Available online at www.equijost.com
Equity Journal of Science and Technology, 2022 9(2): 18 - 27
ISSN 2354-1814; E-ISSN 2683-5961
EQUIJOST An Official Publication of Kebbi State University of Science and Technology, Aliero, Nigeria

Exponential Probability Distribution of Short-Term Rainfall Intensity

[1]Asani M. Afolabi, [2]Oke. Isaiah Adesola
*[1]Department of Urban and Regional Planning, Ladoke Akintola University Technology, Ogbomoso, Nigeria.
[2]Department of Civil Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria
*Corresponding Author's email: maasani@lautech.edu.ng

Abstract
This paper reports statistical goodness-of-fit evaluations of selected rainfall intensity duration data as a follow-up to previous studies on probabilities distribution. It is an application of Microsoft Excel Solver (MES) and the maximum likelihood method (MLM) to establish the performance of the Exponential distribution in predicting the distribution of selected rainfall intensity data. Rainfall intensity data from two locations in Nigeria (Makurdi and Abeokuta) was collected from the literature. The data was used to evaluate the potential of exponential probability distribution to predict and describe rainfall intensity. The constant in the probability distribution was determined using MLM and MES. The numerically determined constant of the density of Exponential distribution was estimated by the MLM and MES. The calculated Exponential probabilities using the estimated parameter were evaluated statistically (analysis of variance (ANOVA), relative error, model of' selection criterion (MSC), Coefficient of Determination (CD) and Correlation coefficient (R). The study established that the Exponential probability distribution's parameter (λ) is the mean of the natural logarithm of rainfall intensity using the MLM estimator. The parameters were 1.665 and 0.783 for Makurdi, and 1.695 and 0.754 for Abeokuta using MLM and MES, respectively. The relative errors were 0.659 and 1.008, and 0.743 and 1.141 for Makurdi and Abeokuta using MLM and MES, respectively. The correlation coefficient for Makurdi and Abeokuta using MLM and MES were 0.826 and 0.800, and 0.470 and 0.344, respectively. It was concluded that the MLM parameter was better than MES based on the values of MSC, CD, relative error and R. MLM predicted Weibull probability of rainfall intensity better than MES. It was concluded that the results are vital ingredients for the designers and managers of urban infrastructures. It was recommended that there is a need to evaluate the application of MLM and other probability distributions in environmental science and engineering.

Keywords: Exponential probability distribution, rainfall intensity, maximum likelihood estimate method (MLM), Analysis of variance (ANOVA), Intensity–Duration–Frequency (InDF)

1. Introduction

Reports of recent major floods not being successfully forecast worldwide are of great concern to civil and environmental engineers, urban and regional planners and hydrologists [1]. One of the significant causes making it hard to predict rainfall intensity is that the literature cannot specifically state the in-depth and the probabilities of rainfall intensity and duration. Plates 1.1 to 1.6 reveal the impacts of floods on the environment as surface water pollution carriers of waterborne diseases, and solid wastes to reservoirs and dams. Rainfall intensities of numerous occurrences and intervals are significant constants for the hydrologic and hydraulic design of culverts, storm sewers, water resources structures, flood control structures, and other water and environmental pollution control structures. These designs and planning can be accomplished by the rainfall Intensity–Duration–Frequency (InDF) relationship, which is resolute through rainfall frequency analysis [2]. These InDF relationships are the most vital tools in hydraulic, water resources, hydrology and environmental engineering for appraising the vulnerability of water resources and environmental structures as well as planning, designing, maintaining and operating the structures. Under the circumstances of planning and designing, a numeral of empirical relationships has been utilized for the probability of rainfall intensities in hydrology, water resources areas, and civil and environmental engineering. Numerous probability distributions are in use over the past three decades for modelling rainfall intensity data in areas of research such as environment, reliability, economics, engineering, biological studies, demography and medical sciences [3].

Two characteristic probability distribution expressions and functions used for rainfall intensity duration data analysis are the exponential and the Weibull probability density functions [4]. These probability distributions are alienated into two portions as follows [5 – 7]:

a. Discrete Probability Distributions (Poisson, Bernoulli, and Binomial Distributions)
b. Continuous Probability Distributions (Normal, Log-Normal, Continuous Uniform, and Exponential Distributions).

Plate 1.1: Front view of a community in Lokoja, Nigeria flooded with water

Plate 1.2: Aerial view of Lokoja City Covered with water

Plate 1.3: Rainfall-runoff with solid waste into a Curvet in Ile-Ife, Nigeria

There are several estimation methods to estimate the reliability of the distribution. These methods include MLM, least square and weighted least square estimation, Percentile estimation, Maximum product of estimation, Minimum spacing distance estimation, Crame´r-Von Mises estimation, Anderson-Darling and Right-tail Anderson-Darling estimation [8,9]. Literature provides information on probability distributions (Weibull, Normal, and log-normal), but there is little or no information on MLM and Exponential distribution. With the recent incidence of earthquakes, floods and other natural disasters [1] there is an urgent need to predict the occurrences of these natural disasters. With this advancement in computer applications and technologies, which makes it possible to collect rainfall intensity-duration data at various stations there is a need to utilize MLM, MES and exponential distribution for rainfall intensity analysis. This study, therefore, focuses on the utilization and evaluation of MLM, MES and exponential probability distribution (EPB) for rainfall intensity-duration data analysis, which can be a vital instrument for urban infrastructure designers, pollution control engineers and managers.

Plate 1.4: Rainfall runoff with plastic waste as floating solids in Ile-Ife, Nigeria

Plate 1.5: Storm runoff as flood along Ede Road in Ile-Ife

Plate 1.6: Storm runoff as flood along link Road in OAU, Ile-Ife

2. Materials and Method

Rainfall intensity-duration data from two stations (Abeokuta (1986 to 2010) and Makurdi (1979 to 2009)) were collected from literature namely David *et al*. [10] and Isikwue *et al*. [11], respectively. The data were analysed statistically using analysis of variance (ANOVA), mean, minimum, maximum, standard deviation and skewness. Skewness was computed as follows (equation 2.1):

$$S_{kr} = \frac{\sum_{i=1}^{N}(R_i - R_{Av})^3}{\delta^3} \qquad 2.1$$

Where; S_{kr} is the computed skewness, R_i is the rainfall intensity (mm), R_{AV} is the rainfall intensity and δ is the variance of the rainfall intensity. The probability of the rainfall intensity was computed using Weibull probability mathematical expression as follows [8,11] Equations 2.2, and 2.3):

$$T_m(x) = \frac{n+1}{m} \qquad 2.2$$

Where; T_m is the return period, n is the sample size and m is the rank.

$$f(x) = P_m(x) = \frac{1}{T_m} \qquad 2.3$$

Where; p_m (x) is the theoretical probability (probability index) and f(x) is the cumulative probability

The Weibull distribution is the most preferred in modelling the rainfall intensity data. The constant of the EPB was calculated using the MLM and MES. The calculated EPB's constant (MLM and MES methods) was used to establish the probability distributions, which were evaluated statistically using analysis of variance ANOVA), Relative error, Model of selection criterion (MSC), Coefficient of Determination (CD) and Correlation coefficient (R). MSC indicates higher precision, soundness, and a goodness fit of the methods. MSC was calculated using equation (2.4) as follows:

$$MSC = \ln\left(\frac{\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{obs})^2}{\sum_{i=1}^{n}(Y_{obsi} - Y_{cali})^2}\right) - \frac{2P}{n} \qquad 2.4$$

Where; Y_{obsi} is the probability value using the Weibull probability mathematical expression; $\bar{Y}_{obs}$ is the average probability value using the Weibull probability mathematical expression; p is the total number of fixed parameters to be estimated in the methods; n is the total number of rainfall intensities calculated, and Y_{cali} is the probability computed using the MLM and MES estimators.

The coefficient of determination (CD) can be taken as the percentage of expected data variation that can be clarified by the obtained data. Higher values of CD indicate higher accuracy, validity and good fitness of the device. CD, correlation coefficient, and relative error can be expressed as follows (Equations 2.5, 2.6 and 2.7):

$$CD = \frac{\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{obs})^2 - \sum_{i=1}^{n}(Y_{obsi} - Y_{cali})^2}{\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{cali})^2} \qquad 2.5$$

Where; $\bar{Y}_{cali}$ is the average probability value calculated using the MLM estimator.

$$R = \sqrt{\frac{\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{obs})^2 - \sum_{i=1}^{n}(Y_{obsi} - Y_{cali})^2}{\sum_{i=1}^{n}(Y_{obsi} - \bar{Y}_{cali})^2}} \qquad 2.6$$

$$R_{el}(\%) = \left(\frac{1}{N}\right)\sum_{i=1}^{N}\left(\frac{Y_{obsi} - Y_{cali}}{Y_{obsi}}\right) \qquad 2.7$$

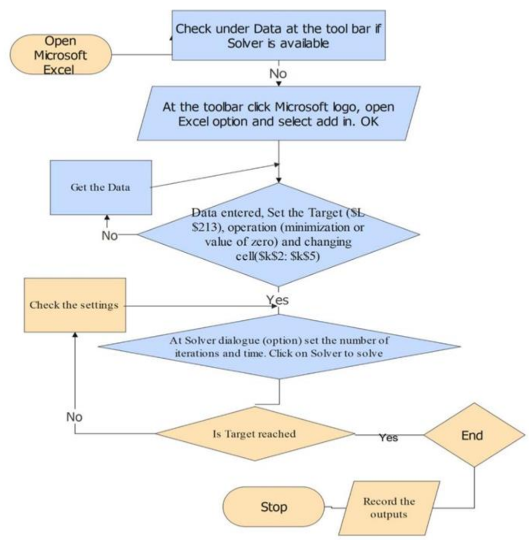

Figure 2.1: Summary of the Microsoft excel solver procedures

Figure 2.1 above, presents the summary of the Microsoft Excel Solver procedures as a flowchart. MES was used for the estimation of these empirically derived parameters based on availability at no additional cost. The procedure used for the Microsoft Excel solver can be summarized as follows:

i. Excel solver was added in Microsoft Excel,
ii. Target of the numerical solution $\sum_{i=1}^{N}(K_{pi} - K_{ti})^2 = 0$, operation, and changing cells were set, where; K_p is the probability value using the Weibull mathematical expression $\left(f(x) = P_m(x) = \frac{1}{T_m}\right)$ and K_t is the Exponential distribution probability calculated using MLM $f(x) = \lambda exp^{-\lambda x_i}$; and
iii. Microsoft Excel Solver was allowed to iterate at 200 iterations with 0.005 tolerance

3. Results and discussion

Figures 3.1 and 3.2 present the rainfall intensity data from David *et al*. [10], while Figure 3.3 shows the rainfall intensity data from Isikwue *et al*. [12]. These Figures established that the highest rainfall-duration-intensity frequency arose when the duration time was 5 min in the year 1 (1979, [11] and 1986, [10] and the lowest rainfall-duration intensity frequency happened when the duration was 1440 min in the 30th year (2009, [11] and 2010, [10]). Table 3.1 presents the statistical properties (average, maximum, minimum, standard deviation and Skewness) of the rainfall intensity data in respect of Abeokuta. Table 3.2 reveals the statistical summary (average, maximum, minimum, and standard deviation) of the rainfall intensity data for Makurdi. From Table 3.1, the averages of rainfall intensity for Abeokuta were 206.40, 164.54, 135.14, 117.83, 85.64, 69.06, 55.33, 41.23, 31.72, 22.81, 19.02, 15.78 and 11.59 mm/h for a duration time of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300 and 420 min, respectively. These results established that heavy rainfalls had the lowest duration and the lowest rainfall intensities had the highest duration. The other statistical properties (maximum, minimum and standard deviation) followed the same trend as the averages. From Table 3.1, the Skewness of the rainfall intensities was between 0.16 and 1.32; all these durations had positive Skewness, which indicated that most of the values of these rainfall intensities concentrated on the right of the mean, with extreme values to the left.

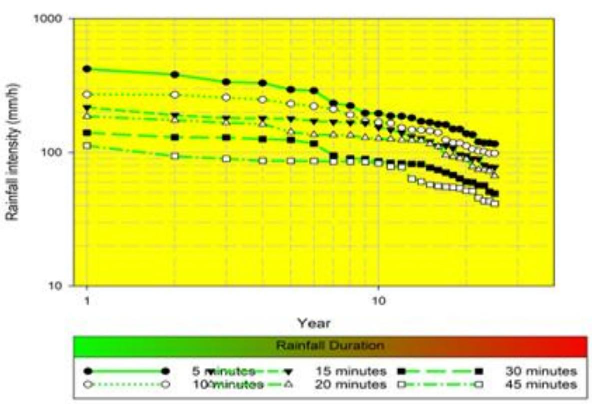

Figure 3.1: Rainfall intensity of Abeokuta (duration of between 5 and 45 min)

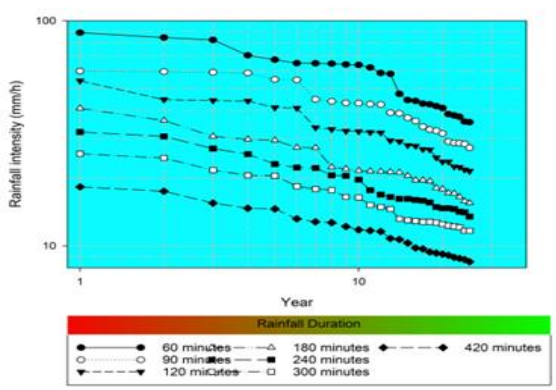

Figure 3.2: Rainfall intensity of Abeokuta (duration of between 60 and 420 min)

Table 3.2 presents the average, maximum, minimum, skewness and standard deviation of the rainfall intensities for Makurdi at different return periods. The averages of the rainfall intensities were 12.119, 22.815, 36.819, 69.320, 11.870, and 180.541 mm/ h for the return period of 100, 50, 25, 10, 5, and 2 years respectively. These statistical trends (average, maximum, minimum, and standard deviation) of rainfall for Tables 3.1 and 3.2 agreed with literature such as Abouammoh [13]; Gupta and Huang [14]; Hassan *et al*. [15]; Lidiya *et al*. [16]; Lihou and Spence [17]; Yoon-Su, *et al*. [18]; Madsen *et al*. [19]; Mahmoudi and Sepahdar [20]; Tramblay *et al*. [21]; Wagh and Kamalja [22]; De Paola *et al*. [23] and Morales and Vicini [24]. Skewness is a measure of the asymmetry of a distribution. A distribution is asymmetrical when its left and right sides are not mirror images. A distribution can have the right (or positive), left (or negative), or zero skewness. A right-skewed distribution is longer on the right side of its peak, and a left-skewed distribution is longer on the left side of its peak. It has been reported that when the bulk of the data is at the left and the right tail is longer, the distribution is skewed right or positively skewed; if the peak is toward the right and the left tail is longer, the distribution is skewed left or negatively skewed. This indicates that the data was non-zero skewness.

In summary, classifications of skewness from the tables can be grouped as follows:

a) skewness of between −1.000 and -0.500 or between 0.500 and 1.000, the distribution can be called moderately skewed.
b) skewness of between −0.500 and 0.500, the distribution can be called approximately symmetric.
c) skewness of less than −1.000 or greater than 1.000, the distribution can be called highly skewed.

These values of skewness for the rainfall can be said to be right-skewed distribution and none was zero-skewed distribution for both Abeokuta and Makurdi. This indicated that in the selection of rainfall intensity for the engineering infrastructure care must be taken in the selection of appropriate skewness.

Table 3.3 reveals the result of an ANOVA of the rainfall-duration intensity frequency (Abeokuta) reverence to the years. From the Table, the $F_{24, 300} = 1.652$ and $p = 3.02 \times 10^{-2}$ for analysis of the rainfall-duration intensity frequency among the years. This result discovered that there were substantial differences between these values within the years at a 95 % confidence level ($p < 0.05$). Table 3.4 presents the outputs from an ANOVA of these frequencies within the duration of the rainfall. The Table reveals that the $F_{12, 312} = 84.32$ and $p = 2.47 \times 10^{-90}$ for analysis of the frequencies amid the duration of the rainfall. This result established that there was a momentous difference between frequency values within these durations at a 95 % confidence level ($p < 0.05$).

Table 3.1: The statistical properties (average, maximum, minimum, standard deviation and Skewness) of the rainfall intensity data in respect of Abeokuta

Duration	5	10	15	20	30	45	60	90	120	180	240	300	420
Average	206.40	164.54	135.14	117.83	85.64	69.01	55.33	41.23	31.72	22.81	19.02	15.78	11.59
Maximum	421.20	271.20	217.20	186.30	140.60	112.40	88.60	59.80	54.20	40.90	32.10	25.70	18.30
Minimum	115.80	98.40	77.40	66.50	49.10	41.30	35.50	27.30	21.50	15.50	13.50	11.70	8.50
Standard deviation	86.80	57.50	40.66	33.57	27.28	19.67	16.08	10.94	8.68	6.46	5.28	4.12	2.77
Skewness	1.14	0.63	0.16	0.25	0.64	0.26	0.50	0.53	0.95	1.32	1.20	1.07	1.00

Table 3.2: The statistical summary (average, maximum, minimum, and standard deviation) of the rainfall intensity data for Makurdi

Return Period	2	5	10	25	50	100
Average	12.119	22.815	36.819	69.320	111.870	180.541
Maximum	24.550	46.220	74.590	140.430	226.640	365.760
Minimum	7.350	13.840	22.340	42.060	67.880	109.540
Standard deviation	4.506	8.484	13.692	25.777	41.599	67.135
Skewness	0.928	0.928	0.928	0.928	0.928	0.928

Table 3.3: The result of an ANOVA of the rainfall-duration intensity frequency (Abeokuta) with respect to the years

Source of Variation	**Sum of Square**	**Degree of freedom**	**Mean Sum of Square**	**F-Value**	**P-value**
Between Years	181955	24	7581.458	1.651739	0.030253
Within Years	1376996	300	4589.985		
Total	1558951	324			

Table 3.4: The outputs from an ANOVA of rainfall-duration-intensity frequency within the duration of the rainfall

Source of Variation	**Sum of Square**	**Degree of freedom**	**Mean Sum of Square**	**F-Value**	**P-value**
Between Rainfall intensity duration	1191745	12	99312.08	84.38	2.47×10^{-90}
Within Rainfall intensity duration	367205.6	312	1176.941		
Total	1558951	324			

Table 3.5: The result of an ANOVA of the rainfall-duration intensity frequency (Makurdi) with respect to the return period

Source of Variation	**Sum of Square**	**Degree of freedom**	**Mean Sum of Square**	**F-Value**	**P-value**
Between Return periods	721710.3	5	144342.1	120.5911	6.21×10^{-59}
Within Return periods	244178.7	204	1196.954		
Total	965889	209			

Table 3.6: The outputs from an ANOVA of rainfall-duration-intensity frequency within the duration of the rainfall

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value
Between durations	147235.3	34	4330.449	0.925701	0.590208
Within durations	818653.7	175	4678.021		
Total	965889	209			

Table 3.5 reveals the result of an ANOVA of the intensities (Makurdi) with reverence to the return period. From the Table, the $F_{5, 204} = 120.59$ and $p = 6.21 \times 10^{-59}$ for analysis of the intensities within the return periods (years). This result exposed that there were noteworthy differences between the intensities values within the years at a 95 % confidence level ($p < 0.05$). Table 3.6 reveals the outputs from an ANOVA of the intensities within the duration of the rainfall. The Table shows that the $F_{34, 175} = 0.926$, and $p = 5.90 \times 10^{-1}$ for analysis of the intensities between the duration of the rainfall. This result revealed that there was no significant difference between the intensity values within these durations at a 95 % confidence level ($p > 0.05$). The results of ANOVA indicated that agreed with previous studies such as Madi *et al*.[44] on Bayesian prediction of rainfall records using the generalized exponential distribution, Pisarenko *et al*.[43] on the application of the theory of extreme events to problems of approximating Probability distributions of water flow peaks, Parisa *et al*.[34] on the climate change impact on short-duration extreme precipitation and intensity–duration–frequency curves, Vivekanandan [2] on the analysis of hourly rainfall data for the development of IDF relationships using the order statistics approach of probability distributions, Tramblay *et al*.[21] on the non-stationary frequency analysis of heavy rainfall events and De Paola *et al*.[23] on the Intensity-Duration-Frequency (IDF) rainfall curves, for data series and climate projection.

3.1 Establishment of exponential parameter using maximum likelihood method

The log-likelihood expression of this random sample is specified as follows [24 - 26], as Equation 2.8):

$$L(x_1, x_2, x_3, x_4, x_5, \dots\dots\dots\dots\dots\dots x_n) = \sum_{i=1}^{n} \ln f(x_i, \theta) \qquad 2.8$$

$X_1, X_2 \dots\dots X_n$ are randomly selected samples of the size of n from a given distribution with the probability density expression f(x, θ), where θ ($\theta_1, \theta_2, \dots, \theta_k$), θ is subset θ, is the unknown parameter. θ is in general vector parameter and assumes X_1, X_2, --- X_n be a comprehension of the random sample. The maximum likelihood estimates (MLE) θ of the parameter θ are the values of θ that maximize (1) with respect to θ. MLE is a systematic practice for estimating values in a probability model from a set of data samples. Suppose the sample size $X_1, \dots, X_n$ has been attained from a probability model specified by mass or density function f (x; θ) dependent on values θ lying in parameter space θ.

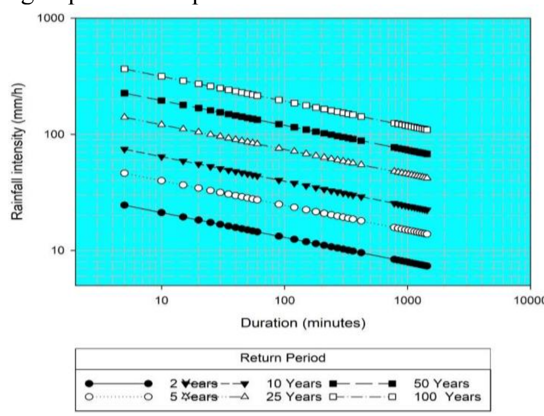

Figure 3.3: Rainfall intensity of Makurdi (return period of between 2 and 100 years)

The MLE is produced as follows [27, 28]:

a. Express the likelihood function (that is, the product of the n mass or density function terms; where the i^{th} term is the mass or density expression evaluated at x_i) observed as an expression of θ., L(θ), as follows (as Equations 3.9 and 3.10):

$$L(\theta) = \prod_{i=1}^{n} fx(x_i, \theta) \qquad 3.9$$

Take the natural log of the likelihood, collect terms involving θ

$$\ln(L(\theta)) = \ln\left[\prod_{i=1}^{n} fx(x_i, \theta)\right] \qquad 3.10$$

Find the value of θ subset of θ, θ, for which log L(θ) is maximized by differentiation. If θ is a single value, find θ by solving numerical equations (3.11 and 3.12), respectively

$$\frac{d}{d\theta}[\ln(L(\theta))] = \frac{d}{d\theta}\left\{\ln\left[\prod_{i=1}^{n} fx(x_i, \theta)\right]\right\} \qquad 3.11$$

In the parameter space θ. If θ is vector-valued, say $\theta = (\theta_i, \dots, \theta_n)$, then find $\theta = (\theta_i, \dots, \theta_n)$ by simultaneously solving the n equations given by

$$\frac{\partial}{\partial\theta_j}[\ln(L(\theta))] = \frac{\partial}{\partial\theta_j}\left\{\ln\left[\prod_{i=1}^{n} fx(x_i, \theta)\right]\right\} = 0; j = 1 \dots . k \qquad 3.12$$

In parameter space θ. The EPB can be articulated as follows [29 -35], as expressed in equations 3.13- 18, respectively):

$$f(x) = \lambda exp^{-\lambda x_i} \quad 3.13$$

$$L(x) = \prod_{i=1}^{n} fx(x_i, \lambda) = \lambda^n exp^{-n\lambda x_i} \quad 3.14$$

$$\ln[L(x)] = \ln\left[\prod_{i=1}^{n} fx(x_i, \lambda)\right]$$

$$= n\ln(\lambda) - \lambda\sum_{i=1}^{n} x_i \quad 3.15$$

$$\frac{d}{d\lambda}[\ln(L(x))] = \frac{d}{d\lambda}\left\{\ln\left[\prod_{i=1}^{n} fx(x_i, \lambda)\right]\right\}$$

$$= \frac{d}{d\lambda}\left[n\ln(\lambda) - \lambda\sum_{i=1}^{n} x_i\right] \quad 3.16a$$

$$\frac{d}{d\lambda}\left[n\ln(\lambda) - \lambda\sum_{i=1}^{n} x_i\right] = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i \quad 3.16b$$

$$\frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0 \quad 3.17$$

$$\lambda = \frac{1}{n}\sum_{i=1}^{n} x_i \quad 3.18$$

Equation (3.18) shows that the EPB's parameter (λ) is the mean of the natural logarithm of rainfall intensity. Table 3.7 presents the values of the EPB's parameter (λ) and statistical evaluation of the performance of the obtained parameter compared with Weibull's probability theory of extreme rainfall events. The values of the EPB's parameter (λ) are between 1.665 and 1.695, 0.783 and 0.754 for MLM and MES methods, respectively. On the statistical evaluation of the performance MSC, error, CD and R for MLM and MES methods are between 1.044 and 1.339, -0.028 and 0.303, 0.659 and 0.743, 1.008 and 1.141, 0.640 and 0.682, 0.118 and 0.221, and 0.800 and 0.826, and 0.344 and 0.470 for Makurdi and Abeokuta, respectively. These results revealed that MLM performed better than MES in predicting the probability distribution of rainfall intensity. These lower performances of this parameter by MES are similar to the performance of negative binomial distribution which agreed with the literature such as Telles [36] on measuring nonlinearity by means of static parameters in Bernoulli binary sequences distribution, Jemilohun and Ipinyomi [37]on the Weibull Poisson Distribution: Properties, Inference, and Applications to lifetime data, Rinne [38] on the Weibull Distribution and Barnett *et al.*[39] with information on Combining Negative Binomial and Weibull Distributions for Yield and Reliability Prediction. In addition, the lower performance of the MES method can be attributed to the weak relationship between Weibull probability and Exponential distribution as agreed with the literature [39 – 44]. Figures 3.4 and 3.5 present the values of EPB's parameters obtained using MLM and MES, and the performance of these methods compared with the standard Weibull method. Figures 3.4 and 3.5 established that the Exponential distribution is a continuous distribution as the probability did not discontinue between certain rainfall intensities for both Abeokuta and Makurdi data. Unlike the Bernoulli distribution in which the probability of the rainfall intensity discontinued for the estimator using the MES method between 104.54 mm/h and 124.82 mm/h. Tables 3.8 and 3.9 provide information on the statistical analysis (ANOVA) of the parameters and statistical evaluations of the effects of the two methods.

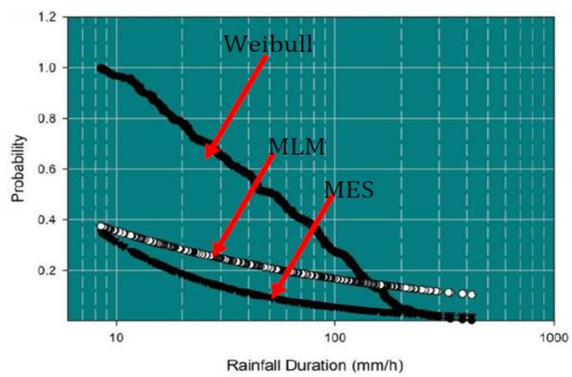

Figure 3.4: Relationship between probabilities obtained using the methods (Abeokuta data)

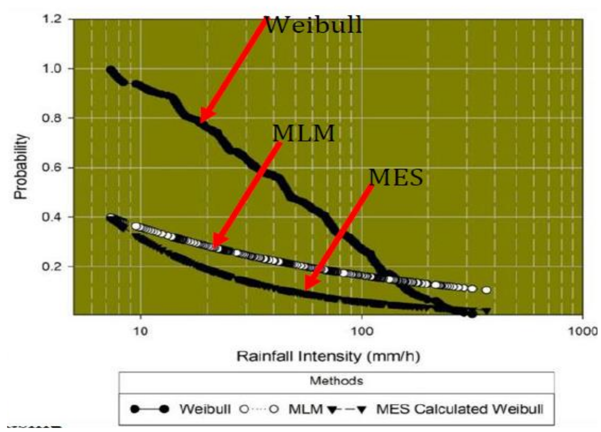

Figure 3.5: Relationship between probabilities obtained using the methods (Makurdi data)

Table 3.7: the values of Exponential probability distribution's parameter and performance of these methods compared with standard Weibull method

Summary	Parameter		Error		MSC		CD		R	
	MLM	MES	MLM	MES	MLM	MES	MLM	MES	MLM	MES
Makurdi	1.665	0.783	0.659	1.008	1.044	-0.028	0.682	0.221	0.826	0.470
Abeokuta	1.695	0.754	0.743	1.141	1.339	0.303	0.640	0.118	0.800	0.344

Tables 3.8 and 3.9 revealed that the values of the parameter were between 0.754 and 1.695 for both MES and MLM estimator methods. These parameter values were similar to those obtained in literature such as Chacko and Mohan [8] and Jemilohun and Ipinyomi. [37]. Results of the ANOVA for these parameters (Table 3.8) established that there was a significant difference between these parameters obtained using the two estimators and methods at a 95 % confidence level ($F_{1,2}$ = 564.098 and p = 0.00177, which is less than 0.05). Tables 3.8, 3.9, 3.10 and 3.11 present the outputs from ANOVA conducted on the statistical evaluation of the effects of selected factors on Exponential distribution.

The Table (Table 9) established that locations had no significant effects on these parameters obtained using the two methods at a 95 % confidence level ($F_{1,2}$ = 0.0070 and p = 0.941, which is greater than 0.05). Table 10 established that the method had significant effects on the exponential distribution of rainfall intensities data at a 95% confidence level ($F_{7,8}$ = 19.306 and p = 2.04 x 10 $^{-4}$, which is less than 0.05). Table 11 established that locations had no significant effects on the exponential probabilities obtained using the two methods at a 95 % confidence level ($F_{1,4}$ = 0117 and p = 0.737, which is greater than 0.05)

Table 3.8: Effects of the methods on the Exponential distribution parameters

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value
Between Methods	2.111383	7	0.30162608	19.30639	0.000204
Within Methods	0.124985	8	0.01562312		
Total	2.236368	15			

Table 3.9: Effects of the Locations on the Exponential distribution parameters

Source of Variation	Sum of square	Degree of freedom	Mean Sum of Square	F- Value	P-Value
Between Locations	2.111383	1	0.01859451	0.11738	0.736983
Within Groups	2.217773	14	0.158412358		
Total	2.236368	15			

Table 3.10: Effects of the parameters on the Exponential distribution

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value
Between Parameters	0.303076	1	0.303076	564.0981	0.001768
Within Parameters	0.001075	2	0.000537		
Total	0.304151	3			

Table 3.11: Effects of the locations on the Exponential distribution

Source of Variation	Sum of Square	Degree of Freedom	Mean Sum of Square	F-Value	P-Value
Between locations	0.001064	1	0.001064	0.007024	0.940841
Within Locations	0.303086	2	0.151543		
Total	3				

4. Conclusions

This study was conducted on the statistical goodness-of-fit evaluations of selected rainfall intensity duration data as a follow-up to previous studies on probabilities distribution. It is an application of MES, MLM and Exponential distribution of selected rainfall intensity data. Rainfall intensity data from two locations in Nigeria (Makurdi and Abeokuta) was collected from the literature. The data was used to evaluate the potential of exponential probability distribution to predict and describe rainfall intensity. The constant in the probability distribution was determined using MLM and MES. The numerically determined constant of the density of Exponential distribution was estimated by the MLM and MES. The calculated Exponential probabilities using the estimated parameter were evaluated statistically (analysis of variance relative error, model of' selection criterion, Coefficient of Determination and Correlation coefficient). It was concluded based on the findings that Exponential distribution performance was better than Bernoulli distribution in describing the rainfall intensities for both Abeokuta and Makurdi; the MLM estimator was better than MES based on the values of MSC, CD, relative error, and R. MLM estimator predicted Weibull probability of rainfall intensity better than MES. It was recommended that there is a need to evaluate the MLM estimator and other probability distributions (such as log Normal, Gamma, and Poisson distributions), and information like this should be available to the designers of urban infrastructures (especially drainage and sewers design) and urban settlement managers for effective integration developments.

References

1. Wu MH, Wang JP, Ku KW. Earthquake, Poisson and Weibull distributions. Physica A: Statistical Mechanics and Its Applications. 2019; 526, 121001
2. Vivekanandan N. Analysis of hourly rainfall data for the development of IDF relationships using the order statistics approach of probability distributions. *International Journal of Management Science and Engineering Management*. 2013; 8(4):283–291
3. Dikko HG, Faisal AM. A New Generalized Exponential Weibull Distribution Its Properties and Application. *Bayero Journal of Pure and Applied Sciences.* 2017; 10(2): 29 – 37.
4. Pierac A. Some useful formulae for Monte Carlo Simulations Relating to The Weibull and to the exponential probability distribution functions. *Fatigue Fract. Engng Mater. Struct*. 1997; 20(1):109-117.
5. Fiondella L, Zeephongsekul P. Trivariate Bernoulli distribution with application to software fault tolerance. *Annals of Operations Research*. 2015; 244(1):241–255.
6. Gao Z, Tao J, Zhou D, Zeng X. Efficient parametric yield estimation over multiple process corners via bayesian inference based on Bernoulli distribution. *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*. 2019; 1–1. Available from: doi:10.1109/tcad.2019.2940682
7. Jones LD, Vandeperre LJ, Haynes TA, Wenman MR. Theory and application of Weibull distributions to 1D peri-dynamics for brittle solids. Comput. *Methods Appl. Mech. Engrg.* 2020; 363, 112903
8. Chacko M, Mohan R. Bayesian analysis of Weibull distribution based on progressive type-II censored competing risks data with binomial removals. *Computational Statistics.* 2017. Available from: https://doi.org/10.1007/s00180-018-0847-2
9. Almarashi AM, Algarni A, Nassar M. On estimation procedures of stress- strength reliability for Weibull distribution with application. *PLoS ONE*. 2020; 15(8): e0237997. Available from: https:// doi.org/10.1371/journal.pone.0237997
10. David AO, Nwaogazie L, Agunwamba JC. Modelling rainfall intensity by optimization technique in Abeokuta, South-West, Nigeria. *Journal of Engineering Research and Reports*. 2019; 6(4): 1-10.
11. Teyabeen AA, Akkari FR, Jwaid AE. Comparison of seven numerical methods for estimating Weibull parameters for wind energy applications. In: *2017 UKSim-AMSS 19th International Conference on Modelling & Simulation.* 2017.p. 173- 180.
12. Isikwue MO, Onoja SB, Laudan KJ. Establishment of an empirical model that correlates rainfall-intensity-duration-frequency for Makurdi area, Nigeria. *International Journal of Advances in Engineering and Technology*. 2012; 5(1): 40-46
13. Abouammoh AM. The distribution of monthly rainfall intensity at some sites in Saudi Arabia. *Environ Monit Assess*. 1991; 17: 89–100
14. Gupta RC, Huang J. Analysis of survival data by a Weibull-generalized Poisson distribution. *Journal of Applied Statistics.* 2014; 41(7):1548–1564.
15. Hassan DS, Ahmed SA, Alwan JK. Estimating three parameters of Weibull — Poisson distribution with the biological application. In: *8th International Conference on Information Technology (ICIT)*. 2017. Available from: doi:10.1109/icitech.2017.8079990
16. Lidiya N, Aleksandrovskaya LN, Ardalionova AE, Papic L. Application of probability distributions mixture of safety indicator in risk assessment problems. *Int J Syst Assur Eng Manag.* 2018. Available from: https://doi.org/10.1007/s13198-019-00760-6
17. Lihou DA, Spence GD. Proper use of data with the Weibull distribution. *Journal of Loss Prevention in the Process Industries.* 1988; 1(2):110–113
18. Yoon-Su J, Seung-Soo S, Kun-Hee H. High-dimensionality priority selection scheme of bioinformatics information using Bernoulli distribution. *Cluster Comput.* 2016. Available from: DOI 10.1007/s10586-016-0622-5
19. Madsen H, Gregersen IB, Rosbjerg D, Arnbjerg-Nielsen K. Regional frequency analysis of short duration rainfall extremes using gridded daily rainfall data as co-variate. *Water Science and Technology*. 2017; 75(8): 1971–1981.
20. Mahmoudi E, Sepahdar A. Exponentiated Weibull–Poisson distribution: Model, properties and applications. *Mathematics and Computers in Simulation*. 2013; 92:76-97
21. Tramblay Y, Neppel L, Carreau J, Najib K. Non-stationary frequency analysis of heavy rainfall events in southern France. *Hydrological Sciences Journal.* 2013; 58(2):280–294
22. Wagh YS, Kamalja KK. Comparison of methods of estimation for parameters of generalized Poisson distribution through simulation study. *Communications in Statistics - Simulation and Computation.* 1–15. Available from: doi:10.1080/03610918.2015.1105971

23. De Paola F, Giugni M, Topa ME. Intensity-Duration-Frequency (IDF) rainfall curves, for data series and climate projection in African cities. Springer-Plus. 2014; 3:133. Available from: https://doi.org/10.1186/2193-1801-3-133
24. Morales FEC, Vicini L. A non-homogeneous Poisson process geostatistical model with spatial deformation. *Advances in Statistical Analysis*. 2020. Available from: doi:10.1007/s10182-020-00373-6
25. Brosius J. An alternative method for the calculation of joint probability distributions. Application to the expectation of the triplet invariant. *Acta Cryst*. 2015: A71: 76–81
26. Couton F, Danech-Pajouh M, Broniatowski M. Application of the mixture of probability distributions to the recognition of road traffic flow regimes. *Transportation Systems*. 1997; 715- 720
27. Kazuki S, Masato K, Ken-Ichi O, Hirotaka T. Probability Distributions of means of IA and IF for Gaussian noise and its application to an anomaly detection. *Advances in Data Science and Adaptive Analysis.* 2017. Available from: doi: 10.1142/S2424922X18500067
28. Pichugina SV. Application of the Theory of Truncated Probability Distributions to Studying Minimal River Runoff: Normal and Gamma Distributions. *Water Resources.* 2008; 35(1): 23–29.
29. Pobočíková I, Sedliačková Z, Michalková M. Application of four probability distributions for wind speed modelling. TRANSCOM 2017: International scientific conference on sustainable, modern and safe transport. *Procedia Engineering*. 2019; 192: 713 – 718
30. Santos LSF. The energy density distribution of an ideal gas and Bernoulli's equations. *Eur. J. Phys.* 2019; 39: 035102. Available from: https://doi.org/10.1088/1361-6404/aaa34c
31. Pichon F. Canonical Decomposition of Belief Functions Based on Teugels' Representation of the Multivariate Bernoulli Distribution. *Information Sciences*. 2017. Available from: doi:10.1016/j.ins.2017.10.030
32. Yashunsky AD. Limit Points of Bernoulli Distribution Algebras Induced by Boolean Functions. *Lobachevskii Journal of Mathematics.* 2016; 40(9):1423–1432
33. Pham H, Pham DH. A Median-Based Machine-Learning Approach for Predicting Random Sampling Bernoulli Distribution Parameter. *Vietnam Journal of Computer Science*. 2019; 6(1):17–28
34. Parisa H, Tabaria PH, Willems P. Climate change impact on short-duration extreme precipitation and intensity–duration–frequency curves over Europe. *Journal of Hydrology*. 2020; 590 5249
35. Sulbhewar LN, Raveendranath P. Performance of consistent through-thickness electric potential distribution for Euler-Bernoulli piezoelectric beam finite elements. *Int. J. Computer Aided Engineering and Technology*. 2017; 9(2):179–198.
36. Telles CR. Measuring nonlinearity by means of static parameters in Bernoulli binary sequences distribution: a brief approach. *International Journal of Modeling, Simulation, and Scientific Computing*. 2017. Available from: doi: 10.1142/S179396232050021X
37. Jemilohun VG, Ipinyomi RA. Alpha Power Extended Inverse Weibull Poisson Distribution: Properties, Inference, and Applications to lifetime data. *International Journal of Statistics and Probability*. 2022; 11(1): 10 – 28
38. Rinne H. The Weibull Distribution A Handbook Chapman & Hall/CRC Taylor & Francis Group 6000 Broken Sound Parkway NW. *Suite 300 Boca Raton, FL.* 2019; 33: 487-2742
39. Barnett TS, Grady M, Singh AD, Purdy KG. Combining Negative Binomial and Weibull Distributions for Yield and Reliability Prediction. *IEEE Design & Test of Computers*. 2006; 06: 0740-7475
40. Eggermont PP, LaRiccia LN. Maximum Penalized Likelihood Estimation Volume II: Regression. Springer Series in Statistics. *Springer Dordrecht Heidelberg.* 2009; 234- 256
41. Ward MD, Ahlquist JS. *Maximum Likelihood for Social Science Strategies for Analysis.* Cambridge: University Printing House; 2018.
42. Wentzel-Larsen T, Anhøj J. Joint distribution for number of crossings and longest run-in independent Bernoulli observations. The R package cross run. PLOS ONE. 2019; 14(10): e0223233. Available from: doi: 10.1371/journal.pone.0223233
43. Pisarenko VF, Bolgov MV, Osipova NV, Rukavishnikova TA. .Application of the Theory of Extreme Events to Problems of Approximating Probability Distributions of Water Flow Peaks. Water Resources. 2019; 29(6): 593–604.
44. Madi MT, Raqab MZ. Bayesian prediction of rainfall records using the generalized exponential distribution. Environmetrics. 2007; 18(5): 541–549. Available from: doi:10.1002/env.826
45. Afify AZ, Gemeay AM, Alfaer NM, Cordeiro GM, Hafez EH. Power-Modified Kies-Exponential Distribution: Properties, Classical and Bayesian Inference with an Application to Engineering Data. *Entropy*. 2022; 24(7):883. https://doi.org/10.3390/e24070883