f)\, A(f) \quad \text{-----}(21)$$

Where $A(f) = \int_{-\infty}^{\infty} y(t)\, e^{-i2\Pi ft}\, dt$ is the frequency spectrum of y (t), and $A^*(f)$ its complex conjugate.

Therefore

$$F_y(f) = 1/T \left| A(f) \right|^2 \quad \text{-----}(22)$$

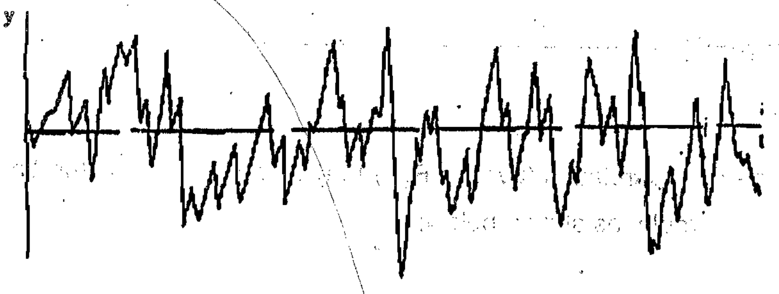

Fig. 1: A Record of Random Vibration

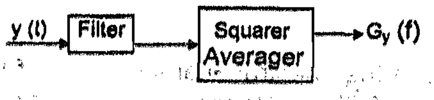

Fig. 2.

The spectral density of an ensemble of random responses is known as the Power Spectral Density $G_y(f)$ and is obtained as the ensemble average of the absolute amplitude squared of the Fourier transform of N - segments of the load history. Therefore, the power spectral density

$$G_y(f) = 1/N \sum_{n=1}^{\infty} 1/T \left|A(f)^2\right| \qquad (23)$$

The energy represented at the various frequencies of a power density spectrum is a quantity that is of interest in random vibration. The energy of the random vibration is given by the area under the power spectral density curve with respect to the frequency variable f(Lathi, 1968)

$$E = 1/2\pi \int_{-\infty}^{\infty} G_y(f)\, df \qquad (24)$$

The power spectral density of a given random load history can be measured electronically by the circuit in Figure 2. The band-pass filter passes the response y(t) in the designated frequency interval and the output is squared and averaged to obtain the power spectral density $G_y(f)$.
The resulting output has a peak for each structural mode and for well separated peaks, the damping ratio at each mode may be obtained by measuring the width of the peak at *half* the peak value

3. THE RANDOM DECREMENT SIGNATURE TECHNIQUE

This is a special averaging technique used to obtain free responses from a number of segments of a random response. In a linear system under random excitation, the average load history converges towards the transient response of the system due to a set of initial conditions. By averaging many samples of the same random response, the random part of the response will average out, leaving the deterministic part of the response. A graphical method illustrating the technique is shown in Figure 3. Figure 3(a) shows the selection of a constant amplitude y_s, which represents a calibrated displacement of the structure. This value gives the free decay step response and makes the resulting signature independent of changes in intensity of the input. Figure 3(b) shows the summation of the first two segments of the curve. First, a segment of the response representing a positive slope above the zero line(where $y_0 = y - y_s$ and $y_0(t) = 0$) is taken and then another segment representing a negative slope below the zero line is also taken from Figure 3(a). These segments represent the free decay positive and negative impulse responses. The average of these two segments(N = 2) is the random decrement signature $\delta(\tau)$ for N = 2. As more samples are taken, the

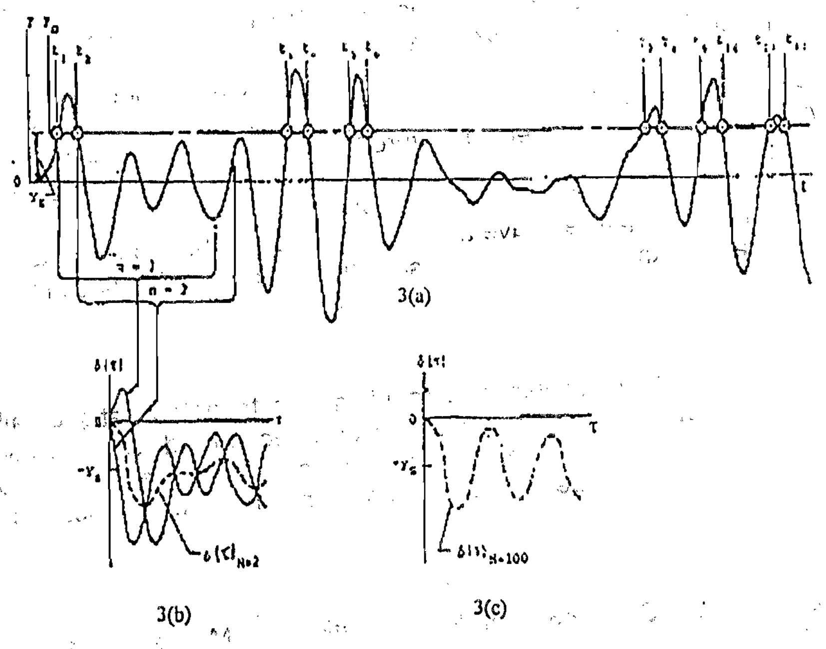

Fig. 3: Graphical illustration of a randon decrement signature technique

signature converges to a free response form as shown in Figure 3(c) for 100 segments. For a single degree of freedom system, the value τ is equal to the period of oscillation ρ of the system.

Analytically, the random decrement signature $\delta(\tau)$ can be computed as follows

$$\delta(\tau) = 1/N \sum_{n=1}^{N} y_0(t_n + \tau)$$

Where $y_0 = y - y_s$ and $y_0(t_n) = 0$

τ is the time difference.

DISCUSSIONS

The time domain identification technique may be useful for analysing linear systems with deterministic responses. The characteristic roots of the system and the number of modes in the free responses are used successfully for failure detection on test machines and structures. The main disadvantage of its use as a failure detector is that the technique depends on the input excitation. The spectral density technique is useful as a failure detector in obtaining a broad picture of the frequencies of the structural modes, the energy in the modes and the approximate damping of the isolated modes. The spectral density technique also dependent on the amplitude and form of the input excitation. The random decrement signature technique is used successfully for failure detection and damping measurements of structures in single-station, single mode response systems. It can also be applied to systems with a multiple of response signals in which case the signature is modified to keep the time-correlation between individual signals unchanged where it exists. However, the technique is most appropriate in situations where controlled or initial excitation cannot be used since it may cause undesirable interruption, or complete knowledge of the excitation not available.

When a fatigue crack develops in a structure, the crack introduces additional degrees of freedom which are excited by the random forces. When the crack is small, small blips would show-up in the hashy, high-modal density region of the spectral density curve, making the detection difficult. As the flaw grows, the frequency of the failure mode would be expected to decrease until it approaches the fundamental modes, at which point fatigue failure is imminent. To detect the failure mode by the spectral density technique, the recorded random response signal is passed through a band-pass filter set at a high frequency. The filter suppresses all frequencies and only information of magnitude of the frequency components of the signal which lie in the narrow band are transmitted intact through the filter. The failure mode could be intercepted at a high enough frequency so that corrective action can be taken and complete failure avoided. With the random decrement signature technique. standard signatures are established for all loading conditions and environments of the undamaged structure. If a failure develops, the failure frequency will dynamically couple with the structural modes within the band-pass frequencies of the filter. Since standard random decrement signatures have already been established, only parts of the signature at these peaks need to be calculated with warning devices sensitive to voltage changes in the peak values

CONCLUSION

The need for an accurate, reliable and rapid means to detect fatigue failure in machines and structures under irregularly varying service loads is widely recognized. Several theories are available to predict the endurance life of a member under such loading conditions. The analyses of the three techniques show how they could be useful in fatigue failure detection in machines and structures.

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