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PREDICTION OF GRAVITY ANOMALIES FOR GEOPHYSICAL EXPLORATION

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ABSTRACT
Interpretation of gravity anomalies (determined on the earth's surface) reveals information on mineral resources beneath the earth. The density of gravity stations (where gravity anomalies are determined) is critical to the successful interpretation of these anomalies. Where the density of the available gravity anomalies is not enough, for a particular purpose of geophysical exploration, more gravity stations can be established within the surveyed area and the gravity anomalies observed for these stations. In some cases, where observations of gravity anomalies are not possible due, probably, to inaccessibility of the newly chosen gravity stations, the required gravity anomalies for such stations can be estimated (predicted). Currently, classical least squares technique is used to accomplish such task. However, the technique does not produce optimum results because its formulation assumes that the observed gravity anomalies, used for the prediction, are error free, whereas, all observed quantities are affected by random errors. Therefore, in this study, an attempt is made to carry out prediction of gravity anomalies for geophysical exploration using least squares collocation technique. This is considered to be a better alternative because its formulation takes the presence of random errors of observations in the observed quantities into consideration and makes provision for filtering out these errors while predicting the signals of interest at the required number of stations.

INTRODUCTION
The search for mineral resources represents the major geophysical exploration activity in the world today. This is because mineral resources are the concealed sub-surface geological features of most economic interest to many countries of the world.

Mineral resources can be described as beneficial subsurface features whose natural habitat is the earth. They include solid metallic minerals such as, iron ore, zinc etc, solid non-metallic minerals like limestone, marble etc., liquid minerals such as oil, water etc., and gaseous minerals like gasses in buried cavity. Mineral exploration has been in existence for a long time. Its method, however, had been only drilling using percussion bits. This had posed a lot of risk as explorer could easily and ignorantly get exposed to dangerous materials underground. Also, it was uneconomical since decision as to where to drill was taken like a gamble because there was no prior information about the actual location of the concealed mineral before drilling. Later, in the early years of twentieth century, the continued efforts by explorers to look for more effective, less risky and more economical technique of sub-surface exploration led to the advent of geophysical exploration.

Geophysical exploration, defined by Reynolds (1998) as "making and interpreting measurements of physical properties of the earth to determine sub-surface conditions, usually with economic objectives such as discovery of mineral

FUTY Journal of the Environment, Vol. 1 No.1, July 2006

depositions", does not dispense with the need for drilling, but if properly applied, it can optimise exploration program by maximizing the rate of ground coverage and minimizing the drilling request. It is broadly divided into two methods: natural-source and artificial-source methods. The natural-source methods are those making use of gravity and magnetic fields of the earth to search for local perturbation, in the natural fields, caused by the concealed geological features while the artificial-source methods involve propagation of artificial waves (e. g. seismic waves) through the earth interior which may be used analogously to natural-source method. Comparing the two methods, it was observed in Kearey and Brooks (1988) that the natural-source method is logistically simpler to carryout than the artificial-source method. Also, in Senti (1988), the unit cost figures of different geophysical surveys carried out by the society of exploration geophysicists in 1987 were compiled to compare the two methods. The results showed that the natural-source method is more cost effective than the artificial-source method. Furthermore, it was reported in Gumert (1992) that the high speed of operation and current level of accuracy of gravity method led to better understanding of regional geology and economically limited the use of more expensive seismic survey for mineral exploration.

Generally, both natural-source and artificial-source methods are used for mineral exploration. However, in some countries such as Nigeria, only the seismic method is being used while the use of gravity method is rarely noticed. One of the reasons for this might not be unconnected with the inadequate gravity data in these countries. This necessitated the plan of Federal Government of Nigeria to set up Nigeria gravity network project committee (NGNPC) to establish various gravity networks of points in Nigeria for geodetic and geophysical studies. Unfortunately, due to inexplicable reasons, the aims of this committee have not been achieved to date (Osazuwa, 1995). Therefore, gravity values and hence gravity anomalies (obtained from gravity values) remain inadequate for geophysical exploration in Nigeria. Acquisition of gravity values for adequate gravity anomalies in these countries can be achieved by direct observation and/or mathematical estimation. Therefore, it is the objective of this paper to apply mathematical estimation technique, that is, least squares collocation technique for the prediction of gravity anomalies for geophysical exploration.

METHODOLOGY

The method used includes the application of least squares collocation technique for the prediction of gravity anomalies at the observation stations. The classical least squares technique, as discussed in Abdelrahman *et al* (1991), is also used to predict gravity anomalies at the same stations. This is for the purpose of comparison of the two techniques. Thereafter, the least squares collocation technique is used to predict gravity anomalies of other stations using the observed gravity anomalies of the observation stations.

Data Acquisition

The data used for the study, as abstracted from SNEPCO (1995), are shown in tables 1 and 2. Columns 1, 2, 3, 4 and 5 of table 1 give the station numbers, gravity anomalies (Δg_i), x-coordinates, y-coordinates and gravity values (g) of thirty gravity observation stations respectively. Columns 1, 2, 3 and 4 of table 2 respectively show the station numbers, gravity anomalies, x-coordinates and y-coordinates of another fifteen gravity stations outside the observation stations whose gravity anomalies are predicted based on the observed gravity anomalies of table 1.

FUTY Journal of the Environment, Vol. 1 No.1, July 2006
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Table 1: Data used for predicting gravity anomalies at observation stations

Station number	Δg_i(mGal)	x (m)	y (m)	g (mGal)
95D0301040	-31.928500	692415.900	1189739.600	978061.4520
95D0301060	-31.772200	692604.800	1190200.600	978061.9706
95D0301100	-31.178600	692487.400	1191183.300	978066.1179
95D0301120	-30.773000	692408.200	1191676.600	978068.5372
95D0301160	-30.350500	692250.300	1192664.300	978071.3239
95D0301200	-29.833000	692092.800	1193652.000	978070.7214
95D0301220	-29.736200	692014.100	1194145.600	978070.3145
95D0301260	-29.421700	691855.600	1195133.000	978074.9719
95D0301300	-29.262800	691698.400	1196120.300	978079.2266
95D0301318	-29.038700	691627.000	1196564.400	978081.4502
95D0301346	-29.010000	691516.500	1197256.300	978084.2498
95D0301381	-28.987000	691378.700	1198120.300	978087.7926
95D0301420	-29.231300	691224.900	1199082.900	978087.7034
95D0301460	-29.609300	691066.600	1200069.900	978088.7919
95D0301480	-29.665300	690987.400	1200564.300	978090.4587
95D0301500	-29.897100	690908.300	1201058.100	978092.1069
95D0301540	-30.442700	690750.600	1202045.100	978092.7586
95D0301580	-30.579400	690593.200	1203032.800	978094.1567
95D0301620	-31.330000	690435.100	1204020.000	978092.1786
95D0301638	-31.223900	690364.100	1204463.600	978093.3157
95D0301660	-31.096500	690276.300	1205008.300	978096.8901
95D0301700	-30.958100	690119.500	1205994.800	978101.5534
95D0301720	-30.586200	690040.200	1206488.800	978103.2666
95D0301740	-30.112100	689961.600	1206982.400	978105.8462
95D0301780	-29.173300	689803.600	1207970.000	978109.4676
95D0301800	-28.809800	689724.300	1208464.300	978110.0709
95D0301840	-27.492100	689566.600	1209451.400	978113.7791
95D0301860	-26.900200	689488.000	1209945.000	978116.5442
95D0301880	-26.256800	689408.700	1210438.500	978118.1695
95D0301920	-25.579700	689251.400	1211426.300	978122.1145

FUTY Journal of the Environment, Vol. 1 No.1, July 2006
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Table 2: Data for other gravity stations where gravity anomalies are predicted

Station number	Δg_i(mGal)	x (m)	y (m)
95D0301080	-31.482100	692566.400	1190689.600
95D0301140	-30.585700	692329.100	1192170.900
95D0301180	-29.958800	692171.700	1193158.300
95D0301240	-29.578400	691934.100	1194638.800
95D0301280	-29.177600	691777.100	1195626.900
95D0301334	-29.024300	691563.900	1196959.900
95D0301360	-29.070900	691461.000	1197601.900
95D0301400	-29.197900	691303.800	1198588.800
95D0301440	-29.462800	691145.300	1199576.600
95D0301520	-30.141500	690829.600	1201551.600
95D0301560	-30.588100	690672.000	1202538.800
95D0301600	-30.817100	690514.000	1203526.600
95D0301680	-31.218200	690199.000	1205500.900
95D0301760	-29.729900	689882.800	1207475.900
95D0301820	-28.225200	689645.300	1208957.800

Least Squares Collocation Technique

Least squares collocation is an advanced least squares method. Schwarz (1976a) used it for the adjustment of a large geodetic network. Rapp (1986) applied it in the prediction of geoidal undulations and components of deflection of verticals. In Ezeigbo (1988), it was described as an appropriate method and a better alternative than other methods in handling gravity depended observations. Its concept combines least squares adjustment to obtain parameters (X), least squares filtering of error (V) and least squares prediction of signals (S) using observed quantities (*l*). As in Rapp (1986), the ultimate generalization and the minimum principle of the linear least squares collocation model are respectively given as (1) and (2).

$$l = AX + S + V \tag{1}$$

$$S^T C_{sl} S + V^T C_{VV}^{-1} V = \min imum \tag{2}$$

Where: C_{sl} = Covariance function between observations and signals

C_{VV} = Covariance matrix of the observations

$A = \partial l / \partial X$

The superscript T indicates the transpose of a vector and/or matrix.

Its theory and detailed proof of equations are fully discussed in Moritz (1972), Moritz (1978), Krakiwsky (1975), Rapp (1986) and Ayeni (2001). However, for easy reference, the step by step applications of the equations are stated here.

$$S = C_{sl} C_{ll}^{-1} (l - AX) \tag{3}$$

$$X = (A^T C_{ll}^{-1} A)^{-1} A^T C_{ll}^{-1} l \tag{4}$$

$$V = C_{VV} C_{ll}^{-1} (l - AX) \tag{5}$$

Where: $C_{ll} = C_{sl} + C_{VV}$ (Covariance function between observations).

FUTY Journal of the Environment, Vol. 1 No.1, July 2006

Other notations used are:

$\sigma_0^2 = 1$ (a-priori variance of unit weight)

$C_{XX} = (V^T C_{ll}^{-1} V)(A^T C_{ll}^{-1} A)^{-1} / (n-m)$ (error covariance matrix of parameters)

$\underset{-ss}{C} = C_{ss} - C_{sl} C_{ll}^{-1} C_{sl}^T + HAC_{XX} A^T H^T$ (error covariance matrix of signals)

$H = C_{sl} C_{ll}^{-1}$

For $A = 0$, (1) reduces to (6), which is the least squares collocation model for the prediction of signals and filtering of errors without determination of parameters.

$$l = S + V \qquad (6)$$

Consequently, (3) and (5) reduce to (7) and (8) respectively.

$$S = C_{sl} C_{ll}^{-1} l \qquad (7)$$

$$V = C_{VV} C_{ll}^{-1} l \qquad (8)$$

While $\underset{-ss}{C} = C_{ss} - C_{sl} C_{ll}^{-1} C_{sl}^T$ (error covariance matrix of signals)

From the above, it can be seen that the covariance function plays a significant role in the concept of least squares collocation technique. Basically, covariance function supplies information on the structure of the gravity field. Therefore, it should be appropriately designed to achieve the objective of least squares collocation technique. Schwarz (1976a) and Moritz (1978) suggested that it should be simple, analytical, isotropic and homogeneous. Isotropic and homogeneous requirements imply that it should be invariant with respect to rotation and translation. Also, it was stated that the matrix formed from such covariance functions must be positive definite. Positive definiteness of a non-singular symmetric matrix is achieved if the eigen values of the matrix are positive. Such matrix has dominant diagonal elements.

There are various mathematical expressions for a covariance function that represent global features of the earth. Detailed discussions on these can be found in Schwarz (1976a), Moritz (1978), Rapp (1986) and Ezeigbo (1988). However, for some limited purposes such as prediction of gravity anomalies, one may approximate the curved surface of the earth locally by a plane surface to find expressions for covariance functions that represent local features. This can functionally be given by (9) as in Moritz (1978):

$$C(r) = C(o) / (1 + (r/a)^2)^{1/2} \qquad (9)$$

Where:
r = distance between two points.
$C(o)$ = covariance function when $r = 0$
a = correlation length

Three parameters ($C(o)$, 'a' and r) are needed, in (9) to evaluate $C(r)$. The value of r is considered known, since it can be measured or computed from the given

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coordinates of gravity stations. However, the values of $C(o)$ and 'a' are unknown hence they have to be estimated. This can be achieved using an optimization technique which is a process of obtaining the optimal values of some parameters. The process is regarded successful if the estimated parameters satisfy as close as possible the objective function designed for such process. In other words, given the objective function, an optimization procedure systematically searches, among the range of possible values of parameters, and selects the best-fit values, which satisfy the given objective function. In this study, an optimisation technique is used to determine the optimal values of $C(o)$ and 'a' for the evaluation of $C(r)$ using (10) as the objective function used in Fajemirokun and Orupabo (1987).

$$R^2 = (F_0^2 - F^2)/F_0^2 \qquad (10)$$

Where: $F_0^2 = \Sigma_{i=1}^{n}(\Delta g_i - \Delta g)^2$

$F^2 = \Sigma_{i=1}^{n}(\Delta g_i - \Delta g_p)^2$

Δg_i = observed gravity anomaly at point i .

Δg_p = predicted gravity anomaly at observation point i.

Δg = mean gravity anomaly

The optimisation process starts by using least squares technique to solve (11) to obtain the values of residual observations (v_i).

$$\Delta g_i = \Delta g_p + v_i \qquad (11)$$

Thereafter, the initial estimates of $C(o)$ and 'a' are determined by solving (12) given by Moritz (1978).

$$(\Sigma_{i=1}^{n-k} v_i v_{i+k})/df = C(o)/(1+(r/a)^2)^{1/2} \qquad (12)$$

Where: df = degree of freedom = n - m

m = number of parameters needed to represent Δg_p in (11)

n = number of observations

k = 0, 1, 2, ..., n-1

The results are given as:

$$C(o) = \Sigma_{i=1}^{n} v_i v_i / df \qquad (13)$$

$$a_i = (r^2/((C(o)/\Sigma_{i=1}^{n-k} v_i v_{i+k})^2 - 1))^{1/2} \qquad (14)$$

$$a = \Sigma_{i=1}^{n} a_i / n \qquad (15)$$

The final values of $C(o)$ and 'a' are obtained by systematically varying their initial values until (10)) is satisfied. The use of (10), as the condition to be satisfied, for the determination of optimum covariance parameters during the prediction of gravity anomalies at the observation stations, is logical. This is because observed gravity anomalies of the observation stations are available. However, where gravity anomalies are being predicted outside the observation stations, the initial values of the covariance parameters can be computed using the above procedure while the final values of the parameters may be obtained by systematically varying their initial values until the trace of the error covariance matrix of the predicted gravity anomalies is

FUTY Journal of the Environment, Vol. 1 No.1, July 2006
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minimum. The trace of a square matrix is the sum of the diagonal elements of such matrix.

NUMERICAL INVESTIGATIONS

The mathematical formulations for the prediction of gravity anomalies have been discussed. Here, numerical investigations are carried out to determine the adequacy or otherwise of these formulations. The investigations include the determination of optimum covariance parameters and prediction of gravity anomalies at the observation stations. This is in addition to the separate determination of optimum covariance parameters and prediction of gravity anomalies at other stations using observed gravity anomalies of observation stations.

Determination of optimum covariance parameters and prediction of gravity anomalies at the observation stations

The process involves two stages:

(i) The values of Δg_p are represented by a third order polynomial function (16).

$$\Delta g_p = b_0 + b_1 g_i + b_2 g_i^2 + b_3 g_i^3 \qquad (16)$$

Where: g_i = gravity values at point i
b_0 , b_1, b_2, b_3 = constant coefficients

Putting these values of Δg_p in (11), the values of v_i are solved for.

The values of v_i are then used in (12), (13), (14) and (15) to obtain the initial values of covariance parameters $C(o)$ and 'a'.

(ii) The parameters obtained are used in (9) to evaluate the covariance function needed for the least squares collocation prediction of Δg_p at the observation stations. The predicted gravity anomalies are functionally given as:

$$\Delta g_p = C_{sl} C_{ll}^{-1} \Delta g \qquad (17)$$

The predicted gravity anomalies are then used in (10) to compute the value of F_i^2.

Thereafter, the values of the covariance parameters are allowed to vary and then used in (10) to compute the value of F_{i+1}^2. For a successful process, F_{i+1}^2 must be less than F_i^2. This is an iterative process, which continues until F_{i+1}^2 is minimum. (10) is satisfied when F_{i+1}^2 is minimum and R^2 is approximately equal to 1.

Determination of optimum covariance parameters and prediction of gravity anomalies at other stations

The covariance parameters obtained, in 3.1(i) above, are used in (9) to evaluate the covariance function required for the least squares collocation prediction of gravity anomalies of other stations using observed gravity anomalies of observation stations. Using (17), the predicted gravity anomalies (Δg_p) are next calculated. Then, the trace (Tr_i) of the error covariance matrix of the predicted gravity anomalies is computed.

FUTY Journal of the Environment, Vol. 1 No.1, July 2006

Thereafter, the values of the covariance parameters are allowed to vary and then used to compute another trace (Tr_{i+1}). For a successful process, Tr_{i+1} must be less than Tr_i. This is an iterative process, which continues until Tr_{i+1} is minimum.

Computer programs written in Fortran 77 language are used for all the computations. The results obtained are presented below:

PRESENTATION OF RESULTS

Extract of the searched covariance parameters used for the prediction of gravity anomalies at observation stations are shown in Table 3. Row 5 of this table shows the optimum values of the parameters. The results of gravity anomaly prediction at the observation stations using classical least squares and least squares collocation techniques are shown in table 4. These include gravity station numbers, observed gravity anomalies (Δg), predicted gravity anomalies using classical least squares technique (Δg_{ls}), predicted gravity anomalies using least squares collocation technique (Δg_{col}), difference between the observed gravity anomalies and predicted gravity anomalies of classical least squares technique (v_{ls}) and difference between observed gravity anomalies and the predicted gravity anomalies by least squares collocation technique (v_{col}).

Extract of the searched covariance parameters and traces used for the prediction of gravity anomalies outside the observation stations are shown in Table 5. Row 7 of this table shows the optimum values of the parameters with the minimum value of the trace. The error covariance matrix, which produced the minimum trace, is shown in table 6. The results of gravity anomaly prediction for these stations using least squares collocation technique are shown in table 7. These include gravity station numbers, observed gravity anomalies (Δg), predicted gravity anomalies (Δg_p) using least squares collocation technique and the differences between the observed gravity anomalies and predicted gravity anomalies (E). The parameters used for the statistical analysis of the results are shown in table 8. These are degree of freedom, upper limit of table statistic, computed statistic, lower limit of table statistic and trace of error covariance matrix respectively.

Table 3: Covariance parameters for prediction at observation stations

C(o) ($mGal^2$)	a (m)	F^2($mGal^2$)	R^2
2356.343089	1012.081477	0.000000000008403	1.000000000000
2359.343089	1016.081477	0.000000000008400	1.000000000000
2362.343089	1020.081477	0.000000000008398	1.000000000000
2365.343089	1024.081477	0.000000000008397	1.000000000000
2368.343089	1028.081477	0.000000000008396	1.000000000000
2371.343089	1032.081477	0.000000000008397	1.000000000000
2374.343089	1036.081477	0.000000000008398	1.000000000000

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Table 4: Comparison of predicted gravity anomalies by classical least squares and least squares collocation techniques at observation stations

Station no.	Δg (mGal)	Δg_{ls} (mGal)	Δg_{col} (mGal)	v_{ls} (mGal)	v_{col} (mGal)
D0301040	-31.928500	-32.162973	-31.928498	-0.234473	-0.000002
D0301060	-31.772200	-31.970325	-31.772200	-0.198125	0.000000
D0301100	-31.178600	-30.727544	-31.178599	0.451056	-0.000001
D0301120	-30.773000	-30.224369	-30.773000	0.548631	-0.000000
D0301160	-30.350500	-29.819916	-30.350499	0.530585	-0.000001
D0301200	-29.833000	-29.892698	-29.833000	-0.059698	-0.000000
D0301220	-29.736200	-29.946280	-29.736200	-0.210080	-0.000000
D0301260	-29.421700	-29.531857	-29.421700	-0.110157	-0.000000
D0301300	-29.262800	-29.469082	-29.262800	-0.206282	-0.000001
D0301318	-29.038700	-29.523856	-29.038700	-0.485156	0.000000
D0301346	-29.010000	-29.651968	-29.010000	-0.641969	-0.000000
D0301381	-28.987000	-29.869728	-28.987000	-0.882728	-0.000000
D0301420	-29.231300	-29.863871	-29.231300	-0.632571	-0.000000
D0301460	-29.609300	-29.935659	-29.609300	-0.326359	-0.000001
D0301480	-29.665300	-30.044471	-29.665300	-0.379171	0.000000
D0301500	-29.897100	-30.145885	-29.897100	-0.248785	-0.000001
D0301540	-30.442700	-30.183086	-30.442699	0.259614	-0.000001
D0301580	-30.579400	-30.255215	-30.579400	0.324185	-0.000000
D0301620	-31.330000	-30.150072	-31.329999	1.179928	-0.000001
D0301638	-31.223900	-30.213197	-31.223900	1.010703	0.000000
D0301660	-31.096500	-30.355059	-31.096500	0.741441	-0.000000
D0301700	-30.958100	-30.338371	-30.958099	0.619729	-0.000001
D0301720	-30.586200	-30.252909	-30.586200	0.333291	0.000000
D0301740	-30.112100	-30.023778	-30.112100	0.088322	-0.000001
D0301780	-29.173300	-29.462521	-29.173300	-0.289221	-0.000000
D0301800	-28.809800	-29.338324	-28.809800	-0.528524	-0.000001
D0301840	-27.492100	-28.356779	-27.492100	-0.864679	-0.000000
D0301860	-26.900200	-27.355042	-26.900200	-0.454842	-0.000000
D0301880	-26.256800	-26.646990	-26.256800	-0.390190	0.000000
D0301920	-25.579700	-24.524175	-25.579699	1.055526	-0.000001

Table 5: Covariance parameters and trace for prediction outside the observation stations

C(o) ($mGal^2$)	a (m)	Tr ($mGal^2$)
32.3430862	1565.0814209	1.7523
27.3430862	1570.0814209	1.4520
22.3430862	1575.0814209	1.1633
17.3430862	1580.0814209	0.8858
12.3430862	1585.0814209	0.6192
7.3430862	1590.0814209	0.3630
2.3430862	1595.0814209	0.1171
-2.6569138	1600.0814209	145.8184

FUTY Journal of the Environment, Vol. 1 No.1, July 2006

Table 6: Error covariance matrix of predicted gravity anomalies ($mGal^2$)

0.0087	0.0040	-0.0017	-0.0004	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0040	0.0095	-0.0063	-0.0018	0.0007	0.0001	-0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0017	-0.0063	0.0095	0.0041	-0.0016	-0.0002	0.0002	-0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0004	-0.0018	0.0041	0.0093	-0.0058	-0.0008	0.0008	-0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0002	0.0007	-0.0016	-0.0058	0.0081	0.0017	-0.0018	0.0011	-0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0001	-0.0002	-0.0008	0.0017	0.0018	-0.0023	0.0017	-0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	-0.0001	0.0002	0.0008	-0.0018	-0.0023	0.0052	-0.0051	0.0020	-0.0003	0.0002	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	-0.0001	-0.0005	0.0011	0.0017	-0.0051	0.0112	-0.0065	0.0011	-0.0006	0.0002	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0002	-0.0004	-0.0006	0.0020	-0.0065	0.0084	-0.0023	0.0012	-0.0004	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	-0.0003	0.0011	-0.0023	0.0086	-0.0071	0.0025	-0.0003	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	-0.0006	0.0012	-0.0071	0.0130	-0.0068	0.0009	-0.0001	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	-0.0004	0.0025	-0.0068	0.0080	-0.0016	0.0003	0.0001
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0003	0.0009	-0.0016	0.0050	-0.0013	-0.0003
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0001	0.0003	-0.0013	0.0054	0.0023
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	-0.0003	0.0023	0.0055

Table 7: Predicted gravity anomalies outside the observation stations by least squares collocation

Station no.	Δg (mGal)	Δg_p (mGal)	E (mGal)
D0301080	-31.482100	-31.495761	0.013661
D0301140	-30.585700	-30.515984	-0.069716
D0301180	-29.958800	-30.064126	0.105326
D0301240	-29.578400	-29.571339	-0.007061
D0301280	-29.177600	-29.395232	0.217632
D0301334	-29.024300	-28.977007	-0.047293
D0301360	-29.070900	-29.029677	-0.041223
D0301400	-29.197900	-29.027411	-0.170489
D0301440	-29.462800	-29.480838	0.018038
D0301520	-30.141500	-30.250076	0.108576
D0301560	-30.588100	-30.428336	-0.159764
D0301600	-30.817100	-31.041928	0.224828
D0301680	-31.218200	-31.102384	-0.115816
D0301760	-29.729900	-29.579175	-0.150725
D0301820	-28.225200	-28.166052	-0.059148

Table 8: Trace, computed and table statistics at α=0.05

Degree of freedom	15
Table statistic (Upper limit)	27.49
Computed statistic	10.0281
Table statistic (Lower limit)	6.26
Trace	0.1171

FUTY Journal of the Environment, Vol. 1 No.1, July 2006

ANALYSIS OF RESULTS

As shown in table 4, it can be seen that the predicted gravity anomalies of least squares collocation are better than those of classical least squares. This appears to confirm the supremacy of the least squares collocation technique over the classical least squares technique. In table 7, the predicted gravity anomalies of other stations compare favourably well with the "true" gravity anomalies for these stations. This tends to indicate a satisfactory level of reliability of the predicted anomalies outside the observation stations.

Statistical investigation is carried out to test the reliability of the predicted gravity anomalies obtained for the other stations (outside the observation stations). This is to show that the procedure used for the prediction has (or has not) introduced distortions in the predicted values. In order words, $E^TC_{ss}^{-1}E$ is statistically examined to know whether it falls within the specified confidence limits or not. This is achieved by means of Chi-Squares (χ^2) test. That is, we test the hypothesis:

Null hypothesis: $H_o : E^TC_{ss}^{-1}E = \sigma_o^2$ ($E^TC_{ss}^{-1}E$ is within the confidence limits)
Alternative hypothesis: $H_1 : E^TC_{ss}^{-1}E \neq \sigma_o^2$ ($E^TC_{ss}^{-1}E$ is outside the confidence limits)

Where $E^TC_{ss}^{-1}E/\sigma_o^2$ is the computed statistic (χ^2).

This is a two-tail test where the Null Hypothesis is rejected if the computed statistic is outside the confidence limits. The confidence limits are the upper limit and the lower limit of the table statistic. They are obtained in the statistical table as $\chi^2_{1-\alpha/2,df}$ for upper limit and $\chi^2_{\alpha/2,df}$ for lower limit, where α is the level of significance and *df* is the degree of freedom (number of observations (n) minus number of predicted gravity anomalies outside the observation stations (u)). From table 8, it can be seen that the value of computed statistic falls within the confidence limits. This suggests that the null hypothesis, that $E^TC_{ss}^{-1}E$ is within the confidence limit, should not be rejected. Therefore, it can be inferred that the technique used for the prediction has not introduced distortion in the values of the predicted gravity anomalies.

Also, the values of E are examined, using statistical t-distribution, to know whether or not they fall within the tolerant error limit (e) for the predicted quantities. The tolerance error limit (e) is defined by (18) and (19) as in Ayeni (2001).

$$e = \pm \sigma t_{u-1,1-\alpha/2} / u^{1/2} \quad (18)$$

$$\sigma = (E^TC_{ss}^{-1}E / (u-1))^{1/2} \quad (19)$$

From the statistical table, also in Ayeni (2001), $t_{u-1,1-\alpha/2} = 2.145$. The computed value of $\sigma = 0.846$. Hence, the value of $e = \pm 0.4687$. It can be seen, from table 7, that all the values of E fall within the tolerant error limit (e) thereby confirming the high level of reliability of the predicted gravity anomalies for geophysical exploration.

CONCLUSIONS AND RECOMMENDATIONS

An attempt has been made to predict gravity anomalies, using least squares collocation technique, for geophysical exploration. When compared with the classical least squares method in the prediction of gravity anomalies, the least squares collocation technique gave the better results. The accurate choice of covariance parameters for the design of covariance functions has helped to ensure the high level of reliability of the predicted gravity anomalies. The predicted gravity anomalies have been found to be reliable at the significance level of 0.05. Therefore, prediction of

FUTY Journal of the Environment, Vol. 1 No.1, July 2006
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gravity anomalies is recommended to improve the density distribution of the observed gravity anomalies of points in a survey area for geophysical exploration.

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