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DEVELOPMENT OF AN INTEGRATED PRODUCTION INVENTORY MODEL FOR A MULTISTAGE SERIAL SYSTEM

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ABSTRACT

This paper presents the development of an integrated production inventory model for multistage serial system. The model seeks to determine simultaneously the optimal production and procurement policies through minimization of time averaged variable cost of production. The cycle concept of instantaneous multistage production inventory system is extended for non-instantaneous production and integrated with concept of inventory models with cost changes. Integer multiple parameters are used as a check factor for cost minimization. A numerical example is also presented.

Key Words : Inventory, Production, Optimization, Modeling

INTRODUCTION

Multistage serial production system exists in both discrete part manufacturing and chemical processing industries. These include our vast biochemical, petrochemical, food and pharmaceutical as well as electronic packaging-industries. In serial systems, a product passes through a series of single processing stages that transform it from raw material to final products. Inventory Management and Production Control are the two subsystems of the total manufacturing systems. These two subsystems interact very closely in real life situation. In spite of this fact, majority of the research work in the past have been towards analyzing them independently probably because such an approach made the study and analysis easier. Thus to deal with production inventory systems the interaction between inventory and production control should be taken into account very closely.

The lot-sizing problem and infact cost minimization (optimization) of production system has received considerable attention for many years. Majority of the research work have been towards the analysis of single item stocked at a single or multi-product stocked at multistage under instantaneous consideration. For instance, Fujita (1978), Bomberger (1966) and Elimarchraby (1978) reported the economic lot-scheduling problem under instantaneous production system using the marginal analysis, basic period and extended basic period approaches respectively. Also non- instantaneous production which is finite production rate have been duly worked upon. For instance, Schwartz and Schrage (1975) presented a branch and bound scheme for Integer Merging Lot (IML) policy. Srendrovits (1998) presented a paper that established that the optimal policy would in general be the non-integer split/Merge Lot sizing (NISMIL) policy. Similarly, Crowston, et al (1973) reported lot sizing for a

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multistage system with converging branches. They accounted for the interdependence of successive stages by deriving an expression for the initial inventory required to ensure no stockout. Bigham and Mogg (1979a, b) studied the same system under non-instantaneous production. They accounted for the interdependence of successive stages by delaying the initial startup at one stage relative to that at its immediate predecessors. They used the lower bound solution and integer multiple assumptions for their first and second works respectively. Their formulations however did not account for the raw material consumption pattern. Gogal (1977) derived an integrated production-inventory policy for single product at single stage under instantaneous production. Karimi (1989, 1992) presented an analytical results for determining stationary cyclic schedule for two-stage serial production system using the Non-Inter Split/Merge Lots Sizing (NISML) policy.

This work presents a formulation for a single production processed through multiple facilities with finite production rates and startup delays for a serial production-inventory system. The concept of multistage serial production of Karimi (1992) is integrated with the concepts of multistage production schedule with startup delays of Bigham and Mogg (1979a,b) and Korgaonke (1979).

MODEL DESCRIPTION

In the N-stage system, the stages are numbered according to sequence of production from raw materials stage to finished product market. Note, that the real production stages start from stage 2, stage 1 denotes the raw material supplier while stage N denotes the final production market. So inventory J = 1 and J = N represents the raw materials inventory and final production inventory respectively. Thus a system with N stages has N-2 real production stages.

SYSTEM ASSUMPTIONS

The following assumptions are made with respect to operational characteristics of the process.

1. A flow shop types production system is assumed where a product is processed on serial of facilities in a sequence on batch basis.
2. Non-instantaneous production for a single product is assumed for which the demand occurs constantly and continuously (that is, the production is sold immediately it passes the final production stage).
3. Each stage cycle consists of the production time and the shutdown period. A stage with production rate equal the average demand rate will not be shut own and must operate continuously within the production hours available. Such stage still be assumed a continuous stage. Thus, by this assumption the market (stage N) is a continuous stage
4. No stockout is allowed, that is materials are feed to each stage from the upstream inventory whenever it needs it. to ensure this, the start of stage J may be delayed by a certain amount of time as compared to preceding stage (J – 1) if the production rate of

stage J is greater or equal to that of J – 1)

5. Each stage operates a fixed cycle time in a day.

SYMBOLS AND NOTATIONS

N = Number of stages
D = Demand rate of product (units/day) for the finished product.
a_j = Setup cost for facility j, this is the sun of one startup and one shut down operations at stage j,
T_j = Cycle time (days) for stage (that is, the duration between successive startups at stage)
H_j = GI = value added inventory cost per unit per day
G_j = value added per unit product at facility j
I = Daily inventory carrying change factor.
P_j = Production rate of production on facility j (Units pay) in units of the final product.
t_j = Production time per lot (days) : $tj = X_i, T_j$ is
M = Number of raw materials for stage j
X_j = D/pj = fraction of circle times of facility j during which it produces.
d_j = Startingup delays of facility j (for j = 3.)
b_j = $1 - X_j$ = a proportion off-time per cycle at stage j.
Q_j = Optional lot size (batch quantity) of product at facility j.
Q_{jL} = Optional lot size (order quality) of raw material per lot.
A_{jL} = Ordinary cost per order
H_{jL} = Inventory carrying cost per unit per day.
K_{jL} = Raw material demands per day.
C = Total variable cost per day of the system.
C_j = Total variable cost per day of the production Inventory system at facility j.
C_{jL} = Total variable cost per day of the inventory system of raw material L at facility j.
S_{jL} = Unit cost of each raw material
R_{jL} = depletion time of each raw material.

Z_j and Z_{jL} are positive integers due to production and ordering respectively.

FORMULATION OF THE MODEL

The formulation of the model follows an approach similar to Karimi (1992), Bigham and Mogg (1979), and Korgaonker (1979) although it is different from them in that;

(1) It considers instantaneous production schedule and inventory control.
(2) It considers a non – instantaneous production [i.e. finite production rate].
(3) It considers a single production through multiple stages in a serial production system.
(4) An enumeration solution procedure has been set up for accurate computation.

Because the average demand rate of the production must be produced at each stage, we can

therefore write that

$$D = P_i X_j \quad (1)$$

The economic batch size of the production on facility is given by

$$Qj = M_j D T_j \quad (2)$$

In which QN-1 = QN – which is the final batch quantity

$$Mj = Z_{j-1}, Z_{j-2}, Z_{j-3} \ldots Z_2, Z_1 \ (j = 1) = \left(\prod_{i=1}^{N-1} Z_i\right) \quad (3)$$

Similarly, the economic order quality is

$$Q_{jl} = Z_{jl} \ldots Q_1 = Z_{jL} \left(\prod_{i=1}^{N-1} Z_i\right) \quad (4)$$

We propose the following defining relationship

$$B_{j1} T_j = B_{j2} T_{j-1} \quad (5)$$

$$X_j = \sum X_j = D/P_j < 1 \quad (6)$$

$$B_{j1} = 1 - X_j \quad (7)$$

Where only production schedules considered are for those in which

$$T_{j+1} = Z_j T_j \quad (8)$$

Where $Z_j = B_{j1}/B_{j2}$ (9)

For rational behind equations 1 to 9 please see Bigham & Mogg (1979a,b) and Karimi (1992). Our problem is that determining B_{j1} and B_{j2} as to calculate the optimal values of Z_j, M_j T, and Z_{jL} needful to minimize the total variable costs. Recall that in our N – stage system, only stages 1 is continuous, we set $B_{11} = B_{12} = 1$ and $T_1 = T_2$. Also stage N is assumed continuous then, $B_{(N-1)\,1} = B_{(N-1),\,2} = 1$ and $T_N = T_{N-1}$. Note that assigning finite cycle times to continuous stages seems arbitrary, it has absolutely no implications because they are not real production stages. We can therefore formulate the variable costs of the multistage serial production system as follows.

1. The total variable cost per day of procuring raw material 1 of the product at facility j is

$$C_{jL} = \frac{a_{jL}}{M_j Z_{jL} T_j} + \frac{Z_{jL} M_j K_{jL} h_{jL} T_j}{2} + S_{jL} K_{jL} r_{jL} \quad (10)$$

$$Cj = \frac{a_j}{M_j Z_{jL} T_j} + \frac{M_j D T_j h_j}{2} + [B_{jL}(b_{j-1}) + B_{j2}(b_j) - 2(1-U_j)] + \sum_{L=1}^{m} C_{jl} \quad (11)$$

$$C = \sum_{j=2}^{N-1} C_j \quad (12)$$

Where

$$Uj = \text{Max}\,[X_j, X_{j+1}] \ldots\ldots j < N-1 \text{ only} \quad (13)$$

$$r_{jL} = \frac{S_{jl2} - S_{jl}}{h_{jl}} + \sqrt{\frac{2h_{jl} 2a_{jL,2}}{K_{jL}(h_{jL})^2}} \ldots \text{ for price change of raw material from } S_{jl} \text{ to } S_{jl.2} \quad (14)$$

$$T_{jL} = \sqrt{\frac{2a_{jL}}{K_{jL}\, h_{jL}}} \quad \ldots \text{When there is no price change} \qquad 15$$

to obtain the optimal values of T, we set to zero the partial derivative of C with respect to T, thus

$$\frac{\delta C}{\delta T} = 0$$

yielding that:

$$T^* = \left\{ \frac{2\left[\sum_j \left(\frac{a}{m}\right) + \sum_j \sum_L \frac{a}{MZ}\right]}{\sum_j M_i Dh_i (B(h_{i-1}) + B_{j2}(b_j) - 2(1-U_i) + \sum_j \sum_j M K h Z} \right\}^{1/2}$$

that is

$$Tj^* = \left\{ \frac{2\left[\frac{a}{m} + \sum_j^m \frac{a}{MZ}\right]}{M\, Dh\, (B(b) + B(b) - 2(1-U) + \sum_j^m K\, h\, ZM} \right\}^{1/2}$$

$$T = \sum_{j=1}^{N-1} Tj \qquad 18$$

ANALYSIS OF THE MODEL

The inter dependence of production stage j is included by calculating the minimum delay in the initial start of production at stage j + 1 relative to that at stage j + L required to ensure no stock out occurs. (Bigham and Mogg 1996) show that of the cycle times chosen for stage j and j + 1 satisfy equation 8 then, the following analytical startup delays expression may be written

$$d_i = B_i + T_j - B_{j2}\, T_{j+1} \qquad 19$$

where B_{j1} and B_{j2} are given by the following

A/. If $P_i \geq P_{j+1}$, then

$$B_{j1} = \begin{cases} b_j - b_{j+1} \ldots & \text{for } T_{j+1} \geq T_j \\ b_j \quad \ldots & \text{for } T_{j+1} < T_j \end{cases}$$

$$B_{j2} = \begin{cases} 0 \quad \ldots & \text{for } T_{j+1} \geq T_j \\ B_{j+1} \quad \ldots & \text{for } T_{j+1} < T_j \end{cases}$$

And

B/. If $P_j < P_{j+1}$, then

$$B_{jL} = \begin{cases} 0 & \dots \quad \text{for } T_{j+1} \geq T_j \\ B_j & \dots \quad \text{for } T_{j+1} < T_j \end{cases}$$

$$B_{j2} = \begin{cases} 0 & \dots \quad \text{for } T_{j+1} \geq T_j \\ B_j & \dots \quad \text{for } T_{j+1} < T_j \end{cases}$$

Recalling that $Z_j = B_{jL}/B_{j2}$, it can be noticed that for $T_{j+L} > T_j$, the value of Z_j yields an infinity or Error in calculation since the denominator is zero. To this effect, we assume that the removal of such restriction has no adverse effect, thus

$$\left.\begin{aligned} B_{jL} &= b_j \\ B_j &= b_{j+L} \end{aligned}\right\} \quad \text{for } P_j > P_{j+1}$$

And

$$B_{jL} = b_{j2} = b_j \qquad \text{for } P_j < P_{j+L}$$

Practically, one can argue that the production rate of a stage (j) should be equal or greater than that of the immediate successor (j + 1), such that the successor stage can be continuously feed without any other delay due to stock out. This will continually ensure availability of products to the product market. By this proposition, we have only dropped the production rate restriction of the type $P_j < P_{j+L}$ as against the entire removal of the restrictions as suggested by Bigham and Mogg, 1996) in their conclusion. Thus we can write that

$$\left.\begin{aligned} B_{jL} &= b_j \\ B_{j2} &= b_{j+L} \end{aligned}\right\} \quad \text{For } P_j > p_{j+L}$$

Similarly, we can define zjl by the expression

$$Z_{jL} = B_{jL1}/B_{jL2}$$

Where

$$B_{jL1} = b_{jL1} = 1 - r_{jL}$$

$$B_{jL2} = b_{jL2} = 1 - r_{jL}$$

And

$$r_{jL} = \sqrt{2a_j / (k_{jL}\, h_{jL})}$$

PREPOSITION

Production (X_j) = Demand (D) / Production rate (P_j)

In raw materials;

Quantity available = Quantity demanded x No of such demand = Quantity demanded x Demand rate.

Also

Depletion time = Quantity available / quantity demanded

Thus, Quantity available = depletion time x Quantity demand

Thus equating (1) and (2) established that the depletion time = demand rate

Thus B_{ji} = 1 – depletion time

= $1 - r_{ji}$

ENUMERATION SOLUTION PROCEDURE FOR THE DETERMINATION OF OPTIMAL PARAMETERS

Firstly, the number of stages N involved in the system are determined; where the actual (real) production stages should be N-2 such that stage 1 and N denotes the raw material supply stage and the final product market stage respectively. Then the:

Values of raw material stage, number of raw material in each stage, and number of production stages are read into the computer and the raw material and production system parameters are inputted.

Then calculate;

- The depletion time which is the demand rate rji for each material from equation 17.
- The multiplier factors Bjl1 and Bjl2 from equation 22 & 23 for J <N-1 only
- The integer factor Zj1 from equation 4 for each raw material at every stage and round off the value to the nearest integer value
- The production rate xj = D/pj for each it stage
- Bj = 1-xj for each stage
- The production multiplier factors Bj1 = bj and Bj2 = bj+1 for J < N-1 only
- The integer factor for production Zj = Bj1/Bj2 for each stage, and the value rounded off to the nearest integer
- The product sum of integer factors Mj from equation 3
- Uj = max (xj, xj+1) for each stage J = N-1
- The cycle time Tj for each stage from equation 17
- The total cycle time T by summation of value in step above, equation 18.
- The variable cost of each raw Cj1 using equation 10
- The total cost for stage J, cj, by summation of values in step above equation 11
- The total product/inventory cost c by equation 12 i.e. summation of cj
- The batch quantity Qj, for each stage from equation 2.
- The economics order quantity Qjl for each raw material from equation 4
- Finally, calculate the delay dj from equation 19 for each stage J >2 that ensures no stockout.

NUMERICAL EXAMPLE

Table 1 below shows data from a real life situation. Application of the model yields table 2 (see Ezema 2000) which gives the optimal batch quantity Qj as 275, optimal procurement quantity as 157 for raw material B of stage 1, 274 for raw material B of stage 2, the optimal total variable cost of production as ₦4,071.00, and the optimal cycle time as 3.2 hours.

Table 1

RAW MATERIAL COST PARAMETERS

Stage	Raw material	Ordering cost	Holding cost	Unit cost	Demand/hour
1	A	40	0.3	30	300
	B	30	0.2	40	600
2	A	35	0.4	20	200
	B	30	0.2	30	400

PRODUCTION COST PARAMETERS

	Demand	Production rate	Setup cost	Holding cost
Stage 1	200	250	60	2
Stage 2	200	300	80	0.2

Table 2

COMPUTED RAW MATERIAL COST PARAMETERS

A	Inputted parameter					Computer parameters					
Stage	a	Kj	h	S	R	Bj11	Bj12	Zj1	Cj1	Qj1	Raw materials
1	40	300	0.3	30	0.9428	0.0572	0.646	1	8571.62	156.45	A
1	30	600	0.2	40	0.7001	0.2929	0.1340	2.1	17101.92	386.25	B
2	35	200	0.4	20	0.9354	0.0646	0.646	1	3821.91	273.08	A
2	30	400	0.2	30	0.8660	0.1340	0.1340	1	10468.89	273.08	B

COMPUTED PRODUCTION COST PARAMETERS

	Inputted parameters				Computed parameters									
Stage	D	Pj	Aj	Hj	Xj	Uj	Bj1	Bj2	Bj	Dj	Zj	Mj	Tj	qj
1	200	250	60	2	0.8	0.8	0.2	0.3333	0.2	0.881	1	1	0.8834	284
2	200	300	80	0.2	0.6667	0.6667	0.333	0.3333	0.333		1	1	2.7756	275.02
T = 3.159					C = 4071.3754									

DISCUSSION OF MODEL AND RESULT

Bj1 and Bj2 are the cost minimization factors representing factors that can boast productivity which means higher profit and minimized variable costs. These include the likes of (1) Environmental factors, factory site, location, excuse of workers while on duty, lack of supervision or planning.

Performance incentives: increase wages and welfare packages; planned activities usually provoke productivity, measuring these factors are rather subjective however, we must realize that

(a) Small value of Bj1 and Bj2 simultaneously denotes when a worker is happy and therefore produces at his best which will invariably increase profit margin, that is Xj will be greater showing utilization of more time in production.

(b) High values denotes increased cost due to delay, laxity – excuse of such may come from delay in pigmenting wages, wages not adequate, medical and eating facilities.

(c) Similarly Bj1 and Bj2 represents procurement equivalents of Bj1 and Bj2 these include names to source of raw material and raw material handling system.

(d) For healthy production: $Bj2 > Bj1$; $Bj2 < Bj1$ giving $j < 1$, $bj + 1 > bj$; $Xj > Xj+1$, $Pj > Pj+1$ and $J1 <$ and $rj+1 < rj$ such that the raw materials for stage $j+$ will finish earlier and $j+1$ which will necessitate action for re-ordering of the first stage raw material and others and thereby avoiding any delay or time wastage.

Again, $bj+L > bj$ (i.e. off time at respective stage) such that stage j is observing off time (idle time) less than thirty the successor $j+1$ giving rise to no laxity or waiting time of $j+1$ with respect to j which ensures productivity and conforms with our adoption that $Pj > Pj+1$. This delay gives the minimum or optimal total variable cost of the production system.

CONCLUSION

A general integrated production inventory formulation for a serial system was presented. The model simultaneously determines the batch quantities of production and raw material procurement policies of any serial production system. This model can be applied to any serial system to check for effectiveness in production and inventory control provided the restriction that $Pj > Pj+1$ is closely observed. It is hoped that if Nigerian companies can keep their production inventory records on daily basis and effectively too, the formulation in this paper would be very helpful to all serial production system and especially those companies which were used as case study following the recommendations made.

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