Coupled Fixed Points Theorem for Mappings Satisfying a Contractive Condition of Integral Type in Cauchy Spaces

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ORIGINAL RESEARCH

Abstract- The contractive-type coupled fixed point theory is a generalization of Banach contraction theory. This research analyses the existence and uniqueness of mappings defined on Cauchy metric spaces via coupled fixed point theorem which satisfies a contractive inequality of integral-type. Furthermore, it generalizes contractive inequality of integral-type of fixed point to coupled fixed point theorem as an improvement to available research in literature. Some illustrative examples to back up our claims are included.

Keywords- integral type, fixed point, contractive mappings, Cauchy metric space.

1 Introduction

Recently, the study of fixed point and common fixed point of mappings which satisfies contractive conditions of integral type has gained several momentums from researchers in the area of study. Aliouche (2006) introduced the notion of a common fixed point for compatible mappings in symmetric spaces of integral type, which was later supported by the work of Jachymski (2009) as a remark on contractive conditions of integral type.

Motivated by the works in Branciari (2002), Altun, Turkoglu and Rhoades (2007), Djoudi and Merghadi (2008), Jachymski (2009), Beygmohammadi and Razani (2010), this work introduce a new coupled fixed points theorem of contractive mappings of integral type and examines its existence and uniqueness in Cauchy spaces. Also, illustrations to complement our claims were given.

2 Preliminary Notes

In this part of the work, we will consider some succeeding definition and theorems.

Theorem 1 (Agarwal *et al.*, 2008): Let $(\mathrm{X},\leq)$ be a partially ordered set and d a metric in χ such that (X,d) is a Cauchy metric space. If there is a non-decreasing function $\Psi:[0,\infty)\to[0,\infty)$ with $\lim_{n\to\infty}\Psi^n(t)=0$ for each $t>0$ and suppose $\Pi:\mathrm{X}\to\mathrm{X}$ is a non-decreasing mapping with

$$d(\Pi\theta,\Pi\lambda)\leq\Psi\big(d(\theta,\lambda)\big),\forall\,\theta\geq\lambda$$

Theorem 2 (Branciari, 2002): Let (X,d) be a complete metric space, $c\in[0,1)$, and let $\Pi:\mathrm{X}\to\mathrm{X}$ be a mapping such that for each $\theta,\lambda\in\mathrm{X}$,

$$\int_0^{d(\Pi\theta,\Pi\lambda)}\Psi(t)dt\leq c\int_0^{d(\theta,\lambda)}\Psi(t)dt$$

Where $\Psi:[0,+\infty)\to[0,+\infty]$ is a Lebesgue-integrable mapping which is summable on $[0,+\infty)$, and such that for each $\varepsilon>0$, $\int_0^{\varepsilon}\psi(t)dt>0$; then Π has a unique fixed point $\alpha\in\mathrm{X}$ such that for each $\theta\in\mathrm{X}$, $\lim_{n\to+\infty}\Pi^n\theta=\alpha$.

Definition 1 (Gnana-Bhaskar & Lakshmikantham, 2009): An element $(\theta,\lambda)\in\mathrm{X}^2$ is called a coupled fixed point of the mapping Π if;

$$\Pi(\theta,\lambda)=\theta,\Pi(\lambda,\theta)=\lambda$$

3 Main Results

This part of the research presents the main results based on the generalization of fixed-point theorem of integral type to coupled fixed points.

Theorem 3: Let (X,d) be a Cauchy space, $c\in[0,1]$, and let $\Pi:\mathrm{X}^2\to\mathrm{X}$ be a mapping such that for $\theta,\lambda\in\mathrm{X}$,

$$\int_0^{d(\Pi(\theta,\lambda),\ \Pi(\lambda,\theta))}\mu(t)dt\leq c\int_0^{d(\theta,\lambda)}\mu(t)dt \qquad (1)$$

and

$$\int_0^{d(\Pi(\lambda,\theta),\Pi(\theta,\lambda))}\mu(t)dt\leq\int_0^{d(\lambda,\theta)}\mu(t)dt \qquad (2)$$

where $\mu:[0,+\infty]\to[0,+\infty]$ is the Lebesgue integrable mapping which is summable on every compact subset of $[0,+\infty]$ such that for each $\theta,\ \lambda\in\mathrm{X}$, $\lim_{n\to+\infty}\Pi^n(\theta,\lambda)=\alpha$ and $\lim_{n\to+\infty}\Pi^n(\lambda,\theta)=\beta$.

Proof:

Step 1

We have;

$$\int_0^{d(\Pi^n(\theta,\lambda),\Pi^{n+1}(\theta,\lambda))}\mu(t)dt\leq C^n\int_0^{d(\theta,\ \Pi(\theta,\lambda))}\mu(t)dt \qquad (3)$$

On iterating (1) n times;

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$$\int_0^{d(\Pi^n(\theta,\lambda),\Pi^{n+1}(\theta,\lambda))}\mu(t)dt \le c\int_0^{d(\Pi^{n-1}(\theta,\lambda),\ \Pi^n(\theta,\lambda))}\mu(t)dt \le c^n\int_0^{d(\theta,\ \Pi(\theta,\lambda))}\mu(t)dt \quad (4)$$

Consequently, for $C \in [0,1]$, we have;

$$\int_0^{d(\Pi^n(\theta,\lambda),\ \Pi^{n+1}(\theta,\lambda))}\mu(t)dt \to 0+ \quad as\ n \to +\infty \quad (5)$$

Similarly,

$$\int_0^{d(\Pi^n(\lambda,\theta),\ \Pi^{n+1}(\lambda,\theta))}\mu(t)dt \to 0+ \quad as\ n \to +\infty \quad (6)$$

Step 2

We have $d(\Pi^n(\theta,\lambda),\ \Pi^{n+1}(\theta,\lambda)) \to 0 \quad as\ n \to +\infty$. Suppose that

$$\lim_{n\to+\infty}\sup d(\Pi^n(\theta,\lambda),\Pi^{n+1}(\theta,\lambda)) = \varepsilon > 0 \quad (7)$$

Hence, there exists $r_\varepsilon \in \mathbb{N}$ and $\{\Pi^{n_r}(\theta,\lambda)\}_{r\ge r_\varepsilon}$ such that $d(\Pi^{n_r}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda)) \to \varepsilon > 0\ as\ r \to +\infty$ and $d(\Pi^{n_r}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda)) \ge \varepsilon/2\ for\ r \ge r_\varepsilon$, then by Step 1, we obtain the following:

$$\lim_{r\to+\infty}\int_0^{d(\Pi^{n_r}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda))}\mu(t)dt \ge \int_0^{\varepsilon/2}\mu(t)dt > 0 \quad (8)$$

$$\lim_{r\to+\infty}\int_0^{d(\Pi^{n_r}(\lambda,\theta),\Pi^{n_r+1}(\lambda,\theta))}\mu(t)dt \ge \int_0^{\varepsilon/2}\mu(t)dt > 0 \quad (9)$$

Step 3

For every $\theta \in X(\Pi^n(\theta,\lambda))_{n\in\mathbb{N}}$ being a Cauchy sequence, then $\forall\ \ \varepsilon > 0$,

$\exists\ r_\varepsilon\ \in \mathbb{N}\ |\ \forall\ m,n \in \mathbb{N}, m > n > r_\varepsilon$, then;

$$d(\Pi^m(\theta,\lambda),\Pi^n(\theta,\lambda)) < \varepsilon \quad (10)$$

If there exists $\varepsilon > 0$ such that for $r \in \mathbb{N}$, there are $m_r, n_r \in \mathbb{N}\ with\ m_r > n_r > r$, such that $d(\Pi^{m_r}(\theta,\lambda),\ \Pi^{n_r}(\theta,\lambda)) \ge \varepsilon,\ then\ \{m_r\}_{r\in\mathbb{N}}$ and $\{n_r\}_{r\in\mathbb{N}}$ are sequences such that m_r is minimal with $d(\Pi^{m_r}(\theta,\lambda),\ \Pi^{n_r}(\theta,\lambda)) \ge \varepsilon$ but $d(\Pi^{r_1}(\theta,\lambda),\ \Pi^{n_r}(\theta,\lambda)) < \varepsilon$ for each $r_1 \in \{(n_r+1),\dots,(m_r-1)\}$.

Now we analyse the properties of $d(\Pi^{m_r}(\theta,\lambda),\ \Pi^{n_r}(\theta,\lambda))$ and

$$d(\Pi^{m_r+1}(\theta,\lambda),\quad \Pi^{n_r+1}(\theta,\lambda))$$

Firstly, we obtain $d(\Pi^{m_r}(\theta,\lambda),\ \Pi^{n_r}(\theta,\lambda)) \to \varepsilon +\ as\ r \to +\infty$, and by applying triangular inequality and the result of Step 2;

$$\varepsilon \le d(\Pi^{m_r}(\theta,\lambda),\Pi^{n_r}(\theta,\lambda)) \le d(\Pi^{m_r}(\theta,\lambda),\Pi^{m_r-1}(\theta,\lambda)) + d(\Pi^{m_r-1}(\theta,\lambda),\Pi^{n_r}(\theta,\lambda)) < d(\Pi^{m_r}(\theta,\lambda),\Pi^{m_r-1}(\theta,\lambda)) + \varepsilon \to \varepsilon + \text{as } r \to +\infty \quad (11)$$

Therefore, there exists $r \in \mathbb{N}$ such that $r > \sigma$ and $d(\Pi^{m_r+1}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda)) < \varepsilon$, that is, there exists $\{r_k\}_{k\in\mathbb{N}} \subseteq \mathbb{N}$ such that $d\left(\Pi^{m_{r_k}+1}(\theta,\lambda),\Pi^{n_{r_k}+1}(\theta,\lambda)\right) \ge \varepsilon$, then;

$$\begin{aligned}\varepsilon \le d\left(\Pi^{m_{r_k}+1}(\theta,\lambda),\Pi^{n_{r_k}+1}(\theta,\lambda)\right) &\le d\left(\Pi^{m_{r_k}+1}(\theta,\lambda),\Pi^{m_{r_k}}(\theta,\lambda)\right)\\ &+ d(\Pi^{m_{r_k}}(\theta,\lambda),\Pi^{n_{r_k}}(\theta,\lambda))\\ &+ d\left(\Pi^{n_{r_k}}(\theta,\lambda),\Pi^{n_{r_k}+1}(\theta,\lambda)\right)\\ &\to \varepsilon\ ask \quad (12)\end{aligned}$$

And from Theorem 3.1;

$$\int_0^{d\left(\Pi^{m_{r_k}+1}(\theta,\lambda),\Pi^{n_{r_k}+1}(\theta,\lambda)\right)}\mu(t)dt \le c\int_0^{d\left(\Pi^{m_{r_k}}(\theta,\lambda),\Pi^{n_{r_k}}(\theta,\lambda)\right)}\mu(t)dt \quad (13)$$

Letting $k \to +\infty$ in both sides of (13), we obtain $\int_0^\varepsilon \mu(t)dt \le c\int_0^\varepsilon \mu(t)dt$ which is a contraction for $c \in [0,)$ and the integral being positive. Hence for a certain $\sigma \in \mathbb{N}$, then $d(\Pi^{m_r+1}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda)) < \varepsilon\ \forall\ r > \sigma$.

Finally, we show the stronger property that there exists a $\delta_\varepsilon \in [0,\varepsilon)$ and $r_\varepsilon \in \mathbb{N}$ such that $r > r_\varepsilon$, we have $d(\Pi^{m_r+1}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda)) < \varepsilon - \delta_\varepsilon$. Suppose there exists $\{r_k\}_{k\in\mathbb{N}}$ such that $d\left(\Pi^{m_{r_k}+1}(\theta,\lambda),\Pi^{n_{r_k}+1}(\theta,\lambda)\right) \to \varepsilon\ as\ k \to +\infty$, then going by (13), we can show that$\{\Pi^n(\theta,\lambda)\}_{n\in\mathbb{N}}$ is Cauchy for each natural number $r > r_\varepsilon$, hence;

$$\begin{aligned}&\varepsilon \le d(\Pi^{m_r}(\theta,\lambda),\Pi^{n_r}(\theta,\lambda))\\ &\le d(\Pi^{m_r}(\theta,\lambda),\Pi^{m_r+1}(\theta,\lambda)) + d(\Pi^{m_r+1}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda))\\ &+ d(\Pi^{n_r+1}(\theta,\lambda),\Pi^{n_r}(\theta,\lambda))\\ &< d(\Pi^{m_r}(\theta,\lambda),\Pi^{m_r+1}(\theta,\lambda)) + (\varepsilon - \delta_\varepsilon)\\ &+ d(\Pi^{n_r}(\theta,\lambda),\Pi^{n_r+1}(\theta,\lambda)) \quad (14)\end{aligned}$$

Thus, $\varepsilon < \varepsilon - \delta_\varepsilon$ this is a contradiction. This proves Step 3.

Step 4

Existence of a fixed point. Since (χ, d) is a Cauchy Metric space, then there exists points $\alpha,\beta\ \in \chi$ such that $\alpha = \lim_{n\to+\infty}\Pi^n(\theta,\lambda)$ and $\beta = \lim_{n\to+\infty}\Pi^n(\theta,\lambda)$, furthermore, α and β are fixed points. Now, suppose that $d(\alpha,\Pi(\alpha,\beta)) > 0$ and $d(\beta,\Pi(\beta,\alpha)) > 0$, thus

$$d(\alpha,\Pi(\alpha,\beta)) \le d(\alpha,\Pi^{n+1}(\theta,\lambda)) + d(\Pi^{n+1}(\theta,\lambda),\Pi(\alpha,\beta)) \to 0\ as\ n \to +\infty \quad (15)$$

By implication, both $d(\alpha,\Pi^{n+1}(\theta,\lambda))$ and $d(\Pi^{n+1}(\theta,\lambda),\Pi(\alpha,\beta)) \to 0$ as $n \to +\infty$, we now have;

$$\int_0^{d\left(\Pi^{n+1}(\theta,\lambda),\Pi(\alpha,\beta)\right)}\mu(t)dt \le c\int_0^{d(\Pi^n(\theta,\lambda),\alpha)}\mu(t)dt \to 0\quad as\ n \to +\infty \quad (16)$$

Now if $d(\Pi^{n+1}(\theta,\lambda),\Pi(\alpha,\beta))$ does not converge to 0 as $n \to +\infty$, then there exists $\{\Pi^{n_r+1}(\theta,\lambda)\}_{r\in\mathbb{N}} \subseteq \{\Pi^{n+1}(\theta,\lambda)\}_{n\in\mathbb{N}}$ such that $d((\Pi^{n_r+1}),\Pi(\alpha,\beta)) \ge \varepsilon$ for a certain $\varepsilon > 0$; thus we obtain the following contradictions;

$$\int_0^\varepsilon \mu(t)dt \le \int_0^{d\left(\Pi^{n+1}(\theta,\lambda),\Pi(\alpha,\beta)\right)}\mu(t)dt \to 0\ as\ r \to +\infty \quad (17)$$

Similarly,

$$\int_0^\varepsilon \mu(t)dt \le \int_0^{d\left(\Pi^{n+1}(\lambda,\theta),\Pi(\beta,\alpha)\right)}\mu(t)dt \to 0\ as\ r \to +\infty \quad (18)$$

Step 5

Uniqueness of the fixed point. Suppose we have two distinct points $\alpha,\beta \in \chi$ such that $\Pi(\alpha,\beta) = \alpha$ and $\Pi(\beta,\alpha) = \beta$, then going by (1), we obtain;

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$$\int_0^{d(\alpha,\beta)} \mu(t)dt = \int_0^{d(\Pi(\alpha,\beta),\Pi(\beta,\alpha))} \mu(t)dt \leq C\int_0^{d(\alpha,\beta)} \mu(t)dt < \int_0^{d(\alpha,\beta)} \mu(t)dt \quad (19)$$

And similarly,

$$\int_0^{d(\beta,\alpha)} \mu(t)dt = \int_0^{d(\Pi(\beta,\alpha),\Pi(\alpha,\beta))} \mu(t)dt \leq C\int_0^{d(\beta,\alpha)} \mu(t)dt < \int_0^{d(\beta,\alpha)} \mu(t)dt \quad (20)$$

Finally, this shows that $\theta,\lambda \in X$, $\lim_{n\to+\infty} \Pi^n(\theta,\lambda) = \alpha = \Pi(\alpha,\beta)$ and $\lim_{n\to+\infty} \Pi^n(\lambda,\beta) = \beta = \Pi(\beta,\alpha)$, which completes the proof.

4 EXAMPLES

In this part of the research, we give some examples and concluding remarks on the contractive mappings of integral type, which clarify the connection between our results and the existing one.

Example 1: Let $\Pi: \mathbb{N}^2 \to \mathbb{N}$ *and* $\mu: \mathbb{R}^+ \to \mathbb{R}^+$ be defined by;

$$\Pi(\theta,\lambda) = \begin{cases} 1 \text{ if } \theta \neq 1 \text{ and } \lambda \neq 1, \\ 2 \text{ if } \theta = 1 \text{ and } \lambda = 1; \end{cases}$$

$$\mu(t) = \begin{cases} e^{1/(1-t)} & \text{if } t>1 \\ 0 & \text{if } t \in [0,1] \end{cases} \quad (21)$$

And let $d: \mathbb{N}^2 \to \mathbb{R}^+$ be the Euclidean metric so that $(\mathbb{N}, d)$ becomes a complete metris space. Now, for each $\theta,\lambda \in \mathbb{N}$, $d(\Pi(\theta,\lambda),\Pi(\lambda,\theta)) \leq 1$, for an arbitrary $c[0,1)$.

$$\int_0^{d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))} \mu(t)dt \leq \int_0^1 \mu(t)dt \leq \int_0^{d(\theta,\lambda)} \mu(t)dt \leq 0 \quad (22)$$

Thus, (1) is satisfied $\forall\, C \in [0,1)$, and Π has no fixed point.

Example 2: Let $\chi := \chi^2 \to \chi$ defined by;

$$\Pi(\theta,\lambda) = \begin{cases} \dfrac{1}{n_1+1} & \text{if } \theta = \dfrac{1}{n_1} \forall\, n_1 \in \mathbb{N}, \\ 0 & \text{if } \theta = 0 \end{cases} \quad (23)$$

Then (1) is satisfied with $\mu(t) = t^{1/t-2}[1-\log t] \forall t > 0$, $\mu(0) = 0$ *and* $c = 1/2$. In this regard, one has $\int_0^{t_1} \mu(t)dt = t_1^{1/t_1}$, so that (1) is equivalent to

$$d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))^{1/d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))} \leq Cd(\theta,\lambda)^{1/d(\theta,\lambda)} \forall\, \theta \neq \lambda \quad (24)$$

Next is to prove the validity of (24).

Let $n_1, n_2 \in \mathbb{N}$ with $n_2 > n_1$ and let $\theta = 1/n_2$ *and* $\lambda = 1/n_1$, then we have;

$$d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))^{1/d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))} = \left|\frac{1}{n_2+1} - \frac{1}{n_1+1}\right|^{1/[1/(n_2+1) - 1/(n_1+1)]} = \left[\frac{n_2-n_1}{(n_1+1)(n_2+1)}\right]^{(n_1+1)(n_2+1)/(n_2-n_1)}$$

On the other hand;

$$d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))^{1/d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))} = \left|\frac{1}{n_1} - \frac{1}{n_2}\right|^{1/|1/n_1 - 1/n_2|} = \left[\frac{n_2-n_1}{n_1n_2}\right]^{n_1n_2/(n_2-n_1)}$$

this shows that;

$$\left[\frac{n_2-n_1}{(n_1+1)(n_2+1)}\right]^{(n_1+1)(n_2+1)/(n_2-n_1)} \leq \frac{1}{2}\left[\frac{n_2-n_1}{n_1n_2}\right]^{n_1n_2/(n_2-n_1)}$$

equivalently,

$$\left[\frac{n_2-n_1}{(n_1+1)(n_2+1)}\right]^{(n_1+1)(n_2+1)/(n_2-n_1)} \cdot \left[\frac{n_1n_2}{(n_1+1)(n_2+1)}\right]^{n_1n_2/(n_2-n_1)} \leq \frac{1}{2} \quad (25)$$

Inequality (25) is indeed true; analysing the second part, we have

$$\left[\frac{n_1n_2}{(n_1+1)(n_2+1)}\right]^{n_1n_2/(n_2-n_1)} \leq 1 \quad (26)$$

Since $n_1n_2 < (n_1+1)(n_2+1)$ and $n_1n_2/(n_2-n_1) > 0$, and also

$$\left[\frac{n_2-n_1}{(n_1+1)(n_2+1)}\right]^{(n_1+1)(n_2+1)/(n_2-n_1)} \leq \frac{1}{2} \quad (27)$$

Since $(n_1+1)(n_2+1)$, while the exponent is greater than 1 (since for all $n_1, n_2 \in \mathbb{N}$, $n_1+n_2+1 > n_2-n_1$ is trivially satisfied). On the other hand, taking $\theta = \frac{1}{n} (n \in \mathbb{N})$ and $\lambda = 0$, we have;

$$d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))^{1/d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))} = \left[\frac{1}{n+1}\right]^{n+1} \leq \frac{1}{2}\left[\frac{1}{n}\right]^n = \frac{1}{2}d(\theta,\lambda)^{1/d(\theta,\lambda)} \quad (28)$$

Because for each $n \in \mathbb{N}$, we have;

$$\left[\frac{n}{n+1}\right]^n \cdot \frac{1}{n+1} \leq \frac{1}{2} \quad (29)$$

Since $n/(n+1) < 1$ and $1/(n+1) \leq 1/2$.

Therefore, Π satisfies condition (24) with $c = 1/2$ and hence (1) with c and for μ defined by $\mu(t) = t^{1/t-2}[1-\log t]$ for $t > 0$ and $\mu(0) = 0$, but

$$Sup_{\{\theta,\lambda\in X|\theta\neq\lambda\}} \frac{d(\Pi(\theta,\lambda),\Pi(\lambda,\theta))}{d(\theta,\lambda)} = 1 \quad (30)$$

Thus, it is not a Banach contraction.

5 CONCLUSION

In this research, as an extension to the work of Branciari (2002), we introduced an integral type coupled fixed point theorem and examined its existence and uniqueness in Cauchy spaces. Hence, the obtained results give a valid proof of the method and also, examples are given to validate our claims. Consequently, as a recommendation for further works, the results herein can be extended to other spaces like Hilbert and Sobolev spaces and also to tripled fixed point theorems.

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