GLOBAL JOURNAL OF EDUCATIONAL RESEARCH VOL. 2, NO. 1 &2, 2003: 23-26

INTERACTION BETWEEN MATHEMATICS LANGUAGE AND NATURAL LANGUAGE: IMPLICATIONS FOR THE TEACHING AND LEARNING OF MATHEMATICS

S. I. BINDA and B. G. ATTAH

ABSTRACT

The language of mathematics is characterized by technical words and symbols, whose meanings and usage in mathematics tend to differ from what obtains in the ordinary language, such as English Language. In this paper, the meanings and usage of few technical words and symbols in mathematics, as well as in the English language have been explained. Since the understanding of the language of mathematics enhances performance in mathematics, it means, therefore, that mathematics teachers need to always explain the meanings and applications of these words and symbols in and outside mathematics. This will help the learners get the exact meanings of these words and symbols, thereby comprehending the mathematics taught them. This comprehension enables them to boost their performance in the subject.

KEYWORDS: Interaction, Language, Mathematics, Teaching, Performance.

INTRODUCTION

Poor performance in mathematics among secondary school students has not only persisted, but seems to defy solution. This has been a source of concern to mathematics teachers, mathematics educators, parents, students, governments as well as the general public (National Council on Education, 1998). One of the factors responsible for the persistent poor performance in mathematics is the failure of mathematics teachers to use the precise language of mathematics in teaching the subject (Thompson and Rubenstein, 2000).

The successful teaching and learning of mathematics largely depends on effective communication between the teacher and students on one hand, and among the students themselves, on the other (Binda, 2004). No wonder, Bromme and Steinbring (1994) have observed that students' ability to constitute mathematics meaning is a function of the process of interaction that goes on in the classroom. In other words, the process of classroom interaction between the teacher and the students (usually *facilitated* through communication) determines which elements of a mathematics problem should have what sort of function in constituting mathematics meaning.

For mathematics teachers to ensure effective communication of what they intend to pass on to the learners, they must pay special attention to the meanings of these words in mathematics as well as outside mathematics. Failure to do this will easily lead to a break in communication which makes it difficult for the students to receive the message from the teacher as intended.

Positive relationship (r=0.301) has been established between the understanding of the language of mathematics and performance in mathematics (Binda, 2005). Although this relationship was weak; it was significant (t=4.96) at the 0.05 level of significant. Similarly, Hall (2004) believes that verbal abilities of students and proficiency in the mathematics subset of the English Language are essential for success in Mathematics. Thus, by using the language of mathematics appropriately in teaching, students can have the opportunity to develop their mathematical understanding, through giving meaning to new experiences and relating them to already held ideas.

Against this background, this paper examines the meanings of few words and symbols in mathematics and in English language.

CONFUSING TERMS IN MATHEMATICS AND ENGLSH LANGUAGES

(a) Difference

The word 'difference' is one of the words often misused in mathematics. Pimm (1990, p.140) gives an instance of a mathematics teacher who asked his students the 'difference' between 328 *and* 49. The students gave the following answers:

49 is small, while 328 is large
49 is odd, while 328 is even
49 is two-digit, while 328 is three-digit
49 is a perfect square, while 328 is not.

Similarly, Backhouse, Haggarty, Pirie and Stratton (1992) recall another instance in which a mathematics teacher asked his students to tell him the 'difference' between 7 and 10. To the astonishment of the teacher, the children gave the following answer: "7 got one figure while 10 has two".

The students' answers in both instances are an indication of their inability to interpret the word 'difference' from

S. I. BINDA, Institute of Education, University of Jos, Jos - Nigeria
B. G. ATTAH, Institute of Education, University of Jos, Jos - Nigeria

the mathematics point of view. As a result, they were interested in the other properties of the numbers involved rather than subtraction. Thus, the 'difference' between 328 and 49 is 328 - 49 = 279; while the difference between 7 and 10 is 10 – 7 = 3. In other words, once the difference between two numbers is being demanded it means subtraction.

In English language, however, 'difference' refers to something that makes one thing or person different from another thing or person. It also means a disagreement that people have, such as in opinion on something. For example if two students give 5 and 6 as answers to a problem, we say these students have different answers. That is, they differ in their solutions.

(b) Function

Neuman (1964, p.227), defines a 'function' in mathematics as the "Numerical dependence of a quantity on one or more other variable quantities". In other words, a function is a quantity whose value depends on the values of other quantities. For example, the area of circle ($A = \pi r^2$) is a function of its radius, since the value of pie (π) is constant. Similarly, the time taken by a motorist to cover a certain distance ($t = \frac{d}{s}$) is a function of the distance (d) to be covered and speed (s) of the motorist.

Mathematically, therefore, y is called a function of x, if a value of y can be calculated or observed which corresponds to each permissible value of x. Thus, we write y = f (x) read 'y is equal to function of x'. In this expression, y is the dependent variable and x the independent variable. This means the values of y depend on the values of x or the values of x determine the values of y. More specifically, if x changes value, y also must assume corresponding value.

Consider for example, the equation $y = x^2+2$. We can say that this is a function y of x, or y is a function of x, which describes a relationship between variables x and y. Thus, if x = 2, y is calculated by substituting in the function, the value of x, and this gives: $y = 2^2 + 2 = 4 + 2 = 6$. Similarly, if x = 3, $y = 3^2 + 2 = 9 + 2 = 11$ etc.

Values of dependent variables in functions are better stated in form of 'table of values'. For the function $y = x^2 + 2$, the table of values of y for values of x ranging from 1 to 4 is presented in Table 1.

Table 1: Table of Values for the Function $y = x^2+2$.

X	1	2	3	4
Y	3	6	11	18

Note: Values of y are obtained by substituting values of x in the function.

In English language, however, the word 'function' according to Hornby (1995,p.480) refers to "a special activity or purpose of a person or thing, ... an important social event, ... any of the basic operations of a computer, ... to work; to operate". Also, Osafehinti (1993,p.74) observes that "the word function to the non-mathematician, means something like an evening get-together, or some special party. More funny is the word function-of-function, which ordinarily would describe the greatest social affair that ever took place".

A 'function' could also mean a certain set of duties or activities an individual performs in a certain capacity. For example, a governor of a state or a chairman of a local government council, commissioning a road project, is said to be performing a function. Similarly, a messenger delivering mails from one office to another is also performing his function.

(c) Root

The root of a number in mathematics is another number, which must be raised to a certain power to obtain the original number. Generally, the nth root of a number 'a' is another number 'b' whose nth power is equal to 'b'. We write as $\sqrt[n]{a} = b$, read the nth root of 'a' equals 'b'. In this example, 'a' is called the radicand, 'n' the index of the root while 'b' is the root.

In particular, the second root of a number is called the square root of the number, the third root is called the cube root, etc. Therefore, the square root of a number is that number which when squared gives the original number, while the cube root is that number which must multiply itself three places to get the original number. For example, the square root of 16 is $\sqrt{16} = 4$, since $4 \times 4 = 4^2 = 16$. Similarly, the cube root of 8 is written as $\sqrt[3]{8} = 2$, since $2 \times 2 \times 2 = 2^3 = 8$. Thus, a root is a quantity which, when multiplied by itself a certain number of times, produces another quantity. This means 16 is obtained by multiplying 4 two times while 8 is obtained from multiplying 3 three times.

In English language, however, the word 'root' reminds us of that part of a plant that grows under the ground, absorbing water and minerals, thereby providing support to the plant. It also refers to the part of a hair, tooth, tongue that attaches it to the rest of the body. In some contexts, the word 'root' is used to mean the basis or origin of something or the part of a word on which its other forms are based, that is, a word from which other words are formed. For example, the word 'walk' is the root of 'walks', 'walked', 'walker' and 'walking'.

(d) Pole

Suppose the value of a function f(x) approaches infinity at a point when x approaches a certain value, then the function is said to have a pole at that point. The particular point we say the function f(x) ceases to be analytic at that point, or f(x) has a singularity at that point (Kreyszig, 1983). Mathematically, therefore, if $f(x) \to +\infty$ as $x \to x_o$, then f(x) has a pole at $x = x_o$. For example, let $f(x) = \frac{1}{x}$. The value of this function approaches infinity as x approaches zero. In other words, whenever x assumes the value zero, the function assumes an undefined value. That is, if x = o, $f(x) = \frac{1}{x}$ becomes $f(o) = \frac{1}{0}$, which is not defined. It is not defined because it has no definite value. That is, $f(x) = \frac{1}{x}$ has a pole at x = 0. Similarly, if $f(x) = \frac{1}{x-1}$, then this function has a pole at x = 1, since substituting this value into the function yields an undefined value. That is, if x = 1, $f(x) = \frac{1}{x-1}$, becomes $f(1) = \frac{1}{1-1} = \frac{1}{0}$, which is undefined. Where a function has a pole (a singular point) such a function becomes, undefined. Thus, it is not differentiable at that point.

In English language, however, a pole simply means a long thin piece of wood or metal, used as a support of something or for pushing boats along. It also means either of the two points at the opposite ends of the line on which the earth or any other Planet turns, hence, north pole and south pole. In National Electric Power Authority, Poles (plural of pole) are used in supporting the cables that transport electricity from one point to another.

(e) Argument:

Given a point (a,b) on the complex plane, the angle which is formed by the line joining the origin (0,0) of the complex axes to the point and the real axis is called the argument of the complex number *a + ib*. *It is to be noted that* whenever a complex number is in the form a + ib where 'a' is the real value while the imaginary part is 'b', by this definition, $a = r\cos\theta$ and $b = r\sin\theta$ where, $r = \sqrt{(a^2+b^2)}$. Figure 1 speaks further on this:

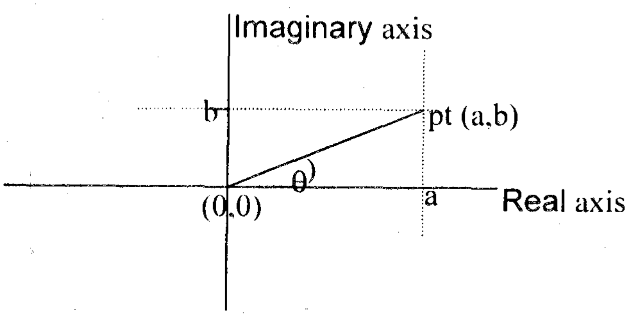

Figure 1: The Complex axes showing the argument of a + ib.

Based on this diagram, the complex number can be expressed in terms of 'r' and the argument, thus: $z = r\cos\theta + ir\sin\theta$. Factorising, this yields $z = r(\cos\theta + i\sin\theta)$.

Since z is constant, the only way to determine the position of pt (a,b) is through the use of the angle which z forms with the real axis. Hence, the angle is referred to as the argument.

In English language, however, the word 'argument' refers to a discussion based on reasoning or a reason put forward. It also means an angry disagreement, such as a quarrel and the like.

SYMBOLS USED IN MATHEMATICS WHICH HAVE DIFFERENT MEANINGS IN ENGLISH LANGUAGES.

There are symbols in mathematics that are used differently in the *natural language*. Few of these have been discussed below:

(a) **The colon (:)** The symbol of a colon (:) in mathematics simply illustrates the concept of ratio. For example ratio 2 to 5 can be written mathematically as 2:5. If for example, certain objects are shared between two people in the ratio 2:5, then it means that the first person will receive 2 out of seven ($\frac{2}{7}$) of all the objects to be shared, while the second person takes 5 out of seven ($\frac{5}{7}$) of the objects to be shared.

In English language, however, the colon (:) signifies that what follows is an example, a list or a summary of what precedes it or a contrasting idea. In another context such as in anatomy, a colon is the lower part of the large intestine.

(b) **The period or full stop (.)**
The symbol (.) in mathematics means different things at different times, depending on the person using it and the situation. Mathematically, a point is an idealised location of a place. It is used when expressing *decimal numbers such as in* 2.5, 3.04 etc. In algebra, this symbol signifies multiplication. For example, 4.5 = 4 × 5 = 20.

But in English language, a point (.) has several interpretations.

According to Hornby (1995, p.890), a point is:

> Any dot used in writing or printing, ... a tiny dot or mark of light or colour, ... a position on a drawing, map, etc ... a particular place, area, ... time or instant, ... a particular stage in a process of change or development, ...

In the commonest use, like in English language, the symbol (.) usually marks the end of a sentence. It is also used in abbreviations.

(c) **The Addition Sign (+).**

This symbol in mathematics tells that numbers should be added. For example, 3+5 = 8, meaning that five is added to three or simply, three plus five, which gives eight (8). This symbol in English language, however, signifies the cross, that is the structure on which Jesus Christ was crucified. No wonder, this symbol is easily seen on church buildings or hospitals.

(d) **Exclamation Mark, (!)**

The exclamation mark in English language is used at the end of a sentence to express a strong feeling toward or about something. Example: It was a good presentation!. It can also be used after a forceful command. Example: Go home immediately!. Also, an exclamation mark is used after a strong interjection or expression, such as what a mess!.

In mathematics, however, this same symbol (called factorial) is used to denote multiplication of consecutive integers (whole numbers). For example, 4! (read as four factorial) means continuous multiplication of all whole numbers from 1 to 4. That is, $4! = 4 \times 3 \times 2 \times 1 = 24$. Similarly, 5! 3! Means $(5 \times 4 \times 3 \times 2 \times 1) \times (3 \times 2 \times 1) = 120 \times 6 = 720$.

CONCLUSION

One of the ultimate goals of mathematics teaching is to communicate mathematical ideas to the students effectively and without distortion. In other words, the message should reach the students as intended. In fact, what learners usually expect of their mathematics teachers is good communication and clear explanation of the mathematics they teach (Backhouse et al; 1992).

Thus, mathematics teachers should teach the meanings and uses of the technical words and symbols the students are likely to come across in their learning They must discuss concepts, ask and answer questions, guide exploration, pose problems, present logical argument and critique the work of their students. In other words, they must endeavour to explain the key words and terms in the mathematics they teach.

The technical terms and symbols explained in this paper are just few of the numerous ones that abound in mathematics. In other to ensure proper communication of mathematics ideas, therefore, a careful examination of mathematics curricular and textbooks at all levels in order to identify the technical words and symbols is hereby recommended. Furthermore, production of dictionaries of mathematics for all levels of education is worthwhile and the right step in the right direction.

REFERENCES

Backhouse, J., Haggarty, L., Pirie, S., and Stratton, J., 1992. *Improving the learning of mathematics*. London: Cassel.

Binda, S. I., August, 2005. *The relationship between understanding the language of mathematics and performance in mathematics. A paper* presented at the 42nd National Conference of the Mathematical Association of Nigeria held at Nnamdi Azikwe University, Awka. *Unpublished.*

Bromme, R. and Steinbring, H. (1994). Interactive development of subject matter in the mathematics classrooms. *Educational Studies in Mathematics, 27* (3): 217 – 248.

Brown, T., 1994. Creating and knowing mathematics through language and experience. *Educational Studies in Mathematics*, 27(1): 79 – 100.

Hall, A., 2004. Mathematics and Language. Retrieved January 4, 2004 from http://mathform.org/vsarah/Discussion.sessions/cocking.html. New York: Oxford University Press.

Kreyszig, E ., 1983. *Advanced Engineering Mathematics*. (5th ed.). New Delhi, India: John Wiley(Eastern Limited). National Council on Education (1998). *Report on May/June 1994, 1995, and 1996 Senior School Certificate Examination (School Candidates).* Lagos: Author.

Neuman, J. R., 1964. *The Universal encyclopaedia of mathematics.* London: George Allen and Unwin Ltd.

Osafehinti, I. O., 1993. Mathematics language and mathematics achievement at pre-university level. *Abacus, 23* (1), 73-87.

Pimm, D., 1990. Mathematics: I can speak it fluently. In F. Ann (Ed.), *Developing Mathematical Thinking* (pp:210-246). Workinghamm, England: Addison – Wesley Ltd.

Thompson, D.R. and Rubenstein, R.N. (2002). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *Mathematics Teacher, 93*(7): 568 – 573.