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# TOOL PATH GENERATION ALGORITHM AND 3D TOLERANCE ANALYSIS FOR FREE-FORM SURFACES

Suresh Babu V[*][1] and Raviswaran N[2]

[1] Department of Mechanical Engineering, Faculty of Engineering, Bahir Dar University, Bahir Dar, Ethiopia
[2] Godrej India Limited, Ambattur, Chennai, TN, India
[*] Corresponding author; E-mail: babu_suresh_123@rediffmail.com, Tel: 0582-20750

**Abstract:** This paper focuses on developing algorithm that generates tool paths for free-form surfaces based on the accuracy of a desired manufactured part. A manufacturing part is represented by mathematical curves and surfaces. Using the mathematical representation of the manufacturing part, we generate reliable and near optimal tool paths as well as cutter location data file for post - processing. This algorithm includes two components. First is the forward-step function which determines the maximum distance, called forward step, between two cutter contact points with a given tolerance. This function is independent of the surface type and is applicable to all continuous parametric surfaces that are twice differentiable. The second component is the side-step function which determines the maximum distance, called side-step, between two adjacent tool paths with a given scallop height. This algorithm reduces manufacturing and computing time as well as the cutter - contact points while keeping the given tolerance and scallop height in the tool paths. The parts, for which the cutter contact points are generated using the proposed algorithm, are machined using a three axes milling machine. As part of the validation process, the tool paths generated during machining are analyzed to compare the machined part and the desired part.

**Key words**: Free form surface, Cutter location data, Cutter contact, Forward step function, Side step function, Tool path.

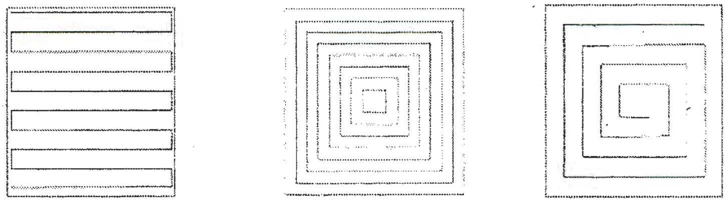
**Abbreviations**: CAD – Computer Aided design, CAM – Computer Aided Manufacturing, MCU – Machine Control Unit, CL – Cutter Location, CC – Cutter Contact, APT – Automated Programmed Tool, NC – Numerical Control, CNC – Computer Numerical Control.

## Introduction

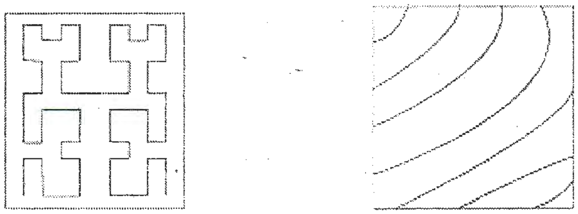
Process - planning is the function within a manufacturing facility that establishes which processes and parameters are to be used to convert a part from its initial form to a final form predetermined in an engineering drawing. Initial material commonly takes the form of bar stock, plate, casting, forging, or may be just a slab of metal. With these raw materials as a base, the process planner must prepare a list of processes to convert this predetermined material into a predetermined final shape. The complete process planning system has different modules viz., surface identification, material selection, process selection, machine selection, tool selection, fixture selection, end effectors selection, process sequencing, cutter path generation and intermediate surface generation. Each module may require execution several times in order to obtain an "optimum" process plan.

The input to the system will be a three-dimensional model from a computer-aided design (CAD) data base. The model contains not only the shape and dimensioning information, but also the tolerance and special features. The process plan can be routed directly to the production planning system and production control system. Time estimates and resource requirement can be sent to the production planning system for scheduling. The part program, cutter location (CL) file, and material handing control program can also be sent to the control system. Process planning is the critical bridge between design and manufacturing. Design information can be translated into manufacturing language only through process planning. Today both computer-aided design (CAD) and manufacturing (CAM) have been implemented. Integrating, or bridging, these functions require automated process planning (Chang *et. al.,* 1998).

The cutter path generation module is the focus of this paper. In this module, numerical control (NC) plays a very important role to transform the raw material into a finished part specified on an engineering design that is either design on paper or in a CAD model. NC system consists of three basic components: i) A program of instructions, ii) A machine control unit and iii) Processing equipment. MCU systems for NC can be divided into two types. a) Point-to-point b) Continuous path *Point-to-point systems,* also called positioning system, move the work table to a programmed location without considering the path taken to get to that location. Once the move has been completed, some processing action is accomplished by the work - head at the location such as drilling or punching a hole. Continuous path systems generally refer to systems that are capable of continuous simultaneous control of two or more axes. The term contouring is used when continuous path control is used for simultaneous control of two or more axes in machining operations. The maximum error between the desired surface and the finished surface can be controlled by the length of the individual line segments, which is one of the main tasks of this paper. The computer's role in computer-aided part programming consists of the following tasks: input translation, arithmetic and cutter offset computations, editing and post-processing. The first three tasks are carried out under the supervision of the language processing program. The fourth task, post-processing, requires a separate computer program. The output of this module is a file called CLFILE, which stands for "cutter location (CL) file". This file consists mainly of tool path data. In the editing phase, the CLFILE is edited, and a new file is generated called CLDATA. CLDATA provides readable data on cutter locations and machine tool operating commands. The output of post-processing is a part program consisting of G-codes, *x*, *y*, and *z*-coordinates, S, F, M, and other functions in word address format (Groover, 1987).



Zig-zag Contour Spiral



Space filling Sequential

Figure 1. Strategies of tool path distribution

The commonly used tool path distribution strategies are 1. Zig - zag or raster curves, 2. Contour curves, 3. Spiral curves, 4. Space filling curves, and 5. Sequential generated curves which are shown in Fig 1.

The designed part can be represented by parametric curves and surfaces such as B´ezier curves and surfaces. The B'ezier curve using de Casteljau algorithm, the blossoms and mathematical representation of Bezier curve are used in the earlier representations. The mathematical representation is based on Foley and Van Dam (1997), Farin (2002), Faux and Pratt (1979), Ait-Haddou and Herzog (2002) and Ramshaw (1989).

Differential geometry of surfaces: A surface may be given by an implicit form *f* (*x*; *y*; *z*) = 0 or by its parametric form. The Cartesian coordinates of a parametric surface x in terms of two parameters are shown below.

$$\mathbf{r}(u,v) = \begin{bmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{bmatrix} ; \quad \mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix} \in [a,b] \subset \mathbb{R}^2$$

Points on the surface S are obtained by varying the parameters *u* and *v*. Where the Cartesian coordinates *x; y; z* of a surface point are differentiable functions of parameters *u* and *v* and [*a, b*] denotes a rectangle in the *u- v* plane. As in the case of a curve, the surface S also has a frame consisting of three orthogonal unit vectors. This frame is referred to as the surface frame and is represented as follows:

$$\mathbb{F}(u,v) = \begin{bmatrix} \hat{t}_1(u,v) \\ \hat{t}_2(u,v) \\ \hat{n}(u,v) \end{bmatrix}$$

The vectors $t_1$ and $t_2$ represent the unit vectors and the vector ˆ01n represents the unit normal vector of the surface *S* at given values of the parameters *u* and *v* as shown below. The surface normal ˆn at an arbitrary point P (*u*, *v*) is expressed as

$$\hat{n} = \frac{X_u \times X_v}{|X_u \times X_v|}$$

An embedded curve P (*u* (*t*); *v* (*t*)) on the designed surface S passing through a point Pp represents a parametric curve on the designed surface. The square of the differential arc length at Pp, along curve P, on the surface can be expressed as follows.

$$I = P^t \cdot P^t = E\frac{du}{dt}\frac{du}{dt} + 2F\frac{du}{dt}\frac{dv}{dt} + G\frac{dv}{dt}\frac{dv}{dt}$$

is referred to as the first fundamental-form, where "t" is the independent variable along the path and information of metric properties of the surface such as measurement of lengths, areas and angles.

$$E = P^u \cdot P^u; \quad F = P^u \cdot P^v; \quad G = P^v \cdot P^v$$

The quadratic

$$II = L\frac{du}{dt}\frac{du}{dt} + 2M\frac{du}{dt}\frac{dv}{dt} + N\frac{dv}{dt}\frac{dv}{dt}$$

is referred to as the second fundamental-form where

$$L = P^{uu} \cdot n, \quad M = P^{uv} \cdot n, \quad N = P^{vv} \cdot n$$

are the coefficients of the second fundamental form. The second fundamental form *II* is of interest in deriving expression for curvature of a surface. The second fundamental form provides measurement of change in the unit normal along the given curve on the surface.

## Methodology

New method developed for generating tool paths for free-form surfaces can be represented by parametric curves and surfaces. It is narrated how the CC point is on a tool and how the CL point is a reference point by which the tool moves along a surface in removing material, then conceptual approach by which generation of tool path is outlined. After this the new methods for tool paths generation is detailed. The algorithm used to calculate forward-step size, and side-step size is explained. A proper tool must be used in machining that covers all the chip-making processes (such as milling, drilling, turning, and boring). When the degree of complexity for machining is increased, the number of tool selections and work-part materials also increases. There are several factors to be considered in machining: first, selecting tool material and geometry involves material, shape, size of the work-part, design requirement, and operation type (roughing or finishing); second, deciding on machining conditions such as feed rate, spindle speed, and depth of cut. The metal cutting process is the removal of work piece materials to obtain a designed part. In this work, we assume that the operation is milling with a ball-end tool. To reduce machining error we have considered two kinds of points on the tool as shown in Fig 3. The proposed approach in this research is derived from an overall conceptual approach as explained briefly below. The overall conceptual approach is also summarized in the flowchart shown in Fig 4, to define the designed surface in the *u-v plane,* calculate the forward-step size with given tolerance, *e,* convert the forward-step size from the physical domain into the parametric domain, convert CC points to CL points, calculate side-step, *g* with given scallop height, *h*, convert side-step the physical domain into the parametric domain, convert CC points to CL points points.

**Calculation of Forward - step:** Each tool path is approximated by a series of line segments whose accuracy of tool path is controlled by deviation (Figure 2). Each segment amount is a forward - step and the maximum deviation is called tolerance. To calculate forward-step, we use first and second derivatives; therefore this function is independent of surface types and is applicable to all continuous parametric surfaces that are twice differentiable.

**Calculation of side - step:** For the purpose of machining, the designed part is approximated by a series of parametric curves and the distance between two adjacent tool-paths is a finite distance called the side-step. The side-step, in general, may vary along the machined surface and the un-machined region between two adjacent tool paths (the scallop or cusp). The upper limit on the height of this scallop is called the scallop-height-allowance. Typically, the desired value of the scallop height is given, from which the side-step, *g*, is determined.

## Implementation

The proposed solution approach was developed and implemented. There are several parts for which the CC points were generated using the proposed algorithm and were subsequently machined using a 3-axis milling machine with different tolerance and scallop height. The hardware and software used to implement proposed algorithms are:

1. **Hardware:**

   HMT Vertical Machining Center with WPLM control Software.
   Roland - 3D Laser scanner and its software.

2. **Software:**

   MATLAB 6.0
   AUTOCAD 2007
   SURVEYOR SCAN CONTROL
   GEOMAGIC QUALIFY 5.0

The proposed algorithms were coded in MATLAB on a personal computer (Pentium IV, 1.33 GHz CPU, 512 Mb of physical memory) operating under Microsoft XP Professional. We machined a free-form shaped part using a block of wax and measured tolerance between the machined surface and the desired surface. After machining, a 3D laser machine (Roland) with Surveyor Scan Control software (point cloud method) was used to scan the machined surface. GEOMAGIC QUALIFY 5

## Results

In the first column, *"Scallop"* and *"Tolerance"* represent maximum allowed tolerance and scallop height. The second column, SE, represents the maximum scallop height on the machined surface. TE stands for maximum error of tolerance. "Average" in the fourth column represents the average tolerance and scallop height over the surface. The fifth column, "Number of CL points", is for CL points used to create the NC-code by which the machined surface was generated. The last column, "Total Length" shows the total length of tool paths. The total length of tool paths generated by proposed approach was compared with total length of tool paths of efficient iso-scallop approach proposed by Lin and Koren (1994). They also showed the efficient iso-scallop approach is efficient machining compared with iso-parametric machining. The total length of the tool paths generated by proposed approach is shorter than the total length of the tool paths for the efficient iso-scallop approach. Therefore, the proposed approach is efficient machining compared with the iso-scallop and iso-parametric approach. The total lengths of tool paths of the proposed approach are 1705.356 and 955.04 mm and they are 1717.548 and 977.138 mm of the iso-scallop approach with tolerance and scallop height of 1.25 and 0.254mm, respectively. According to the Table 1. the proposed algorithm reduced CL points significantly and almost all areas of tolerance and scallop height were within the given maximum allowed tolerance and scallop height for the free-form surface. However, the tolerance around a few CC points is a little higher than the given tolerance and some scallop heights are a little higher as well. In Table I, the maximum error of scallop height and tolerance are not greater than 0.43*mm* and 0.25*mm*. We conclude that the proposed approach is efficient machining compared with the iso-scallop approach and the maximum machining error is from 0.254mm to 0.4318mm for the tolerance and scallop height.

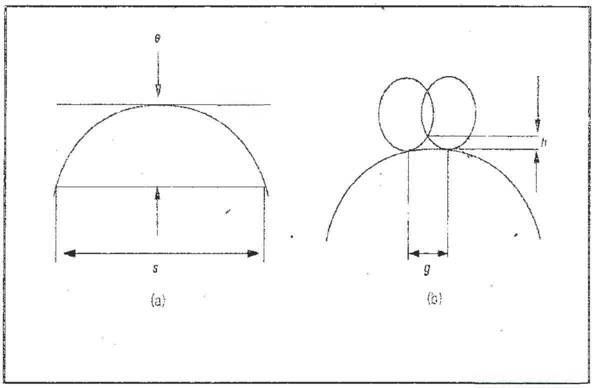


Figure 2. Forward and side step

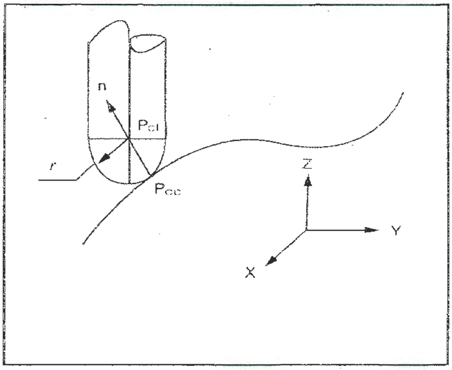


Figure 3. The CC and C L point

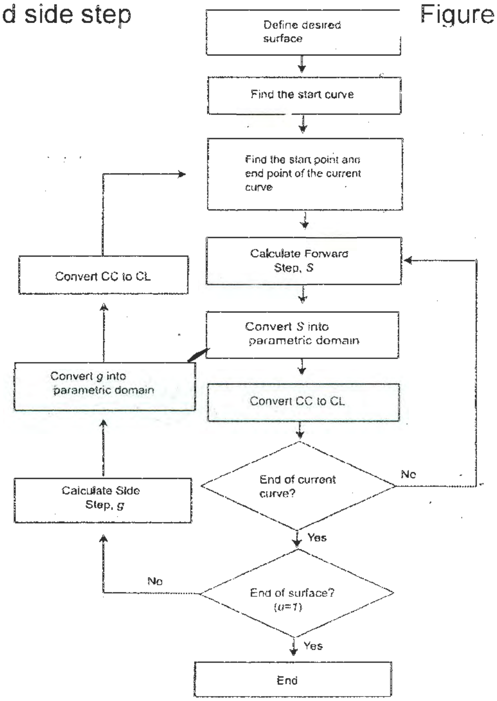


Figure 4. The overall procedure

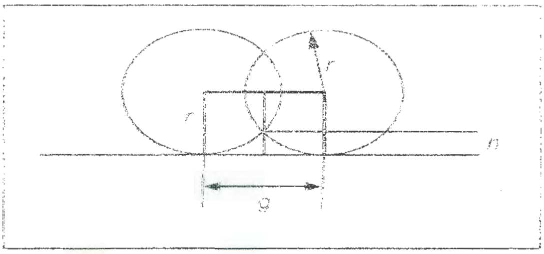


Figure 5. Tool position on flat surface

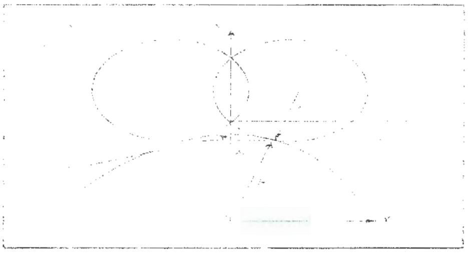


Figure 6. Tool position on curved surface

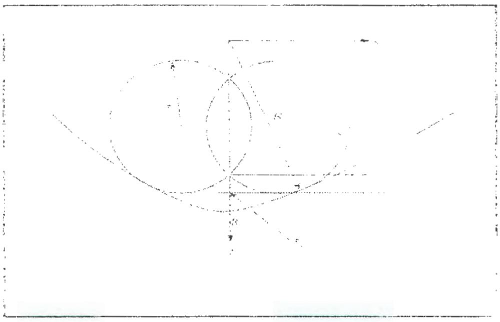


Figure 7. Tool position on curved surface

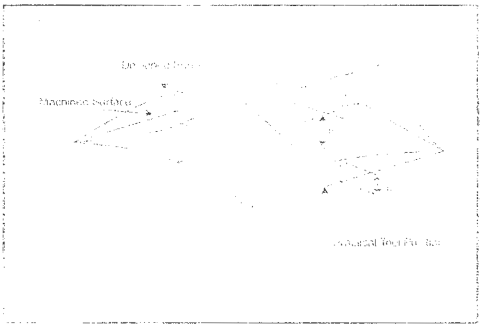


Figure 8. practical tool position on convex surface

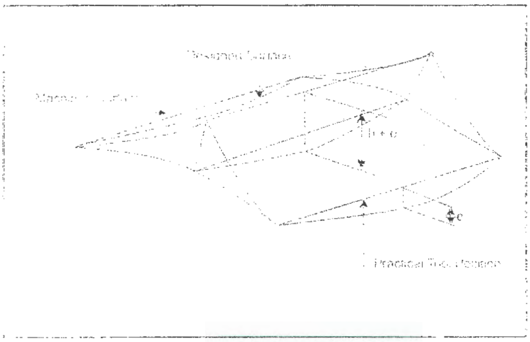


Figure 9. Practical Tool position on concave surface

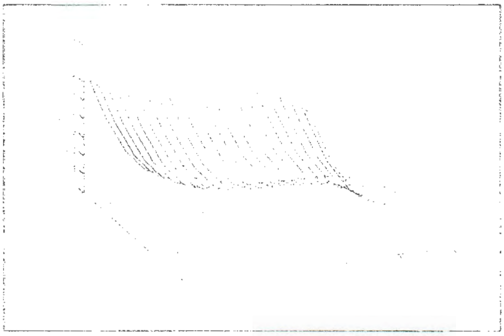


Figure 10.Tool path

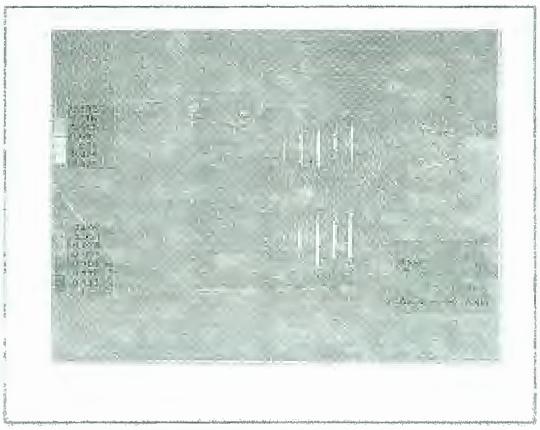


Figure 11. Comparision between desired part and tolerance 1.25mm

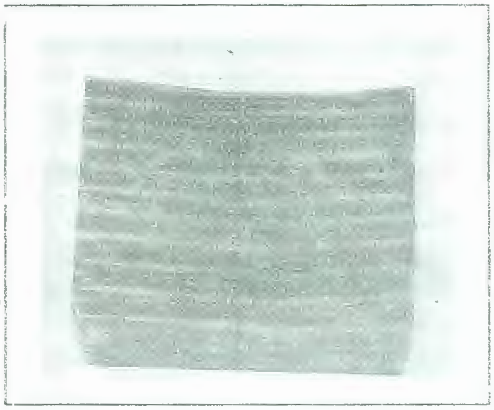


Figure 12. Machined part with tolerance 1.25mm

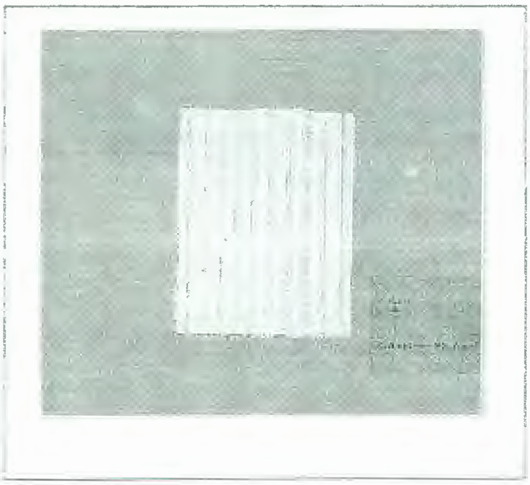


Figure 13. Scanned surface with tolerance 1.25mm

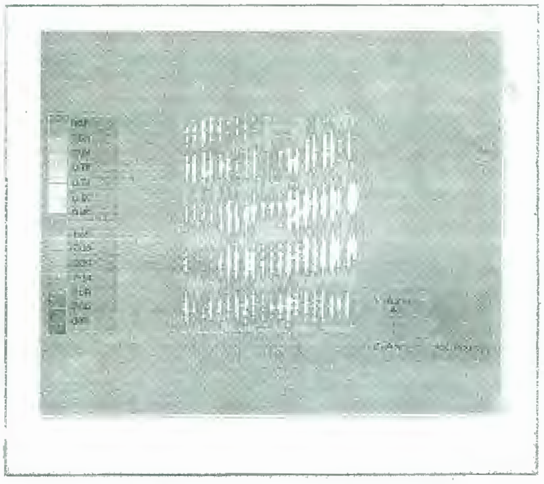


Figure 14. Comparision between desired part and Machined part with tolerance 0.25mm

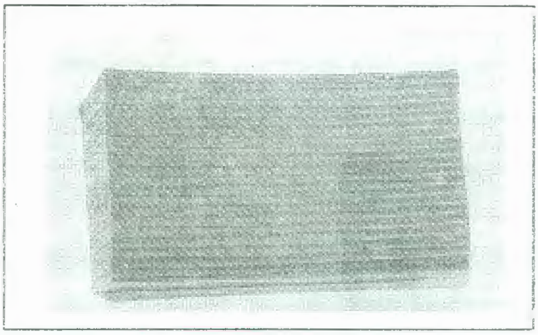


Figure 15. Machined part with tolerance 0.25mm

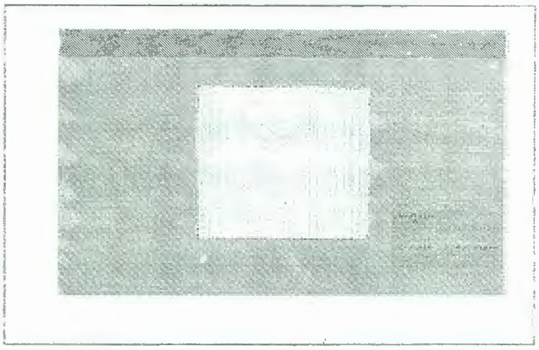


Figure 16. Scanned surface with tolerance 0.25mm

## Discussions

The proposed algorithm for tool path generation was developed and implemented successfully through the integration of mathematical modeling used for calculating forward and side-step size into the core of the our algorithms. The present study was contributed towards developing a new method for tool path generation in milling operations and verifying true machining error in milling operations. Using the mathematical representation, we determined forward-step size; from there we have developed a method for side-step size by studying the geometry of the tool and the differential geometry of the designed part. It is then verified for true machining errors by comparing machined and designed surfaces using the point cloud method. The implementation of this algorithm shows that it is very efficient for finish machining and the algorithm involved one iteration compared to existing methods. Additional contribution is related to mathematical representation of manufactured parts through the use of parametric curves and surfaces. As a conclusion, we list some advantages of this work. 1. We reduced CL points significantly (by which NC code was generated). For example, the designed part was generated by 100 x 100 points. However, we generated machined surface by less than 160 CL points with predetermined scallop height and tolerance for small test problems. As a result of this, the manufacturing data generated from machining also decreased significantly, that is we reduce cost of data manipulation as well as storage. 2. We verified true machining errors by comparing designed surface and machined surface using the point cloud method. Previous efforts have relied upon computational approach (Huang and Oliver, 1994) and our work provides superior verification via true machining. 3. Our method is independent of surface types and is applicable to all continuous parametric surfaces that are twice differentiable. Therefore, this approach is well suited for sculptured and analytic surfaces.

## References

Ait-Haddou, R. and Herzog, W. (2002). Convex subdivision of a Bezier curve. *Computer Aided Geometric Design*. 19: 663-671

Bobrow, J., E. (1985). NC machine tool path generation from CSG part representations. *Computer Aided Geometric Design*, 17(2): 69-76.

Chang, T., C., Wysk, R., A., and Wang, H., P. (1998). Computer-aided Manufacturing, Prentice-Hall, Upper Saddle River, NJ.

Foley, J., D., and VanDam, A. (1997). Computer Graphics: Principles and Practice. Addison-Wesley Publishing Company, 2nd edition, New York.

Faux, I., D., and Pratt, M., J. (1979). Computational Geometry for Design and Manufacture. Ellis Horwood Limited, Chichester, West Sussex, England.

Farin, G. (2002). Curves and Surfaces for CAGD. Academic Press, 5th edition. San Diego, CA.

Groover, M., P. (1987). Automation, Production Systems, and computer Integrated Manufacturing. Prentice-Hall, Upper Saddle River, NJ.

Huang, Y., and Oliver, J. (1994). Non-constant parameter NC tool path generation on sculptured surfaces. *International Journal of Advanced Manufacturing Technology*, 9(2): 281-290.

Lin, R., S., and Koren, Y. (1994). Efficient tool-path planning for machining free-from surfaces. *Transactions of the ASME*, 118: 20-28.

Ramshaw L (1989). Blossoms are polar forms. Computer Aided Geometric Design, 6: 323-358.

Sarma, R. (1996). NC tool path synthesis. Ph.D. dissertation. University of Michigan, Ann Arbor, MI.