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DETERMINATION OF BENCHMARKS STABILITY WITHIN AHMADU BELLO UNIVERSITY, ZARIA, NIGERIA

Ojigi, M. L.
Department of Surveying & Geoinformatics,
Federal University of Technology, Minna, Nigeria
E-Mail: lmojigi@yahoo.com , lmojigi@futminna.edu.ng

ABSTRACT

Heights of six geodetic benchmarks over a total distance of 8.6km at the Ahmadu Bello University (ABU), Zaria, Nigeria were recomputed and analysed using least squares adjustment technique. The network computations were tied to two fix primary reference pillars situated outside the campus. The two-tail Chi-square hypothesis test of the a-posterior variances of the benchmark heights at 5% and 1% levels of significance were carried out in order to determine the reliability of the computed heights. This is meant to establish whether the observations used truly represent the heights of the Benchmarks in the study area. The results showed that, the estimated variances for fore, back and mean (of fore and back) leveling computations were 3.17mm, 1.77mm and 2.93mm respectively. The tests of these variances showed that the results, obtained from fore and mean leveling computations, do not truly represent the heights of the benchmarks. On the other hand, the back leveling computed variance passed the confidence limits test which tends to infer that the heights obtained significantly represent the heights of the benchmarks. That is, only the back leveling observation truly represent the benchmarks' heights in the study area. Four out of the six benchmarks investigated for stability within the period of study showed displacement values ranging between 4.14mm and 12.42mm over a period of 17years. This may not be alarming but effort should be made toward the continuous monitoring of the benchmarks.

Key words: Benchmark Stability, Leveling Network, Benchmark Monitoring and Least Squares Technique

Introduction

Studies related to vertical displacement of survey controls and engineering structures require precise leveling measurements for optimal accuracy (Lachepelle, 1979). The specifications for field procedures in leveling are usually based on a minimum permissible discrepancy between two independent (Back and fore) leveling of the same time (Blachut, *et al,* 1979).

In survey measurements, it is practically impossible to obtain values which are absolutely correct. The measurement of any quantity such as land, sea, buildings and other natural and man-made features can only be made to a certain degree of accuracy which is governed by the instruments used, external conditions and the individuals who make the observations. No matter how good the instruments may be or how careful and meticulous the observer may be, there will always be presence of small random errors between repeated observations of the same quantity or variable quantities. Every observer in a survey measurement has his or her own personal style and limitation which could be described by his or her personal errors and which may propagate as random or systematic errors depending on its magnitude. The multiplicity of personal errors from different observers in a survey is a major source of error in survey data integration and applications hence methods of adjustment computation and analysis for managing these errors become imperative. Adjustment computation and analysis provide the best known statistical and mathematical solutions for the survey network analysis to determine the reliability of survey data and results.

Vertical displacements of Benchmarks (BMs) may be due to subsidence or physical impact on the BMs. Subsidence in this context refers to the vertical deviation of the geometrical and structural shapes of grounds or buildings from the original or designed geo-referenced height due to various load factors or crustal movements. Geodetic leveling has been used independently and integrated with Global Positioning System (GPS) to determine a subsidence of about 0.20m/year in an area of about 50 x 50km along east coast of Lake Maracaibo in Venezuela (Chrzanowski et al, 1989).

The reference well-monumented benchmarks in ABU were established using geodetic leveling by Arinola (1976). Aladewolu (1991), Sule (1992) and Ojigi (1993) respectively later used geodetic techniques to monitor the stability of the benchmarks. The preliminary computations of the heights of BMs by Ojigi (1993) showed that there was indication of benchmarks instabilities in ABU, though not at critical level. The studies on ABU Benchmarks did not rule out the possibility of effect of personal observation errors in the estimated benchmarks instabilities but recommended further studies and more rigorous adjustment and analysis to further determine the integrity and reliability of the data and results.

In order to provide strong basic data for any benchmark stability and deformation analysis, new technologies must be employed in order to integrate all types of measurements into comprehensive network of observables (Chrzanowski et al, 1986). There are short-term and long-term instabilities and deformations that occur to benchmarks, small or large structures and building should be investigated and re-investigated time after time (Teskey, 1988b).

Therefore, to provide reliable reference vertical controls for development and structural monitoring purposes at the Samaru Campus of ABU, a comprehensive computation and analysis of the most probable values (mpv) of differences in heights and absolute heights of BMs becomes imperative. Therefore, the method of least squares is used for geodetic leveling computations in order to provide the most probable values of the observations as a strong basis for data analysis for vertical control or benchmarks stability in ABU and its environs.

Aim and Objectives

The aim of the study is to provide a comprehensive height computations and analysis of Benchmarks (BMs) stability in ABU, Zaria which could be used as references and models for control extension and research in Zaria and its environs. The objectives of the study are to:

i. Determine the most probable values of heights, differences in heights and heights variances of six geodetic benchmarks in ABU using least squares technique.
ii. To carry out Pearson's Chi-Square two-tail statistical test and analysis of the results obtained at 5% and 1% levels of significance in order to determine the reliability of the results.
iii. Carry out numerical and graphical analysis of the benchmark instabilities between 1976 and 1993 for the six vertical controls and hence make relative benchmark displacement projection in the study area for the year 2010.

Study Area

Samaru Campus of ABU, Zaria is situated along Gusau-Sokoto road in Zaria, Zaria local government area of Kaduna State, Nigeria. It geographically located on latitude 11° 09′ 04″N and longitude 07° 39′ 20″E and on average terrain elevation of 673m above MSL. Figures 1 and 2 show the geospatial coverage of the Campus. Figure 1 is the central area while figure 2 is the entire extent of the Campus.

Figure 1: Central Part of ABU-Main Campus Samaru Zaria Nigeria (Study Area).

Figure 2: Satellite Imagery (May 1st 2008) of Samaru Campus of ABU, Zaria
(Source: http://www.googleearth.com/)

Methodology

The method adopted is the least squares adjustment technique for the comprehensive computation of the leveling network. Also, Pearson's Chi-Square two-tail statistical test was used to determine the reliability of the results.

Data Acquisition

The data used for the study, as abstracted from Ojigi (1993) covers heights of six (6) previously established benchmarks. Four of the benchmarks (i.e. BM1, BM3, BM5 and

BM9) are on building structures while the remaining two benchmarks (CP12 and CP13) are on the ground.

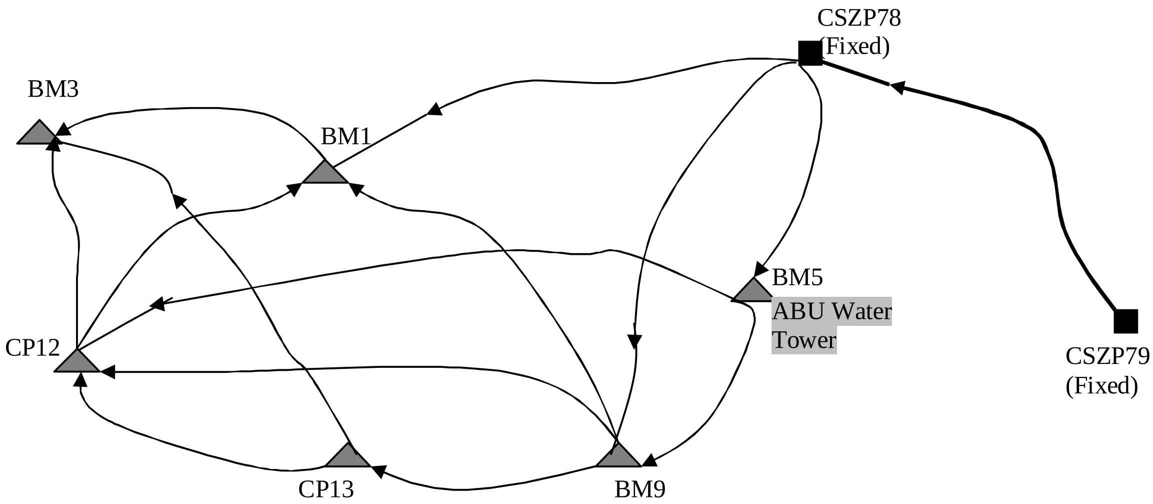

Figure 3: Sketch of the Precise Leveling Network in ABU, Samaru Campus, Zaria.

The leveling network was tied to two fixed primary reference pillars (CSZP78 and CSZP79) situated outside the campus. In table 1, column 7 is the arithmetic difference between columns 4 and 5, while column 8 is the measures of the accuracy as against the standard specification. Column 3 is the mean distance whose inverse values are used as the weights of the respective observations.

Table 1: Precise Leveling data for Subsidence Network at the Study Area

S/N	Leveling Line (From →To)	Leveling Length (m)	Diff in Height (fore)(m)	Diff. in Height (back)(m)	Mean Diff in Height (m)	Observed Misclosure (fore+back)	Standard Specification ±2.8mm√k
1	*CSZP79→ *CSZP78	481.407	3.14165	-3.14204	3.14185	-0.39	1.94
2	*CSZP78→BM5	600.811	-3.45271	3.45251	-3.45251	-0.20	2.17
3	BM5 →BM9	361.312	-0.14467	0.14371	-0.14419	-0.96	1.68
4	BM9 →CP13	602.407	-1.16023	1.16041	-1.16032	0.18	2.17
5	CP13 →CP12	240.934	1.59117	-1.59150	1.59134	-0.33	1.37
6	CP12→BM3	361.118	-0.36961	0.36950	-36956	-0.11	1.68
7	BM3→BM1	962.933	5.39076	-5.39266	5.39171	-1.90	2.75
8	*CSZP78 →BM1	842.878	1.85774	-1.85734	1.85734	0.80	2.57
9	*CSZP78→BM9	962.573	-3.59558	3.59392	-3.59475	-1.66	2.75
10	BM1→CP12	601.748	-5.02138	5.01994	-5.02066	-1.44	2.17
11	CP13→BM3	602.052	1.22237	-1.22200	1.22219	0.37	2.17
12	BM1 →BM9	962.794	-5.45289	5.45119	-5.45204	-1.70	2.75
13	CP12→BM5	843.040	-0.29029	0.29162	-0.29096	1.33	2.57
14	CP12→BM9	602.036	-0.43065	0.43101	-0.43083	0.36	2.17
	Total Distance = **8.6km**						

Note: *In this study, due to the fact that line 1 is the fixed levelling baseline, the leveling lines 2 to 14 are assigned lines 1 to 13.*

Models and Specifications Used

The standard first order leveling misclosure specification used was $\pm 2.8\text{mm}\sqrt{k}$, where k is the distance leveled in kilometers (Blachut et al, 1979). The basic quantity of physical heights are the potentials or the potential difference in heights which is represented with a gravity model as in (1)

$$\Delta H = \int_A^B g.dx = \int_A^B g dh \approx \sum_i g_i \Delta h_i \quad (1)$$

The Δh_i are the leveled height increments. Using gravity measurements g_i along the leveling line gives a geopotential difference which can be transformed into an orthometric height difference (Sneeuw, 2006). However, without the measurements of g_i, the incremental differences in heights Δh_i could be added to an absolute pre-determined orthometric height (H) to provide densification of height control for subsidence studies and other geodetic analysis.

Observation Equation Method and Least Squares Models

Least squares adjustment models consist of two important components: *the functional and the stochastic models* (ESRI, 2007). The functional model is a set of relations between the measurements and the unknown parameters. The stochastic model describes the expected error distribution of the measurements. In a one-dimensional network, such as in leveling, only one point needs to have an elevation (CDT, 2006; ESRI, 2007). In this study, each leveling line of the leveling network between points i and j supplied one observation equation to be used in the least squares adjustment of the form iŸ j to determine the most probable values (mpvs) of heights and differences in height of six geodetic benchmarks. A way of expressing the vector model between any two occupied benchmark in a leveling observation is given by (2).

$$-dH_i + dH_j - (h_{ob} - h_{ap}) = V \quad (2)$$

Where, dH_i and dH_j are unknown corrections to the approximate heights of the new points, h_{ob} = *measured difference in height*, h_{ap} = *computed difference in height* between benchmark *i* and *j* calculated from their approximate heights, and *v* = *residuals* which is a correction to h_{ob} after the adjustment (Blachut et al, 1979; Cooper, 1982). The value of dH_i or dH_j for fixed points equal zero, while the minus (-) and plus (+) represents the sign notation for the matrix-element position for observation equation of the starting and ending benchmark in the network. Equation (2) will later yield the *Ax-b = v* required for least squares solution.

The observation equations relating the adjusted observations $L^a{}_i$ with the adjusted parameters X^a may be expressed as Ayeni (2001):

$$L^a{}_1 = f_1(X^a) \quad (3)$$

For the purpose of generalization, equation (3) is treated as a non-linear equation. The linearised form of (3) through Taylor's series is modified after Ayeni (2001) as:

$$v_1 = A_1 x + b_1 \quad (4)$$

Where $A_1 = \frac{\partial f_1(X^a)}{\partial(X^a)}$, $b_1 = f_1(X^0) - L^0_1$ and $x = X^a - X^0$

The principles of least squares is based on the fact that, in a set of measurements having unequal or equal weights, the most probable values are those which make sum of the products of the weights and squares of the residuals minimum (Moffitt and Bouchard, 1975). This is represented by equation (5).

$$\sum wv^2 = \text{minimum} \quad (5)$$

For observations of unequal weights as in the present leveling network, the least squares condition given by equation (5) is expressed in matrix form as follows:

$$v^T wv = \text{minimum} \quad (6)$$

v^T is the transpose of the column matrix of residuals (v).
The value of v from equation (4) is substituted into equation (6), giving

$$v^T wv = (Ax + b)^T w(Ax + b) = \min imum \quad (7)$$

Expanding (7) gives equation (8)

$$v^T wv = x^T A^T wbx + x^T A^T wb + b^T wAx + b^T wb \quad (8)$$

Since $b^T wAx = x^T A^T w^T b$, $w^T = w$ and $b^T wAx = x^T A^T wb$
(Modified after Cooper,1982); hence equation (8) can be re-written as equation (9)

$$v^T wv = x^T A^T wbx + 2x^T Awb + b^T wb \quad (9)$$

In order to make the weighted sums of squares of the residuals a minimum, we take the partial derivative of $v^T wv$ with respect to x and set equal to zero. Thus from equation (9)

$$\frac{\partial v^T wv}{\partial x} = 2A^T wAx + 2A^T wb = 0 \quad (10)$$

$$A^T wAx = -A^T wb \quad (11)$$

$$x = -(A^T wA)^{-1} A^T wb \quad (12)$$

The computation for the determination of x was achieved using MATLAB 2007b Software.

Adjustment Computation of the Leveling Network

In the determination of the parameters, the *b* matrices in equation 13 were used in turn (i. e. one after the other) with the *Ax* matrices for the generation of the residuals for mean, fore, back leveling operations respectively.

$$[A] \quad x \quad = \quad [b_{mean}], \quad [b_{fore}], \quad [b_{back}], \; + \; [v] \qquad (13)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta BM_5 \\ \delta BM_9 \\ \delta CP_{13} \\ \delta CP_{12} \\ \delta BM_3 \\ \delta BM_1 \end{bmatrix} = \begin{bmatrix} -0.00132 \\ -0.00278 \\ 0.00146 \\ -0.00123 \\ 0.00102 \\ -0.00028 \\ 0.00048 \\ 0.00059 \\ 0.00075 \\ 0.00020 \\ 0.00016 \\ -0.00158 \\ -0.00004 \end{bmatrix}, \begin{bmatrix} 0.00201 \\ -0.00310 \\ 0.00109 \\ -0.00140 \\ 0.00099 \\ -0.00111 \\ 0.00151 \\ 0.00071 \\ -0.00011 \\ 0.00040 \\ -0.00037 \\ -0.00061 \\ 0.00060 \end{bmatrix}, \begin{bmatrix} -0.00212 \\ 0.00321 \\ -0.00109 \\ 0.00107 \\ -0.00107 \\ -0.00054 \\ 0.00055 \\ -0.00121 \\ -0.00161 \\ 0.00000 \\ -0.00143 \\ 0.00105 \\ -0.00006 \end{bmatrix} + [v]$$

$$\therefore \; x = (A^T wA)^{-1} (A^T wb) = Q(A^T wb) \qquad (14)$$

Where $Q = (A^T wA)^{-1}$ = the variance-covariance matrix and w is the weight matrix of the observations. The weight of a leveling network is calculated by taking the reciprocal or inverse of the length of the leveling line (*Moffitt & Bouchard, 1975; Cooper, 1982*), where the distance or length of leveling line is represented by S (in kilometers). The weight matrix expression is given by:

$$W_i = \frac{1}{S_i} \qquad (15)$$

Variance Factor and Standard Deviation

A basic procedure in error analysis is finding the variance factor ($\hat{\sigma}_o^2$) as derived from the adjustment (Blachut et al, 1979). To determine the variance factor equation (16) was used for *a posteriori* variance factor of the observations:

$$\hat{\sigma}_o^2 = \frac{v^T wv}{m-n} \qquad (16)$$

Where m is the number of observations and n is the number of unknowns. Also, m-n = degree of freedom). In order to determine the reliability of the variance-covariance estimation, it is important to test the estimated variance factor.

Test of Hypothesis

The two hypotheses set for the study include:

Null hypothesis H_0: $\frac{v^T wv}{m-n} = \sigma_o^2$ [i.e. $\frac{v^T wv}{m-n} = \sigma_o^2$ is within the confidence limits]

Alternative hypothesis H_1: $\frac{v^T wv}{m-n} \neq \sigma_o^2$ [i.e. $\frac{v^T wv}{m-n} = \sigma_o^2$ is outside the confidence limits]

Where σ_o^2 is the *apriori* variance of unit weight. If the mathematical models of the network and the *apriori* estimation of the accuracy of observations are correct, the calculated variance ($\hat{\sigma}_o^2$) value should agree with the *apriori* variance (σ_o^2) value within the confidence interval at a specified probability level (Blachut et al, 1979). This requires a two-tail test by Chi-Square (χ^2) distribution; determined by equation (15) at 95% and 99% probability levels.

$$\frac{(m-n)\hat{\sigma}_o^2}{\chi^2_{p_2}} \leq \sigma_o{}^2 \leq \frac{(m-n)\hat{\sigma}_o^2}{\chi^2_{p_1}} \qquad (17)$$

Where $\chi^2_{p_2}$ and $\chi^2_{p_1}$ are two-tail percentile values in the Chi-Square distribution table. The required percentile values (in Table 2) for the Chi-square distribution according to Pearson and Hartley (1966) were used for the agreement test between the *apriori* and the *aposteriori* variances; meant to establish whether the parameters used in the determination truly represents the heights of the benchmarks in the study area.

Table 2: Chi-Square Percentile Probability Values for the Observed Redundancies

Leveling direction	df (m-n)	95% Prob.		99% Prob.	
Chi-Square Value		$\chi^2_{p_2}$	$\chi^2_{p_1}$	$\chi^2_{p_2}$	$\chi^2_{p_1}$
Fore	7	16.0	1.69	20.3	0.989
Back	7	16.0	1.69	20.3	0.989
Mean	7	16.0	1.69	20.3	0.989

Determination of Accuracy by Variance-Covariance Estimation

Variance-covariance matrix of coordinates of a network is the basis for calculating the absolute and relative positional error and accuracy. This is computed using equation (18).

$$C_x = \hat{\sigma}_o^2 (A^T W A)^{-1} \qquad (18)$$

C_x is more explicitly expressed as equation (19):

$$C_x = \hat{\sigma}_o^2 \begin{bmatrix} \sigma^2_{x_1} & \sigma_{x_1x_2} & \sigma_{x_1x_3} & \sigma_{x_1x_4} & \sigma_{x_1x_5} & \sigma_{x_1x_6} \\ \sigma_{x_2x_1} & \sigma^2_{x_2} & \sigma_{x_2x_3} & \sigma_{x_2x_4} & \sigma_{x_2x_5} & \sigma_{x_2x_6} \\ \sigma_{x_3x_1} & \sigma_{x_3x_2} & \sigma^2_{x_3} & \sigma_{x_3x_4} & \sigma_{x_3x_5} & \sigma_{x_3x_6} \\ \sigma_{x_4x_1} & \sigma_{x_4x_2} & \sigma_{x_4x_3} & \sigma^2_{x_4} & \sigma_{x_4x_5} & \sigma_{x_4x_6} \\ \sigma_{x_5x_1} & \sigma_{x_5x_2} & \sigma_{x_5x_3} & \sigma_{x_5x_4} & \sigma^2_{x_5} & \sigma_{x_5x_6} \\ \sigma_{x_6x_1} & \sigma_{x_6x_2} & \sigma_{x_6x_3} & \sigma_{x_6x_4} & \sigma_{x_6x_5} & \sigma^2_{x_6} \end{bmatrix} \qquad (19)$$

Where $\sigma^2_{x_i}$ [where i =1, 2, 3…6] are the variances of the 6 benchmarks and $\sigma_{x_ix_j}$ [where i,j = 1,2; 1,3;….; 2,3; 2,4…;5,6] are the fifteen (15) covariances of the 6 benchmarks.

Comparison with Previous Determinations

The statistical relationship with apriori observations were done in order to establish the relative stability status of the benchmarks. The results of 1993 observations were compared with those of 1976 to ascertain the differential displacements for each benchmark and to make projections for benchmark displacement up to year 2010. The estimated displaced values with respect to the base benchmark heights as at 1976 were computed and plotted respectively.

Results and Discussion

The Most Probable Values of Height and Standard Deviations

According to Blachut et al (1979), first order leveling network usually consists of control points spaced at 2km to 4km interval, and the second order network consist of benchmark spacing between 0.5km and 1km, while the third order network is between 0.1km to 0.3km respectively. By this classification, the network in this study averagely falls in the 2^{nd} order category, whose standard deviation results must satisfy the $2mm\sqrt{k}$ specification. The most probable values of the benchmark heights are as contained in Table 3.1

Table 3.1: The computed corrections and Most Probable Heights of the Benchmarks for Fore, Back and Mean Leveling.

Bench mark Name	Appr. Elevation Values(m)	Correction (x) (mm)			MPV of Heights (m)		
		Fore	Back	Mean	Fore	Back	Mean
BM5	667.98920	*0.17*	***2.43***	*2.37*	667.98937	667.99163	667.99157
BM9	667.84763	*-0.95*	***0.47***	*0.33*	667.84668	667.84810	667.84796
CP13	666.68631	*0.92*	***2.05***	*1.76*	666.68723	666.68836	666.68807
CP12	668.27888	*-0.04*	***0.97***	*0.95*	668.27884	668.27985	668.27983
BM3	667.90828	*0.84*	***2.16***	*1.42*	667.90912	667.91044	667.90970
BM1	673.30015	*-0.26*	***1.13***	*-0.17*	673.29989	673.30128	673.29998

Table 3.2: The standard deviations of the Benchmark Heights (σ_{H_i}) for Fore, Back and Mean Leveling computations.

Benchmark Name	(σ_{H_i}) (m)		
	Fore ($\pm$)	Back ($\pm$)	Mean ($\pm$)
BM5	0.00103	0.00077	0.00099
BM9	0.00104	0.00078	0.00100
CP13	0.00126	0.00094	0.00121
CP12	0.00116	0.00087	0.00112
BM3	0.00133	0.00099	0.00128
BM1	0.00114	0.00085	0.00110

The estimated standard deviations for the fore, back and mean computation of the leveling observations were 1.78mm, 1.33mm and 1.71mm respectively.

3.2 The Standard Deviations of the Difference in Heights ($\sigma_{\Delta h_i}$) and Residuals (v_i).

The results of determination of the three phases of computations (fore, back and mean) are shown in Table 3.3 and 3.4 respectively.

Table 3.3: The estimated residuals of fore, back and mean leveling computations and their corresponding standard deviations.

Leveling line	v_i : ***Fore*** (mm)	v_i : ***Back*** (mm)	v_i : ***Mean*** (mm)	σ v_f: (mm) ±	σ v_b (mm) ±	σ v_m (mm) ±
*CSZP78Ÿ BM5	1.49	0.42	-0.25	*0.91*	*0.68*	*0.88*
BM5 Ÿ BM9	1.66	1.14	-1.21	*0.65*	*0.49*	*0.63*
BM9 Ÿ CP13	0.41	0.49	-0.30	*1.04*	*0.78*	*1.00*
CP13 Ÿ CP12	0.27	0.31	-0.26	*0.51*	*0.38*	*0.49*
CP12Ÿ BM3	-0.14	0.20	0.59	*0.69*	*0.52*	*0.66*
BM3Ÿ BM1	-0.83	0.08	2.13	*1.39*	*1.03*	*1.33*
*CSZP78 Ÿ BM1	-0.74	-0.38	-0.38	*1.17*	*0.88*	*1.13*
*CSZP78Ÿ BM9	-1.54	-0.24	0.84	*1.40*	*1.05*	*1.35*
BM1Ÿ CP12	-0.52	-0.05	0.50	*1.03*	*0.77*	*0.99*
CP13Ÿ BM3	-0.28	-0.29	0.34	*1.04*	*0.78*	*1.00*
BM1 Ÿ BM9	-0.84	-0.28	0.89	*1.44*	*1.08*	*1.38*
CP12Ÿ BM5	1.79	2.08	-2.47	*1.30*	*0.97*	*1.25*
CP12Ÿ BM9	-0.87	-1.09	0.64	*1.11*	*0.83*	*1.06*

Note: v_i := residuals of the difference in height observations, σ v_f: = standard deviation of residual for fore leveling, σ v_b = standard deviation of residual for back leveling and σ v_m = standard deviation of residual for mean leveling.

Table 3.4: Standard Deviation of *Fore*, *Back* and *Mean* Leveling

Leveling Line No.	Leveling line	$\sigma_{\Delta h_i}$ (±mm) *fore*	$\sigma_{\Delta h_i}$ (± mm *back*	$\sigma_{\Delta h_i}$ (±mm) *mean*
1	*CSZP78Ÿ BM5	1.03	*0.77*	0.99
2	BM5 Ÿ BM9	0.85	*0.63*	0.81
3	BM9 Ÿ CP13	0.91	*0.68*	0.88
4	CP13 Ÿ CP12	0.71	*0.53*	0.68
5	CP12Ÿ BM3	0.82	*0.61*	0.79
6	BM3Ÿ BM1	1.06	*0.80*	1.02
7	*CSZP78 Ÿ BM1	1.14	*0.85*	1.09
8	*CSZP78Ÿ BM9	1.04	*0.78*	1.00
9	BM1Ÿ CP12	0.92	*0.69*	0.89
10	CP13Ÿ BM3	0.91	*0.68*	0.87
11	BM1 Ÿ BM9	0.99	*0.74*	0.95
12	CP12Ÿ BM5	0.99	*0.74*	0.95
13	CP12Ÿ BM9	0.82	*0.62*	0.79

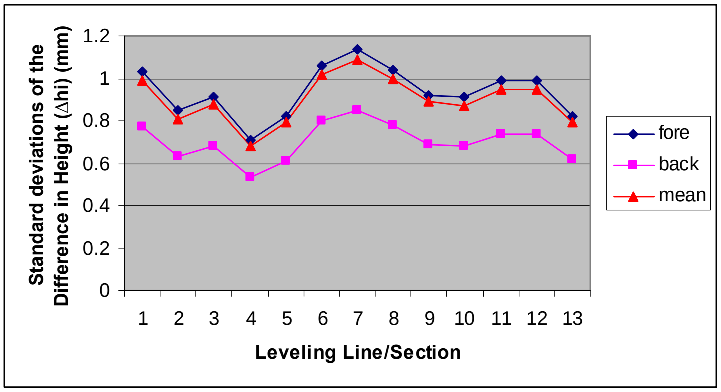

Figure 3.1: Standard Deviations of the differences in Height (fore, back and mean Observations)

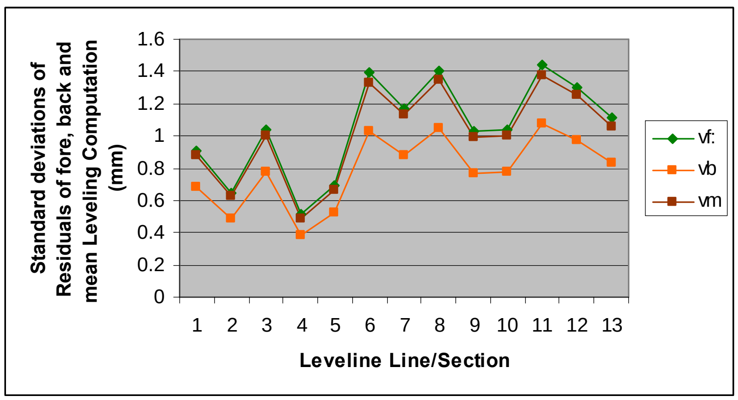

Figure 3.2: Standard Deviations of the Residual (fore, back and mean Observations)

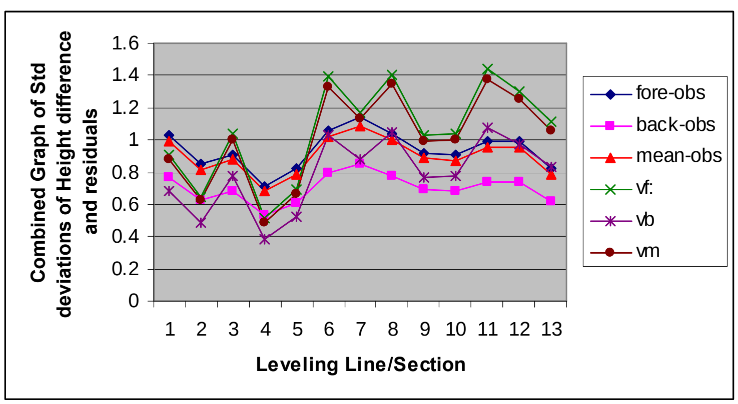

Figure 3.3: Standard Deviations of the difference in Height and residual (fore, back and mean Observations)

Error Analysis of the Adjusted Network

Table 3.5 shows the test parameters and results of the two-tail Chi-square test of the *a posteriori* variance ($\hat{\sigma}_o^2$) at 95% and 99% confidence levels, with an *apriori* variance factor (σ_0^2) of 1**.** The results show that the back leveling alone passed the test; hence its values seem to have produced the most accurate variance. However, the fore and mean leveling computation failed the test respectively. A projection of percentile probability values shows that the larger the degree of freedom in an observation the higher the percentile values for both 95% and 99% probability levels; which consequently enhances the accurate estimates of variances.

Table 3.5: The apriori and estimated variance factors and the computed lower and upper critical values for the observations at 5% and 1% Significance Levels

Leveling direction	Degree of freedom (df)	Apriori variance factor (σ_o^2)	Estimated variance factor ($\hat{\sigma}_o^2$) (mm²)	95% Probability		99% Probability	
				Lower Critical Value (α = 0.05)	Upper Critical Value (α = 0.95)	Lower Critical Value (α = 0.01)	Upper Critical Value (α = 0.99)
Fore	7	1	3.17	1.387	13.130	1.093	22.437
Back	7	1	1.77	0.774	7.331	0.610	12.528
Mean	7	1	2.93	1.282	12.136	1.010	20.738

The results of the test show that, the estimated variances for the fore and mean leveling computations failed to agree with the apriori variance factor. This implies that, $\hat{\sigma}_o^2$ was outside the confidence limits of 99% and 95% respectively, hence the null hypothesis (H_o) is not accepted. The geometric implication of these results is that the parameter in the fore and mean leveling does not truly represent the Benchmarks' heights in the study area whereas the back leveling passed the confidence limits test showing that the null hypothesis (H_o) is accepted. The back leveling, therefore, provides the true representation of the Benchmarks' heights in the study area.

Final Adjusted Leveling Observation and Subsidence Analysis

Tables 3.7 and 3.8 represent the best estimate of the difference in height and Benchmark Heights of the leveling network and their standard deviations in the study area. The values having satisfied the null hypothesis for this study stand as the most reliable elevation coordinate parameters in the study area. Figure 3.5 shows the subsidence values for the six (6) benchmarks in the leveling network in the study area between 1976 and 1993. The breakdown shows that BM5 showed a downward movement of -12.42mm, BM9 showed an upward movement of +0.57mm, CP13 had the least displacement of +0.44mm. Others are CP12, BM1 and BM3 with displacement values of -6. 77mm, +4.81mm and +4.14mm respectively.

Table 3.7: Adjusted difference in heights of the Leveling Network and their standard deviations, loop specifications and misclosures.

Leveling Line No.	Appr. difference in height ($a\Delta h_{ij}$): (m)	Residual (mm)	Adjusted difference in Height (Δh_i): (m)	Observed Loop Misclosure (±mm)	Standard Specification ±2.8mm√k	% of misclosures w.r.t. std specif.
1	3.45251	0.42	3.45293 ±0.77mm	-0.39	1.94	20
2	0.14371	1.14	0.14485±0.63mm	-0.2	2.17	9.2
3	1.16041	0.49	1.16090 ±0.68mm	-0.96	1.68	57.1
4	-1.59150	0.31	-1.59119±0.53mm	0.18	2.17	8.3
5	0.36950	0.20	0.36970 ±0.61mm	-0.33	1.37	24.1
6	-5.39266	0.08	-5.39258±0.80mm	-0.11	1.68	6.5
7	-1.85734	-0.38	-1.85732±0.85mm	-1.9	2.75	69.1
8	3.59392	-0.24	3.59368±0.78mm	0.8	2.57	31.1
9	5.01994	-0.05	5.01989±0.69mm	-1.66	2.75	60.4
10	-1.22200	-0.29	-1.22229±0.68mm	-1.44	2.17	66.4
11	5.45119	-0.28	5.45091±0.74mm	0.37	2.17	17.1
12	0.29162	2.08	0.29370±0.74mm	-1.7	2.75	61.8
13	0.43101	-1.09	0.42992±0.62mm	1.33	2.57	51.8

Δh_{ij}: = difference in height between two benchmarks;
* = These are the fixed/reference primary pillars

Table 3.8: Adjusted Heights and Displacements of the Leveling Benchmarks (BMs) in Part of ABU, Zaria for 1976 and 1993 and projected figures 2010.

Benchmarks	Adjusted Height (1976) (m)	Adjusted Height(1993) (m)	BM Height Displacements (mm) (1976-1993)	Yearly BM Displacement (mm)	Projected BMs Displacement by Year 2010
BM5	668.00405	667.99163	-12.42	-0.73	-24.84
BM9	667.84753	667.84810	+0.57	0.03	1.14
CP13	666.68792	666.68836	+0.44	0.03	0.88
CP12	668.28662	668.27985	-6.77	-0.40	-13.54
BM3	667.90630	667.91044	+4.14	0.24	8.28
BM1	673.29647	673.30128	+4.81	0.28	9.62

(Modified after Arinola, 1976; Ojigi 1993)

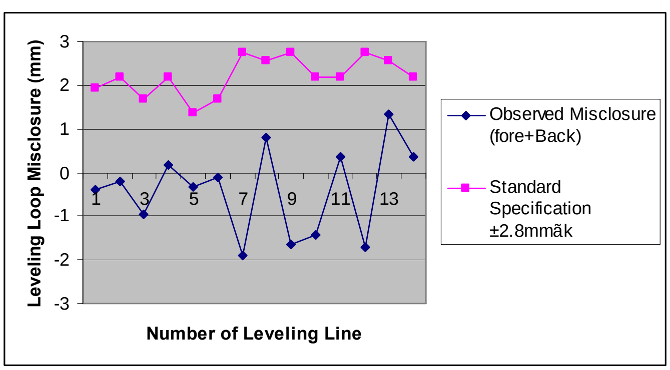

Figure 3.4: Comparison of Leveling Loop specifications and Misclosures

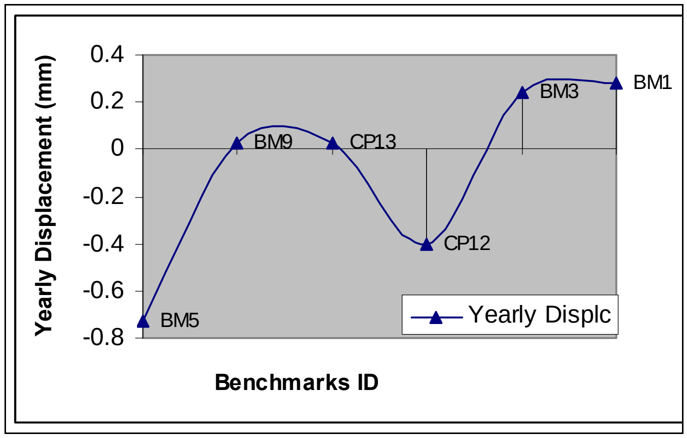

Figure 3.5: Yearly Displacement of the Benchmarks between 1976 and 1993 (mm)

Summary of Findings

The following findings were made in this study:

1. The fore leveling was in error, hence produced outliers in the estimated height parameters.
2. It is possible to propagate errors in a lop observation of this nature if average of fore and back observations are considered to be a blanket solution without proper 'stand-alone' or 'independent' analysis of the phase observations. The consequence of this is that a more accurate phase observation could be corrupted and degraded.
3. The size of the degree of freedom is a significant factor in the precision and accuracy of variance estimation.
4. All the six benchmarks in the leveling network showed various levels of displacements over a period of 17 years. The values range from 0.44mm to 12.42mm, which is considered not to be a major instability considering the period; hence the benchmarks are adjudged to be fairly stable.

Conclusions and Recommendations

This study used the method of least squares to carry out the computation and analysis of a geodetic leveling network of six benchmarks over part of the Samaru Campus of Ahmadu Bello University, Zaria, Nigeria, with the sole aim of determining their stability. The most probable values of the Benchmark heights and differences in height in the network were adequately determined and their reliability tested at 5% and 1% significance level. The importance of rigorous adjustment computation and network analysis to this study is to ensure that random errors are effectively constrained and managed in order to avoid spurious results and false alarm concerning stability of the Benchmarks in the study area.

The statistical tests helped to detect gross errors and provided estimates of the precision of the network's Heights and allowed the reliability of the network and individual measurements to be determined. Based on the above, the following recommendations are hereby made:

i. The two phases (fore and back) of a geodetic leveling operation should always be computed as stand-alone system, tested and statistically certified to be the true representation of the reference surface(s) before it is combined for a final analysis.

ii. Though the finding in this study does not present an alarming scenario, but a periodic monitoring of all beacons/benchmarks at the Ahmadu Bello University should be encouraged by the University Authority for preventive reason and for environmental management information database development strategy.

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