A comparison of different models to characterize lactation curves of Friesians in a humid forest zone

B. K. AHUNU, J. D. KABUGA[+] & P. K. KARIKARI
(B. K. A.: Department of Animal Science, University of Ghana, Legon, Ghana; J. D. K.: Department of Animal Science, KNUST, Kumasi, Ghana; P. K. K.: Dairy Beef Cattle Research Station, KNUST, Kumasi, Ghana)

SUMMARY

A study was undertaken of 110 lactations of a herd of Friesians in the humid tropical zone of Ghana to investigate the goodness of fit and other milk production variables estimates by six published lactation curves. These were $Y_n = a - bn + cn\ ln\ (n)$, (Singh); $Y_n = n/(a + bn + cn^2)$, (Nelder); $Y_n = a - bn - ae^{-cn}$, (Cobby); $Y_n = an^be^{-cn}$, (Wood); $Y_n = an^b/cosh\ (cn)$, (Papajcsik); and $Y_n = n/ae^{-bn}$, (Jenkins); where Y_n is the milk yield on day *n* and *a*, *b* and *c* are the parameters estimated iteratively from the solution to the equations. Season of freshening did not influence peak day, but significantly influenced peak production and 300-day lactation in favour of cows freshening in the major rainy season. Lactation yield was best estimated by Wood model which also gave the highest average R^2 values as well as the closest estimate of day of peak production. However, Wood model overestimated peak yields and required the highest number of iterations for convergence. The Jenkins equation imposed a lactation curve that was mostly inconsistent with the data. Except for Jenkins, all models estimated mid to late lactation very well, but differed significantly in their abilities to estimate initial lactation. The relative rankings of the six models by various indexes ranked Singh and Wood models favourably with Nelder as intermediate while the models of Cobby, Papajcsik and Jenkins were ranked poorly.

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RÉSUMÉ

AHUNU, B. K., KABUGA, J. D. & KARIKARI, P. K.: *Une comparaison des différents modèles pour caractériser les courbes de lactation des Frisonnes dans une zone de forêt humide.* Une étude était entreprise de 110 lactation d'un troupeau des Frisonnes dans la zone tropicale humide du Ghana pour examiner la convenance d'ajustement et d'autres estimations de la variable de production laitière par six courbes de lactation publiées. Elles comprennent Y_n = a-bn + cn ln (n) (Singh); Y_n = n/ (a + bn + cn^2) (Nelder); Y_n = a-bn-ae^{-cn} (Cobby); Y_n = a n^b e^{-cn} (Wood); Y_n = a n^b/cosh (cn) (Papajcsik); et Y_n = n/a e^{-bn} (Jenkins); où Y_n représente le rendement laitier au jour *n* et *a*, alors que *b* et *c* représentent les paramètres estimés itérativement de la solution aux équations. La saison de rafraîchissement n'a pas influencé le jour de record mais influençait considérablement la production maximum et la lactation de 300-jour en faveur des vaches rafraîchissant dans la saison des pluies majeures. Le rendement de lactation était estimé le meilleur par le modéle de Wood qui enregistrait également les valeurs R^2 moyennes les plus élevées ainsi que les estimations les plus près du jour de la production maximum. Cependant, le modèle de Wood surestimait les rendements maximums et exigeàit le nombre des itérations le plus élevé pour une convergence. L'équation de Jenkins imposait une courbe de lactation qui était la plus souvent en contradiction avec les données. A l'exception de Jenkins, tous les modèles estimaient très bien du milieu à la dernière lactation mais différaient considérablement dans leurs capacités d'estimer la lactation première. Les classements relatifs des six modèles par les différents indexes classaient les modèles de Singh et Wood favorablement avec Nelder comme l'intermédiaire alors que les modèles de Cobby, Papajcsik et Jenkins étaient classés médiocrement.

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Introduction

When a known functional form of a milk yield of a lactating cow is known, the milk production at any given stage of lactation can be predicted. Such predictions, if accurate, form the basis for important decisions to cull or retain in the breeding herd. The application of functions derived in exotic data may invariably have little or no relevance to local production records which are highly subject to environmental influences. In a review of 20 different models to compare and evaluate for accuracy of prediction, Papajcsik & Bodero (1988) indicated that different models may rank differently depending on seasonal effects and levels of production. Such seasonal effects are important in the tropics and in cattle operations where there is no restricted breeding season whereby lactation could commence any time of the year. Furthermore, when exotic dairy cattle are imported to the tropics, there is usually a lowering of milk production. There is the need, therefore, to investigate their production functions, and to provide estimates of parameters for some of the known standard lactation curves which could aid managerial decisions. Consequently, prediction equations developed from available data are necessary for the local dairy enterprise.

This study aimed at examining the suitability of six published models to fit lactation curves to milk yield records of imported Friesian cows in the humid forest zone of Ghana.

Materials and methods

Data on 110 lactations investigated in the study were collected during routine herd recording at the dairy project of Kwame Nkrumah University of Science and Technology (KNUST), Kumasi. Herd recording was done on daily basis for all lactating cows. The aim and history of the management of the KNUST project have been previously described by Kabuga & Agyeman (1984) who determined the mean lactation length for the herd to be 322 days. Recent management practices have also been described by Osei, Effah-Baah & Karikari (1991). In brief, in 1974 through the assistance of the Canadian International Development Agency (CIDA), 35 Holstein-Friesian heifers and five bulls were introduced to KNUST to show that, with proper management and nutrition, exotic breeds of dairy cattle could survive and produce satisfactorily in the humid forest zone of Ghana. The rainfall distribution in the forest zone is about bimodal with nearly 55 per cent of the rains in the major rainy season (April-July), and a further 30 per cent in the minor rainy season (September-November) with the remaining months dry. Total rainfall averages about 1500 mm per annum which ensures production of a more adequate and high quality herbage throughout the year (Asare, 1970).

The management practices were such that after milking at 0530 h cattle were released to graze on pasture till about 1100 h and returned to the barns in the afternoons to avoid excessive exposure to the sun. While in the barn, animals were given freshly cut forage *ad libitum*. Predominant forages grazed or fed included elephant grass (*Pennisetum purpureum*), guinea grass (*Panicum maximum*), gamba grass (*Andropogon gayanus*), giant star grass (*Cynodon plectostachyus*) and Centro (*Centrosema pubescens*).

Animals were milked twice a day at 0530 and 1630 h. The herd was machine-milked until 1980 when hand-milking was started. Lactating animals received concentrate during milking at the rate of 1 kg of concentrate per 2.5 kg of milk produced. Cows calved throughout the year with fairly even distribution in all three seasons. Lactation records included in the analysis were those which lasted at least 210 days (about 7 months), but where lactation continued for a period longer than 300 days, only the first 300-day yields were used in the analysis. However, records shorter than 300 days were not adjusted to 300-day basis. This criterion gave the data distribution as follows: 110 lactations to 210 days, 101 lactations to 240 days, and 79 lactations each at 270 and 300 days.

Six algebraic models, all non-linear, were fitted to the milk yield data for each of the 110 individual lactations and also to the mean observed data. Individual cow's lactation curve parameter estimates for each model were obtained by using the computer software Statgraphics® (STSC, 1989). All records were fitted by using intrinsically non-linear regression techniques without linearization, since linearization of models before fitting is known to affect the error structure (Kellog, Urquhart & Ortega, 1977). The six models chosen for this study were designated as follows:

Singh: the linear cum log model (Singh & Gopal, 1982)

Nelder: the inverse polynomial function (Nelder, 1966)

Cobby: the exponential plus linear decline (Cobby & Le.Du, 1978)

Wood: the incomplete gamma function (Wood, 1967)

Papajcsik: the modified Wood equation (Papajcsik & Bodero, 1988)

Jenkins: the inverse exponential (Jenkins & Ferrell, 1984)

Table 1 describes the six models and relevant statistics.

Individual cow's parameter estimates for each model were then applied to evaluate individual production variables, namely: peak day, peak yield, and 300-day lactation. In addition, the number of iterations required for convergence, the proportion of variation explained by the fitted model (R^2), and residual error mean square (EMS) were recorded for each cow's data for each of the six models. For each model, means, standard errors

TABLE 1

Equations and Derived Traits for Six Lactation Models

Model.	*Equation (Y_n)*	*Peak day*	*Peak yield*
Singh	$a - bn + c\ ln\ (n)$	c/b	$a - c + ln\ (c/b)^b$
Nelder	$n/(a + bn + cn^2)$	$\sqrt{(a/c)}$	$1/(2\sqrt{(ac)} + b)$
Cobby	$a - bn - a\ e^{-cn}$	$c^{-1}\ ln\ (ac/b)$	$c^{-1}\ (ac - b - ln\ (ac/b)^b)$
Wood	$an^b e^{-cn}$	b/c	$a(b/c)^b e^{-b}$
Papajcsik	$an^b/cosh\ (cn)$	b/c	$2a(b/c)^b e^{-b}/(1 + e^{-2b})$
Jenkins	$n/(ae^{-bn})$	b^{-1}	$(ab)^{-1}\ e^{-1}$

*Y_n - daily milk yield

of the estimated parameters, and descriptive statistics for the milk production variables were also calculated separately for parity (1, 2 and ≥ 3), season of freshening (major rains, minor rains and dry season), and for period (year of freshening) classified as 1 (1977-1980), 2 (1981-1983), and 3 (1984 -1987). The estimated production variables were subjected to regular analysis of variance. The assumed statistical model was as follows:

$$Y_{ijklm} = \mu + T_i + S_j + P_k + M_l + \varepsilon_{ijklm}$$

where Y_{ijklm} = the observed lactation variable; μ = the overall mean; T_i = the effect of the i^{th} period of calving (i = 1, .. 3); S_j = the effect of the j^{th} season of calving (j = 1, .. 3); P_k = the effect of the k^{th} parity of calving (k = 1, 2, ≥ 3); M_l = the effect of the l^{th} model on the observation (l = 1, .. 6); and ε_{ijklm} = the random error term assumed to follow normal distribution with zero mean and constant variance. Interactions terms of the effects were not considered important.

Results and discussion

Shape of the lactation curve

Table 2 shows parameter estimates from fitting the various models to the milk yield records and Fig. 1 depicts the fitted curves. The suitability of each of the different models in describing the lactation curve depends on the initial rise in milk yield, the position of the peak, and the rate of decline (Rolands, Lucey & Russel, 1982). The lactation curve estimated by the Jenkins equation differed considerably from the curves by the other five models (Fig. 1). Also, Singh and Cobby models did not seemingly show the characteristic monotonous increase in daily yields with stage of lactation to a peak, which is then followed by a gradual decline. On the other hand, the Singh and Cobby models declined from near peak production at the onset of lactation. Table 2 shows that these two models estimated days of peak production

TABLE 2

Lactation Curve Parameter Estimates and Milk Production Variables Estimated from Pooled Lactation

	Parameter estimates			Milk production variables		
Model	*a*	*b*	*c*	*Peak day*	*Peak yield*	*300-day lactation*
Singh	13.91 ± 2.77	0.0258 ± 0.0061	-0.0394 ± 0.7546	2	13.9	2943.4
Nelder	0.682 ± 0.129	0.0450 ± 0.0039	0.00033 ± 0.00002	45	13.3	2953.7
Cobby	7.232 ± 0.262	0.0473 ± 0.0131	-0.0021 ± 0.0009	1	14.4	2983.0
Wood	9.786 ±1.888	0.1164 ± 0.0543	0.0036 ± 0.0005	32	13.1	3034.5
Papajcsik	6.177 ± 1.155	0.0332 ± 0.0495	0.0032 ± 0.0004	19	13.2	3051.7
Jenkins	2.438 ± 0.085	0.0112 ±0.0010		89	13.5	2787.8

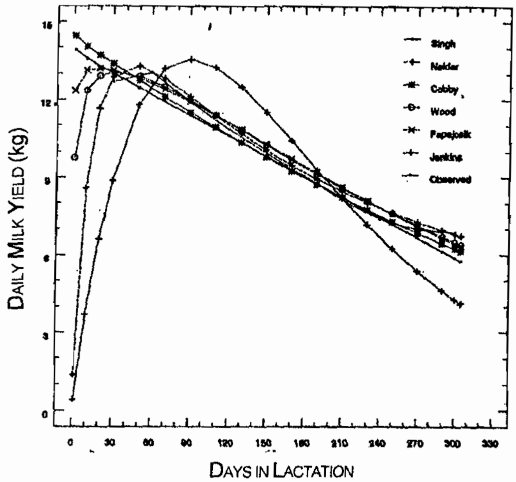

Fig. 1. Lactation models fitted to the mean observed data.

at days 2 and 1, respectively, which were rather too early to impact any noticeable effect on the overall curvature of the graph. The Jenkins model, on the other hand, estimated day of peak production rather very late (89 days), and daily milk yield dropped off more rapidly after peak production to a lower value at the end of lactation.

There were obvious differences among models in their abilities to estimate early lactation. Milk yield estimates at the onset of lactation were 14.4, 13.4 , 12.4, and 9.8 kg by the Cobby, Singh, Papajcsik and Wood models, respectively; the inverse functions, Nelder and Jenkins, estimated, very low initial productions of only 1.4 and 0.4 kg, respectively (Fig. 1). The Cobby and Singh models tended to overestimate initial production while the Nelder and Jenkins models underestimated it. All models, with the exception of the Jenkins, estimated mid to late lactation very closely to observed values, and differences among models were more subtle for mid-lactation yields. For the Jenkins model, however, daily production in early and late lactations were grossly underestimated while mid-lactation yields were overestimated.

In individual cases, however, the best fitting curve was not always the same model; different models often varied in their fits and ranked differently for individual lactations. These differences were due mainly to variations in the positions in the peaks and rate of decline. These observations agree with those of Singh & Bhat (1978) and Rolands *et al.* (1982). In general, the lactation curve estimated by the Jenkins model differed markedly from the curves estimated from the other five models. It consistently gave poorer fits to the individual lactation yields as evidenced by the low average R^2 value of only 37.2 per cent and high residual mean squares (Table 3). These observations agree with those of Hohenboken, Dudley & Moody (1992) who stated that in general the Jenkins equation imposed a shape on the lactation curve that was predominantly inconsistent with milk yield data. The remaining five models gave fairly good fits to the milk yield data, with the Cobby model being slightly less

TABLE 3

Mean Computation Indices of the Fitted Models to Individual Cow Data

Model	R^2	*EMS*[1]	*Iterations*
Singh	84.4 ± 1.4[a]	1.255 ± 0.146[b]	4.14 ± 0.03[c]
Nelder	83.4 ± 1.3[a]	1.476 ± 0.189[b]	4.67 ± 0.08[c]
Cobby	80.4 ± 1.8[a]	1.679 ± 0.199[b]	7.35 ± 0.43[b]
Wood	84.5 ± 1.4[a]	1.251 ± 0.149[b]	12.67± 0.67[a]
Papajcsik	84.1 ± 1.4[a]	1.274 ± 0.149[b]	11.22 ±0.64[a]
Jenkins	37.2 ± 3.3[b]	6.014 ± 0.566[b]	4.95 ± 0.07[a]

[1]EMS = residual error mean square

efficient (80.4 per cent) than the remaining four models which had higher R^2 values ($83.4 \leq R^2 \leq 84.5$ per cent). Thus, with the exception of the Jenkins model which has been noted for its inconsistencies (Hohenboken *et al.* 1992), the result generally show good fit of the algebraic models to the milk yield data. This suggests that the shape of the lactation curves of these animals has been unaffected by the stressful environmental conditions of the humid tropics.

Milk production variables

Table 4 shows the outline of analysis of variance for the milk production variables and Table 5 shows the means of the variables analyzed. Season effect was not significant in both peak day and peak yield analyses. Also, the models did not differ significantly in estimating 300-day yield. Period effects were such that in the early beginnings of the programme when only the imported animals were in production, milk yields were high. The high production era was associated with high peak yields even though peak day remained relatively unchanged. The period effects also partly reflect the year-to-year variation in standard of management and decline in level of nutrition (Kabuga & Agyeman, 1984).

TABLE 4

Analysis of Variance of Milk Production Variables

		Mean squares		
Source of variation	*d.f.*	*Peak day*	*Peak yield*	*300-day yield*
Period	2	466.04	5343.15**	2.9886×10^{8}**
Season	2	1911.87	146.58	8.4431×10^{6}**
Parity	2	5200.52**	1009.62**	1.8334×10^{7}**
Model	5	74038.22**	549.07**	1.1497×10^{6}
Residual	648	679.42	52.82	0.6390×10^{6}

**Significant, $P<0.01$.

While seasonal effects did not significantly influence peak day, there was the tendency for peak yield to increase during the major rainy season, with a gradual shortening of peak day through the minor rains to the dry season. These were paralleled by significant increases in 300-day lactation during the major rains and gradual decline in 300-day lactation through the dry season. These observations can be explained by the fact that during the rainy seasons, luxuriant forage enabled the cows freshening to produce near optimum levels, as their feeding requirements were expected to be more readily met. Lactating cows, therefore, had higher production levels though not necessarily earlier than cows freshening in the drier seasons when herbage quality was comparatively poorer (Asare, 1970).

Parity effects were such that, compared to older dams, heifer dams reached peak production late and at a lower plane of production, resulting in a significantly lower 300-day lactation. These agree with results reported by others (Wood, 1967; Nwakalor *et al.*, 1988; Collins-Lusweti & Mpofu, 1989; Hohenboken *et al.*, 1992). The six models did not differ significantly in their abilities to predict 300-day lactation. However, there was a slight tendency for the Jenkins and Nelder equations to underestimate total lactation while the Papajcsik, Cobby and Singh models showed some propensity to overestimate 300-day lactation.

Peak yields estimated by all six models were similar, ranging from only 13.1 to 14.4 kg (Table 2). Even though there were no significant differences among the six models in predicting 300-day lactation, there was the tendency for the inverse functions (Nelder and Jenkins) to underestimate total production, while the Papajcsik, Cobby and Singh models tended to slightly overestimate total

TABLE 5

Least Squares Means of the Milk Production Variables

		Peak day	*Peak yield*	*300-day yield*
Period				
1	264	41.8 ± 2.2	19.9 ± 0.6[a]	4245.7 ± 65.8[a]
2	186	41.3 ± 2.5	12.5 ± 0.5[b]	2347.5 ± 48.2[b]
3	210	42.8 ± 2.5	9.4 ± 0.5[c]	1918.7 ± 44.7[c]
Season				
1	222	39.7 ± 2.5	16.0 ± 0.7[a]	3421.8 ± 98.4[a]
2	270	44.1 ± 2.1	15.1 ± 0.5[a]	2895.5 ± 77.2[b]
3	168	41.5 ± 2.6	11.6 ± 0.6[b]	2494.0 ± 86.3[c]
Parity				
1	180	47.4 ± 2.8[a]	11.0 ± 0.6[b]	2402.0 ± 69.4[b]
2	204	41.5 ± 2.5[ab]	16.0 ± 0.7[a]	3625.4 ± 102.6[a]
3	276	38.8 ± 2.0[b]	15.7 ± 0.5[a]	3122.9 ± 83.4[a]
Model				
Singh	110	43.8 ± 3.5[b]	15.1 ± 0.7[a]	3031.9 ± 128.2
Nelder	110	41.0 ± 2.4[bc]	10.4 ± 1.1[b]	2905.2 ± 130.6
Cobby	110	5.1 ± 0.1[d]	14.7 ± 0.6[a]	3036.9 ± 131.8
Wood	110	44.0 ± 3.1[b]	16.4 ± 1.0[a]	3006.1 ± 129.0
Papajcsik	110	32.3 ± 2.9[c]	16.4 ± 0.9[a]	3050.7 ± 131.4
Jenkins	110	85.6 ± 1.2[a]	14.0 ± 0.6[a]	2791.3 ± 122.5
Overall	660	42.0 ± 1.0	14.5 ± 0.3	2970.3 ± 30.1

[a,b,c,d] subclass means bearing different superscripts are significantly different (P<0.05).

production. The Wood model's estimate of total production was closest to observed data. Considering the number of iterations required for convergence as an index of the difficulty of computation (Table 3), the Singh, Nelder, and Jenkins models were relatively easy to fit, requiring on the average about five iterations. The Cobby model was intermediate, requiring on the average 7.4 iterations which was significantly higher than the first three. The Papajcsik and Wood models did not converge readily, averaging 11.2 and 12.6 iterations, respectively. These were significantly higher than the other four models.

Milk production variables estimated from individual application of the six models to separate data from individual cows (Table 5) differed in many instances from those estimated simultaneously from all 110 cows (Table 2). This may have arisen because daily yields of some individual cows did not closely conform to the shape of the typical lactation curve, leading in some cases to unreasonably high or sometimes negative estimates of peak day. This was particularly so in estimates of peak yield by Singh. All such negative estimates were, therefore, set to day 1.

Peak day estimates by Jenkins were the highest in both approaches to the data analyses, with values for the Nelder, Wood and Papajcsik models being intermediate while Cobby estimates peaked very early. Except for Nelder, estimates from individual plots for peak production were slightly higher than those from the pooled data analyses. In 300-day lactation analyses, estimates by Papajcsik and Jenkins were the most consistent, giving almost exact values in both approaches to the data analyses.

However, compared to estimates from individual application, pooled data estimates were slightly higher for the Cobby and Singh models, but slightly lower for the Nelder and Wood models. In both approaches, the inverse functions (Nelder and Jenkins) underestimated 300-day lactation, but this was more pronounced in the Jenkins model. Conversely, the Cobby, Wood, and Papajcsik models tended to overestimate 300-day lactation in both instances, while the Singh model which overestimated individual lactation variables now underestimated 300-day lactation in the pooled data analysis. This may be because negative estimates of peak days in this model were set to day 1.

Table 6 presents correlations established among the three milk production variables by the six models. Peak day estimates of production for the Wood and Papajcsik models were highly correlated ($r = 0.78$). Estimates by the Singh model were also moderately correlated with those of the Wood and Papajcsik models ($0.44 \leq r \leq 0.58$). Estimates by Nelder and Wood as well as by Nelder and Papajcsik were moderately correlated ($r = 0.55$). Peak day estimates by Jenkins did not

TABLE 6

Correlations Among Milk Production Variables Estimated from Six Lactation Curve Equations[1]

	Peak yield					
Peak day						
Singh		0.73	0.75	0.56	0.60	0.71
Nelder	0.24*		0.96	0.71	0.67	0.93
Cobby	0.13^{ns}	0.25		0.72	0.72	0.94
Wood	0.44	0.55	0.20*		0.88	0.69
Papajcsik	0.58	0.56	0.14^{ns}	0.78		0.66
Jenkins	0.17^{ns}	0.02^{ns}	0.29	0.14^{ns}	0.08^{ns}	
300-day yield						
Nelder	0.92					
Cobby	0.95	0.92				
Wood	0.92	0.90	0.92			
Papajcsik	0.94	0.92	0.94	0.96		
Jenkins	0.97	0.95	0.96	0.95	0.96	

ns = not significant; * = significant (P<0.05); all other correlations are highly significant (P<0.01).

correlate with estimates by other models. Correlations among peak yields were all moderate to high ($0.56 \leq r \leq 0.96$) and highly significant while 300-day lactations were all highly and significantly correlated ($0.90 \leq r \leq 0.97$). In particular, estimates by the Wood and Papajcsik models were more closely associated for all three variables while peak yield estimates by Nelder and Cobby associated closely with estimates by Jenkins. The correlations observed among the production variables were slightly stronger than those observed by Hohenboken *et al.* (1992), but this may be because different lactation models were used in their analysis or that they worked with milk yield data in beef and dairy × beef cattle data. The implications for these observations, as noted also by Hohenboken *et al.* (1992) who had similar results, are that each of the six models would have ranked the 110 cows similarly for estimated 300-day lactation but not for time and level of peak yield.

Ranking of models

The six models were compared by ranking them with overall average R^2, the number of iterations required for convergence, and closeness of the three milk production variables (peak day, peak yield, and 300-day lactation) to observed data. Since the overall objective of a model is to predict lactation yield, it was felt that the ability of a model to predict closely to observed data and goodness of fit (R^2) might be weighted favourably. On the other hand, with the use of computers, the number of iterations required for convergence might not be considered too important a statistic. For this reason, the following five indexes were considered:

Index 1. Ranking based on the two most important measures (R^2 and predicted 300-day lactation).

Index 2. Ranking of models based on magnitude of the five enumerated statistics.

Index 3. Ranking of models based on significant differences between the statistics, (models were assigned different ranks only when they differed significantly, $P < 0.05$).

Index 4. Weighted ranking (as for *Index 2* but statistics were assigned weight relative to their importance: rankings for R^2 and 300-day yield were weighted 2 ×, while ranking for number of iterations was weighted 0.5. Weight for other statistics remained unity).

Index 5. Weighted ranking for *Index 3.*

Table 7 shows the relative rankings for the various indexes. The Singh model was ranked the

TABLE 7

Rank[1] *Order of the Six Lactation Curves for the Derived Indexes*

Model	*Index1*	*Index2*	*Index3*	*Index4*	*Index5*	*Aggregate*
Singh	2	1	1	2	1	7
Nelder	3	3	2	3	3	14
Cobby	5	5	5.5	4	5	24.5
Wood	1	2	3	1	2	9
Papajcsik	4	6	4	5	4	23
Jenkins	6	4	5.5	6	6	27.5

[1]Rank: 1 = Best, ..., 6 = Worst.

best by three of the five indexes. It was closely followed by the Wood model which was ranked best by two of the indexes and was also ranked the second best by two others. Nelder was a close third in most of the rankings. The remaining three models, Cobby, Papajcsik, and Jenkins did not rank favourably, especially the Jenkins model which was ranked poorly by nearly all five indexes.

Conclusion

Of the six models considered, the Jenkins model was the least appropriate for describing lactation curve of Friesians under study. The Singh and Wood models were about equally appropriate, but the Wood model gave the best overall fit (R^2) as well as the closest predicted 300-day lactation to observed data which may be regarded as the two most important features in successfully fitting a lactation curve. Moreover, considering the tendency of the Singh model to sometimes result in negative estimates of peak day, one may prefer the Wood model.

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