Volume and Implicit Taper Functions For *Cupressus Lusitanica* and *Pinus Patula* Tree Plantations in Ethiopia

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Abstract

Data from Cupressus lusitanica and Pinus patula were used to develop total and exponential form merchantable volume models, and implicit taper functions. The exponential form merchantable volume model to a specified top diameter limit showed marked improvement compared with the unbounded non exponential form merchantable volume model of Burkhart (1977). Implicit taper functions derived from the exponential form merchantable volume models were found superior to taper functions obtained from the non exponential merchantable volume models. In general, these models are essential management tools for the plantation of the species and in particular provide stock volume estimates by end use type.

***Key words**: C. lusitanica, Merchantable volume models, Taper functions, Total tree volume models, P. patula.*

Introduction

Tree volume function is a basic tool in quantifying volume and value of forest stands. It is also important for growth and yield studies and for evaluating response to silvicultural treatments. Hence, it is an essential tool in forest planning and management processes. Individual tree volume usually refers to the volume of the commercially marketable portion of the tree. As a result, the central activities of both researchers and managers in plantation forests give due focus on the production and precise estimation of the merchantable stem volume of the trees.

For prediction of total tree stem volume, a multitude of tree volume functions are published in forestry literature, usually by species type. Because of inherent morphological differences among tree species, it is generally necessary to develop separate standard volume equations for each species or closely related species group (Burkhart and Gregoire, 1994). In tree volume models, diameter at breast height (D) usually at 1.3 meter height from the ground, and total tree height (H) tend to account the greatest proportion of the variability in volume. Commonly used total tree bole volume (V_t) models (see, for example, Avery and Burkhart (2002); Clutter *et al.* (1983)) are (1) by Spurr (1952) and (2) by Schumacher and Hall (1933):

$$V_t = \beta_1 + \beta_2 D^2 H + \varepsilon \quad (1)$$

$$V_t = \beta_1 D^{\beta_2} H^{\beta_3} + \varepsilon \quad (2)$$

where β_i's are parameters to be estimated from the data , ε is error and the rest as defined previously. The *D*, *H* and V_t measurements for estimating parameters in (1) and (2) are obtained from felled sample trees representing the full range of the population of interest. Models are fitted to measurements conducted on felled trees, so as to minimize measurement error and its consequent effect on parameter estimation. Kozak and Smith (1993) recommended that trees should be selected in such a way that the sample will cover the whole range of diameters at breast height and tree height, with more or less uniform frequency. They noted that sample data selected in this way yields much more stable models relative to random sample.

It has been a common practice to develop a new tree volume equation, as required, in response to changes in the upper-bole merchantability diameter limit. However, such costly and perhaps duplicative effort was eliminated since Burkhart (1977) introduced a merchantable volume ratio equation based on upper stem diameter (*d*). Assuming that total volume (V_t) is given from reliable total volume models such as model (1) or (2), merchantable volume to any top diameter or height may be obtained as

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$V_m = V_t R + \varepsilon$ (3)

where V_m is merchantable volume to a specified top diameter (*d*) or height (*h*) limit, R is monotonic function that describes the ratio of V_m to V_t.

Different exponential and non exponential forms of R functions have been proposed by several authors (Burkhart, 1977; Van-Deusen *et al.*, 1981; Alemdag, 1988; Clark and Saucier, 1990; Tasissa *et al.*, 1997). Teshome (2005) used model (4) of Burkhart (1977) and a modified Burkhart model (5) by Cao and Burkhart (1980) in developing merchantable volume equations to upper *d* and *h* limits, respectively for *C. lusitanica* tree plantation of Shashemene Forest Industry Enterprise (SFIE) in Ethiopia:

$$V_d = Vt \quad (1+\beta_1 \, d^{\beta_2} D^{-\beta_3}) + \varepsilon \qquad (4)$$

$$V_h = Vt \ (1+\alpha_1 \, (H-h)^{\alpha_2} H^{-\alpha_3}) + \varepsilon \qquad (5)$$

where V_d and V_h are merchantable volume models to *d* and *h* limits, respectively, α_i's are parameters and others as defined before. These models (4) and (5) are referred to non exponential form merchantable volume ratio (NEMV) models in this study. In the NEMV models, if *d* is equal to zero such as in (4), and *h* is equal to *H* in (5), the ratio becomes one and thus merchantable volume is equal to total stem volume. However, as noted in Van-Deusen *et al.* (1981) and Tasissa *et al.* (1997), model (4) is unbounded and yields illogical volume estimates as *d* tends to approach stump diameter. This was also noted as a cautionary remark in Teshome (2005).

Van-Deusen *et al.* (1981), Clark and Saucier (1990), and Tasissa *et al.* (1997) presented exponential form merchantable volume ratio (EMR) models for the prediction of tree merchantable volumes. Tasissa *et al.* (1997) developed merchantable volume to any top diameter outside bark (*Vd*) and height from the ground (*Vh*) using equations (6) and (7), respectively for loblolly pine trees.

$$Vd = Vt \quad \exp(\beta_4 \, d^{\beta_5} D^{-\beta_6}) + \varepsilon \qquad (6)$$

$$Vh = Vt \ \exp(\alpha_4 \, (H-h)^{\alpha_5} H^{-\alpha_6}) + \varepsilon \qquad (7)$$

In this study models (6) and (7) are referred to exponential form merchantable volume ratio (EMV) models. The EMV models possess desirable properties such that, as diameter outside bark approaches infinity, the ratio goes to zero, ensuring that predicted merchantable volume goes to zero thereby avoiding illogical negative volumes at the lower portion of the tree bole.

For utilization purposes, it is desirable to merchandize trees into multiple products which necessitates the development of a taper function (McTague and Bailey, 1987). Knoebel *et al.* (1984) derived implicit taper functions from the NEMV models. Such implicit taper functions can be derived by equating the NEMV models (4) and (5). The derivation of these implicit taper functions is based on the assumption that merchantable volumes to a specified *h* and its corresponding *d* limit are equal. Hence, by equating the NEMV models and with some algebraic manipulation, taper functions (8) and (9) can be obtained for predicting diameter outside bark (d_r) and height up the stem (h_r), respectively:

$$d_r = \left(\frac{\alpha_1}{\beta_1}\right)^{\frac{1}{\beta_2}} (H\text{-}h)^{\frac{\alpha_2}{\beta_2}} \, H^{\frac{-\alpha_3}{\beta_2}} \, \mathrm{D}^{\frac{\beta_3}{\beta_2}} + \varepsilon \quad , (8)$$

$$h_r = H\text{-} \left(\frac{\beta_1}{\alpha_1}\right)^{\frac{1}{\alpha_2}} d^{\frac{\beta_2}{\alpha_2}} \, D^{\frac{-\beta_3}{\alpha_2}} \, \mathrm{H}^{\frac{\alpha_3}{\alpha_2}} + \varepsilon \qquad (9).$$

Similarly, by equating the EMV models (6) and (7) and with some algebraic manipulation, taper functions (10) and (11) can be obtained for predicting diameter outside bark (d_e) and height up the stem (h_e), respectively:

$$d_e = \left(\frac{\alpha_4}{\beta_4}\right)^{\frac{1}{\beta_5}} (H\text{-}h)^{\frac{\alpha_5}{\beta_5}} \, H^{\frac{-\alpha_6}{\beta_5}} \, \mathrm{D}^{\frac{\beta_6}{\beta_5}} + \varepsilon \quad ,(10)$$

$$h_e = H\text{-} \left(\frac{\beta_4}{\alpha_4}\right)^{\frac{1}{\alpha_5}} d^{\frac{\beta_5}{\alpha_5}} \, D^{\frac{-\beta_6}{\alpha_5}} \, \mathrm{H}^{\frac{\alpha_6}{\alpha_5}} + \varepsilon \quad .(11)$$

The taper functions (10) and (11) were used by Tasissa *et al.* (1997) to develop taper equations for thinned and unthinned *loblolly pine* trees in cutover, site-prepared plantations in USA.

Teshome (2005) has used NERM models to construct merchantable volume ratio models and their associated taper functions for *C. lusitanica*, one of the two species considered in this study. The accuracy and precision of such taper functions are direct result of the accuracy and precision of the merchantable volume equations from which they are derived (Clutter, 1980). In light of the unboundedness problem of the NEMV (Van-Deusen *et al.*, 1981; Tasissa *et al.*, 1997) and possible effect on their implied taper functions (Clutter, 1980), this study is motivated to fit EMV models and their associated taper functions. To this end, the NEMV model (4) was compared with EMV model (6). Similarly, the EMV models associated taper functions (10) and (11) were compared with their corresponding taper functions (8) and (9) derived from the NEMV models .Thus, the objective of this study was to develop total tree volume, merchantable volumes and associated taper functions for *C. lusitanica* and *P. patula* plantations of the Shashemene Forest Industry Enterprise (SFIE) in Ethiopia. To date, no such effort has been made to *P. patula* while Teshome (2005) has developed NEMV models and their derived taper functions for *C. lusitanica*. It is believed that such models are important tools for the forest planning and management of the SFIE as well as other training and research institutions.

Materials and Methods

Data

For this study, 204 *C. lusitanica* and 196 *P. patula* sample trees were taken from the SFIE plantation in Ethiopia. SFEI is one of the major lumber and wood products supplier in the country and is located in the Oromia region about 250 kms south of Addis Abeba, Ethiopia. *C. lusitanica* and *P. patula* are the major lumber plantations of the SFEI.

Age and diameter distribution as well as site factors were taken into account in the sampling process based on records available and information from the technical staff of the SFEI. Before felling, the diameter at breast height (D) and other lower bole portion diameters at 0.2 (stump height), 0.35, 0.50, 0.65, 0.80, and 1 m were measured. After felling, total height (H), and diameters at one meter intervals from D to top of the tree were measured. Diameter records were the average of two measurements taken at perpendicular position to each other along the axis of the tree bole. For computing total tree volume, log volume between consecutive diameter measures was calculated using Smalian's formula while the top section was computed from a cone formula. To develop merchantable volume models, 5124 *C. lusitanica* and 5022 *P. patula* pairs of diameter and height measurements were taken. Table 1 presents descriptive statistics of the data.

Models

The widely used total tree bole volume models (1) and (2) (see Avery and Burkhart (2002); Clutter *et al.* (1983)) were evaluated to develop the total tree volumes for *C. lusitanica* and *P. patula* tree plantations of the SFIE. To construct merchantable volume model for these tree species, NEMV model (4) and EMV model (6) were compared. These models estimate tree merchantable volume to any upper diameter limit. Such models are the most practical and commonly used in practice as compared to those merchantable volume models which predict tree volume to upper height limit. Model (4) was used by Teshome (2005) for *C. lusitanica* plantation after comparing several models (Alemdag, 1988; Burkhart, 1977; Cao and Burkhart, 1980; Van-Deusen *et al.*, 1981). Model (6) is one of the widely used and accepted equation (Jordan *et al.*, 2005; Tasissa *et al.*, 1997; Clark and Saucier, 1990). To construct taper functions for the plantations, the EMV and NEMV derived taper functions are compared.

Model selection

Model selection refers to choosing the most appropriate model to describe given data in mathematical form. Model selection methods rank candidate models relative to each other. The commonly used model selection methods are Akaike Information Criteria (AIC) (Akaike,1974), Bayesian Information Criteria (BIC) (Schwarz, 1978), and Cross Vvalidation (CV) (Stone, 1974). However, there are also several others and modifications of these methods. For details, refer to Burnham and Anderson (2000). For valid use of information-theoretic methods, models must have the same response variable which the models in this study have met as requirement. The AIC attempts to find the model that best explains the data with a minimum of free parameters. The preferred model is the one with the lowest AIC value.

For comparing regression models, usually with different response variables in forest growth models, Kozak and Kozak (2003) identified two procedures which are based on an examination of the prediction errors or fit statistics computed from ordinary residuals. The first procedure compares models on basis of statistics obtained directly from models built from entire data sets while the second does on the bases of the validation data set which normally accounts for less than or half of the entire data set. On the basis of a simulation study, Kozak and Kozak (2003) concluded that the validation data procedure provides little, if any, additional information in the process of evaluating regression models relative to the procedure which is based on the entire data set for computing comparison statistics. Accordingly, the authors recommended the first procedure. In the present work the method recommended by Kozak and Kozak (2003) and the AIC criterion, when appropriate, were used for comparing models.

The statistics used to compare the models were bias (B), standard error of estimate (SEE), mean of absolute value of the difference (MAD), and estimated coefficient of determination, also known as correlation squared index (I^2). These statistics used for comparison are defined as follows:

$$\text{AIC} = 2k - 2\ln L,$$

$$\text{B} = \frac{\sum_{i=1}^{n} \varepsilon_i}{n},$$

$$\text{SEE} = \sqrt{\frac{SSR}{n-k}},$$

$$I^2 = \frac{SST - SSR}{SST}, \text{ and}$$

$$\text{MAD} = \frac{\sum_{i=1}^{n} |\varepsilon_i|}{n}$$

Where

$\varepsilon_i = Y_i - \hat{Y}_i$, SSR = $\sum_{i=1}^{n} (\varepsilon_i)^2$ and

SST = $\sum_{i=1}^{n} (Y_i - \overline{Y})^2$.

Similarly, *lnL* is log likelihood function, *n* is the number of observations, *k* the number of estimated parameters, Y_i the dependent variable, $\hat{Y}_i$ predicted value and $\overline{Y}$ the average of the Y_i. The R statistical software (R Development Core Team, 2007) was used for estimating the parameters of the models and computing performance statistics.

Result and Discussion

Total volumes

After estimating the total tree volume model (1) parameters by least squares and model (2) by nonlinear least squares method for both *C. lusitanica* and *P. patula* data, the performance statistics were computed (Table 2). The result indicated that both total volume models were reasonable and, according to the AIC, SEE and MAD values, model (2) of Schumacher and Hall (1933) showed marginal improvement over model (1) of Spurr

(1952). As expected, the B value for least square estimation method in the Spurr (1952) model is zero. Otherwise, both are worthy for estimating the total volume of the species. Their parameter estimates (all significant with $p < 0.001$) along with their estimated standard error are presented in Table 3.

Merchantable volume models

The NEMV model (4) and EMV model (6) were compared and evaluated in this section. These models predict merchantable volume to any upper outside bark diameter limit d. The fit statistics of these models are shown in Table 4. The values of the fit statistics in Table (4) revealed that the EMV model (6) was more precise compared to that NEMV model (4). The AIC, SEE, B and MAD values for NEMV model (4) were much more in size relative to EMV model (6) which was a clear indication that the NEMV model was of poor performance. The estimated coefficient of determination (I^2) of the EMV model was higher than the Burkhart model for both *P. patula* and *C. lusitanica* trees. In addition to the overall goodness of fit comparison of these NEMV and EMV models, they were also evaluated for predicting volume along various sections of the tree bole by relative diameter class on the basis of the SEE, B, and MAD statistics for both *C. lusitanica* and *P. patula* tree species (Fig 1 (a)-(c)). For both species, the EMV model (6) overwhelmingly outperformed the NEMV model (4) in estimating merchantable volume all over along the tree stem with exception to the relative diameter classes d/D <0.1 and 0.8 < d/D< 0.9 where similar performance is observed. Figure 1 also showed that the EMV model (4) performed better in estimating the volume with decreasing merchantable diameter d as compared to its performance with the increasing size of d.

In addition to its poor performance, the NEMV model (4) resulted in negative volume estimates (Fig 2 (a) and (c)) at the lower portion of the tree bole while the EMV model (6) predicted no illogical values (Fig 2 (b) and (d)) confirming findings by Van-Deusen *et al.* (1981), Tasissa *et al.* (1997) and Jordan *et al.* (2005). Therefore, the EMV model (6) has shown considerably better fit for both tree species compared to the NEMV model (4).

For deriving implicit taper functions and their evaluation in the next section, the parameter estimates of the EMV model (7) used for predicting merchantable volume to any upper merchantable height and EMV model (6) are required. Thus performance statistics of These models were presented in Table 4 also creating a comparison study among themselves.

The results of the comparison of the merchantable volume models (Table 4) were consistent with the research reports by McTague and Bailey (1987), Tasissa *et al.* (1997) and Teshome (2005) who noted that models predicting merchantable volume to upper height show better fit particularly for the *P. patula* tree species in this study. However such models are less important in practice as tree volumes are normally assorted and merchandized by diameter size. The parameter estimates for the EMV models were presented in Table 5.

Taper functions

In this section, implicit taper functions derived from the ERM ((10) and (11)) and NEMV ((8) and (9)) models were evaluated and compared for both *C. lusitanica* and *P. patula*. It is believed that the precision and accuracy of the taper functions are determined by the precision and accuracy of the merchantable volume equations from which the taper models are derived (Clutter, 1980; Jordan *et al.*, 2005). Accordingly, in this section, the overall performance statistics of the NEMV derived taper models (8) and (9) versus their respective EMV derived taper models (10) and (11) for predicting diameter and height, respectively were shown in Table 6.

On average, the SEE, B and MAD estimates of the taper model (8) has shown an increase of 43, 129, and 51 percent, respectively for *C. lusitanica* tree over the corresponding statistics estimates of the taper model (10) in estimating d (Table 6). For *P. patula*, the taper model (8) has shown an average of 58, 151, and 59 percent increase of SEE, B and MAD, respectively, in estimating diameter compared to the taper model (10). For estimating merchantable height of *C. lusitanica* (Table 7), the taper model (9) has shown an average of 14, 63, and 23 percent increase of SEE, B and MAD, respectively over the taper model (11). In estimating height for *P. patula*, an average increase of 12, 53, and 18 percent of SEE, B and MAD, respectively were shown in the taper model (9) compared to taper model (11).

Although single indices of SEE, B and MAD are good indicators of the effectiveness of the taper functions, they may not clearly indicate the best equation for practical purpose (Kozak and Smith, 1993). Hence, it is advisable to compute the performance statistics for different sections of the tree usually by relative height class along the bole of the tree (Sharma and Zhang, 2004; Newnham, 1992; Kozak, 1997; Kozak and Smith, 1993; Muhairwe, 1999). Such statistics allow us to evaluate the performance of the taper function at various height of the tree from the ground which could not be revealed by the overall performance statistics such as in Table 6. Accordingly, the performance statistics (SEE, B and MAD) of the EMV and NEMV derived taper models for both tree species were calculated along the tree bole by relative height class ($z = h/H$) and displayed in Figure 3 for diameter predicting taper models (8) and (10) and Figure 4 for height predicting models (9) and (11). In predicting diameter, with the exception of the lower relative height classes (lower section of the stem) in both tree species where both (8) and (10) models resulted in about similar performance, the taper model (10) outperformed the model (8) in all other sections of the tree (Figure 3 (a)-(c)). Similar evaluation of the height predicting taper models (9) and (11) also referred as merchantable height equations by McTague and Bailey (1987), in estimating height along the bole section by relative height class (Figure4 (a)-(c)) has also confirmed the superiority of the EMV derived taper model (11) over the NEMV derived taper model (9) particularly for $z > 0.4$. However, the differences of the performance statistics (Table 6) of model (9) and (11) seem to be not large enough as compared to their wide range differences in the taper model (8) and (10). Such narrowing gap of the average performance statistics (Table 6) of the models (9) and (11) was due to poor performance of model (11) for the lower section of the bole ($z < 0.1$) and about its comparable performance with the model (9) for $0.1 < z < 0.4$ section of the tree.

Otherwise, Figure 4 ((a)-(c)) shows that the model (11) was overwhelmingly more precise over the model (9) in estimating merchantable height at the upper section ($z > 0.4$) of the tree. Hence, this study recommends the EMV derived taper models for practical use as compared to the NEMV derived taper models.

Figure 5 (a)-(d) revealed that both taper models (10) and (11) reasonably predict diameter and height, respectively at the upper section of the tree compared to the bottom section of the tree. This observation was also reported by Tasissa *et al.* (1997). However, as noted in Amateis and Burkhart (1987), optimal prediction is not normally expected from models (10) and (11) since the optimization of the parameters is for volume rather than tree profile. Accordingly, estimates of these models could be unreasonably biased in the lower section of the tree. Particularly, model (11) is unbounded and likely to yield illogical height predictions at the very bottom section of the tree as observed in Figure 5 (b) and (c). Accordingly, as noted also in Tasissa *et al.* (1997), these implied taper functions provide reasonable estimates in the main bole portion of the tree.

Summary and application of the models

For simplicity to users, the recommended models in this study are summarized by species type along with application example. The input values used (when appropriate) in the application example for *C. lusitanica* models are: H = 19.85 m ; D = 20.90 cm; h= 7.3 m; and d = 16.4 cm . Similarly the input values for *P. patula* are: H = 24.22 m ; D = 28.2 cm; h= 12.3 m; and d = 20.5 cm .

1. *C. lusitanica*

1.1 Total volume:

$$V_t = 0.00005944\ D^{1.757} H^{1.087}$$

By substituting the input values for D and H in the total volume model V_t yields total volume for the tree with the specified input values as follows:

$$V_t = 0.00005944\,(20.90^{1.757} 19.85^{1.087})$$
$$= 0.31933\ m^3$$

1.2 Merchantable volume to upper diameter limit:

$$V_d = V_t \exp(-0.39895\ d^{5.1073} D^{-4.63996})$$

By substituting the input values for V_t , d and D in this V_d model yields merchantable volume to upper diameter d for the tree with the specified input values as follows:

$$V_d = 0.31933 \exp(-0.39895\ 16.4^{5.1073} 20.90^{-4.63996})$$
$$= 0.19785\ m^3$$

1.3 Merchantable volume to upper height limit:

$$V_h = V_t \exp(-2.81541\ (H-h)^{3.82969} H^{-3.86491})$$

By substituting the input values for V_t , h and H in this V_h model yields merchantable volume to upper height h for the tree with the specified input values as follows:

$$V_h = 0.31933 \exp(-2.81541\ (19.85-7.3)^{3.82969} 19.85^{-3.86491})$$
$$= 0.20611\ m^3$$

1.4 Taper model for predicting diameter:

$$d = \left(\frac{-2.81541}{-0.39895}\right)^{\frac{1}{5.1073}} (H-h)^{\frac{3.82969}{5.1073}} H^{\frac{-3.86491}{5.1073}} D^{\frac{4.63996}{5.1073}}$$

$$= 1.466084\ (H-h)^{0.7498463} H^{-0.7567423} D^{0.9084957}$$

By substituting the input values for h, H and D in this taper model gives diameter at height h for the tree with the specified input values as follows:

$$d = 1.466084\ (19.85-7.3)^{0.7498463}\ 19.85^{-0.7567423}\ 20.9^{0.9084957}$$
$$= 16.1157\ cm$$

1.5 Taper model for predicting height:

$$h = H - \left(\frac{-0.39895}{-2.81541}\right)^{\frac{1}{3.82969}} d^{\frac{5.1073}{3.82969}} D^{\frac{-4.63996}{3.82969}} H^{\frac{3.86491}{3.82969}}$$

$$= H - 0.6003568\ d^{1.333607} D^{-1.211576} H^{1.009197}$$

By substituting the input values for d, H and D in this taper model yields height at diameter d for the tree with the specified input values as follows:

$$= 19.85 - 0.6003568\,(16.4^{1.333607}\ 20.90^{-1.211576}\ 19.85^{1.009197})$$
$$= 7.00387 \text{ m}$$

2. *P. patula*

2.1 Total volume:

$$V_t = 0.00004425\, D^{1.950} H^{1.011}$$

By substituting the input values for D and H in the total volume model V_t yields total volume for the tree with the specified input values as follows:

$$V_t = 0.00004425\,(28.20^{1.950}\,24.22^{1.011})$$
$$= 0.74696 \text{ m}^3$$

2.2 Merchantable volume to upper diameter limit:

$$V_d = V_t \exp(-0.63345\, d^{6.04690} D^{-5.70453})$$

By substituting the input values for V_t, d and D in this V_d model yields merchantable volume to upper diameter d for the tree with the specified input values as follows:

$$V_d = 0.74696 \exp(-0.63345(20.50^{6.04690}\,28.20^{-5.70453}))$$
$$= 0.55953 \text{ m}^3$$

2.3 Merchantable volume to upper height limit:

$$V_h = V_t \exp(-2.64901\,(H-h)^{3.45301} H^{-3.46576})$$

By substituting the input values for V_t, h and H in this V_h model yields merchantable volume to upper height h for the tree with the specified input values as follows:

$$V_h = 0.74696 \exp(-2.64901\,(24.22-12.30)^{3.45301}\,24.22^{-3.46576})$$
$$= 0.59950 \text{ m}^3$$

2.4 Taper model for predicting diameter:

$$d = \left(\frac{-2.64901}{-0.63345}\right)^{\frac{1}{6.04690}} (H-h)^{\frac{3.45301}{6.04690}}\, H^{\frac{-3.46576}{6.04690}}\, D^{\frac{5.70453}{6.04690}}$$

$$= 1.266948\,(H-h)^{0.571038}\, H^{-0.5731466}\, D^{0.943381}$$

By substituting the input values for h, H and D in this taper model gives diameter at height h for the tree with the specified input values as follows:

$$d = 1.266948\,(24.22-12.3)^{0.571038}\, 24.22^{-0.5731466}\, 28.20^{0.943381}$$
$$= 19.59549 \text{ cm}$$

2.5 Taper model for predicting height:

$$h = H- \left(\frac{-0.63345}{-2.64901}\right)^{\frac{1}{3.45301}} d^{\frac{6.04690}{3.45301}} D^{\frac{-5.70453}{3.45301}} H^{\frac{3.46576}{3.45301}}$$

$$= H\text{-}0.6607686\,(d^{1.751197}\, D^{-1.652046}\, H^{1.003692})$$

By substituting the input values for d, H and D in this taper model yields height at diameter d for the tree with the specified input values as follows:

$$h = 24.22 - 0.6607686\,(20.5^{1.751197}\, 28.20^{-1.652046}\, 24.22^{1.003692})$$

$$= 11.31984 \text{ m}$$

Conclusion

Total and Merchantable volume models were presented for *C. lusitanica* and *P. patula* tree plantations of the SFEI in Ethiopia. The EMV model (6) was found more precise as compared to the unbounded NEMV model (4) of Burkhart (1977). From the EMV models (6) and (7), implied taper functions were developed. These taper models overwhelmingly outperformed taper models derived from the NEMV models. Diameter to desired upper height and height to desired top diameter can be obtained by evaluating these taper models.

Therefore, this study overcomes the shortcomings of Teshome (2005) who constructed the NEMV models and their associate taper functions for *C. lusitanica* while provided total volume, merchantable volume and taper models for *P. patula*. However, since the optimization of the parameter estimates is for the merchantable volume models, the taper functions do not rovide optimum prediction (Amateis and Burkhart (1987). Hence, while the total and merchantable volume models presented in this study are very reliable estimation tools, the implied taper functions are meant only to provide estimates in the main bole portion of the tree and should not be thought as substitute for tree taper models directly developed from stem analysis data

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Table 1: Data summary

	No Trees	Diameter (D)			Height (H)		
		Mean	Range	sd [1]	Mean	Range	sd
C. lusitanica	204	21.14	5.35-47.60	8.69	19.04	5.40-34.30	6.34
P. patula	196	20.39	4.15-48.00	9.54	19.00	4.70-35.10	6.89

[1] sd= standard deviation

Table 2: Total volume models statistics

	C. lusitanica		*P. patula*	
	Schumacher and Hall (2)	Spurr (1)	Schumacher and Hall (2)	Spurr (1)
AIC	-674.7364	-626.1592	-615.96	-613.62
I^2	0.9868	0.9831	0.99023	0.99003
SEE	0.0457	0.0516	0.05124	0.05167
B	-0.0015	0.0000	-0.00197	0.00000
MAD	0.0305	0.0366	0.03324	0.03368

Table 3: The parameter estimates of Spurr and Schumacher and Hall total volume models with standard error in parentheses

	C. lusitanica	*P*. patula
Spurr (1)		
β_1	0.02681 (0.005083)	0.003521 (0.0005397)
β_2	0.00003246 (0.0000002993)	0.00003818 (0.0000003162)
Schumacher and Hall (2)		
β_1	0.00005944 (0.00000689)	0.00004425 (0.000005191)
β_2	1.757 (0.02687)	1.950 (0.03402)
β_3	1.087 (0.04190)	1.011 (0.04966)

Table 4: Fit statistics of the NEMV (4) and EMV (6) Models.

	C. lusitanica		*P. patula*	
	(4)	(6)	(4)	(6)
AIC	-14388.55	-21841.29	-11182.34	-16836.23
I^2	0.9710	0.99435	0.96941	0.99317
SEE	0.05938	0.02623	0.07787	0.03678
B	0.00697	-0.00129	0.00228	-0.00283
MAD	0.03618	0.01456	0.04351	0.01921

Table 5: Parameter estimates (with standard errors in parentheses) and performance statistics of the EMV models.

			C. lusitanica	*P. patula*
V_d Model (6)	Parameter estimates	β_1 β_2 β_3	-0.39895 (0.01763) 5.10730(0.02137) 4.63996 (0.02218)	-0.63345 (0.03770) 6.04690 (0.03237) 5.70453 (0.03433)
	Performance statistics	AIC I^2 SEE B MAD	-21841.29 0.99435 0.02623 -0.00129 0.01456	-16836.23 0.99317 0.03678 -0.00283 0.01921
V_h Model (7)	Parameter estimates	α_1 α_2 α_3	-2.81541(0.17053) 3.82969(0.01439) 3.86491(0.02311)	-2.64901 (0.14208) 3.45301 (0.01248) 3.46576 (0.01997)
	Performance statistics	AIC I^2 SEE B MAD	-22145.77 0.99364 0.02786 -0.00605 0.01659	-19891.19 0.99443 0.03338 -0.00533 0.01907

Table 6: The overall performance statistics of the NEMV derived ((8) ,(9)) and EMV derived ((10),(11))taper models.

Models	Performance Statistics	Species	
		C. lusitanica	*P. patula*
(8)	I^2	0.9389	0.9163
	SEE	2.4850	3.0039
	B	0.9024	1.0628
	MAD	1.8667	2.1704
(9)	I^2	0.9365	0.9120
	SEE	1.9187	2.3270
	B	0.7300	0.8230
	MAD	1.4376	1.7306
(10)	I^2	0.9702	0.9667
	SEE	1.7343	1.8963
	B	0.3934	0.4220
	MAD	1.2340	1.3618
(11)	I^2	0.9515	0.9301
	SEE	1.6770	2.0735
	B	0.4481	0.5376
	MAD	1.1674	1.4447

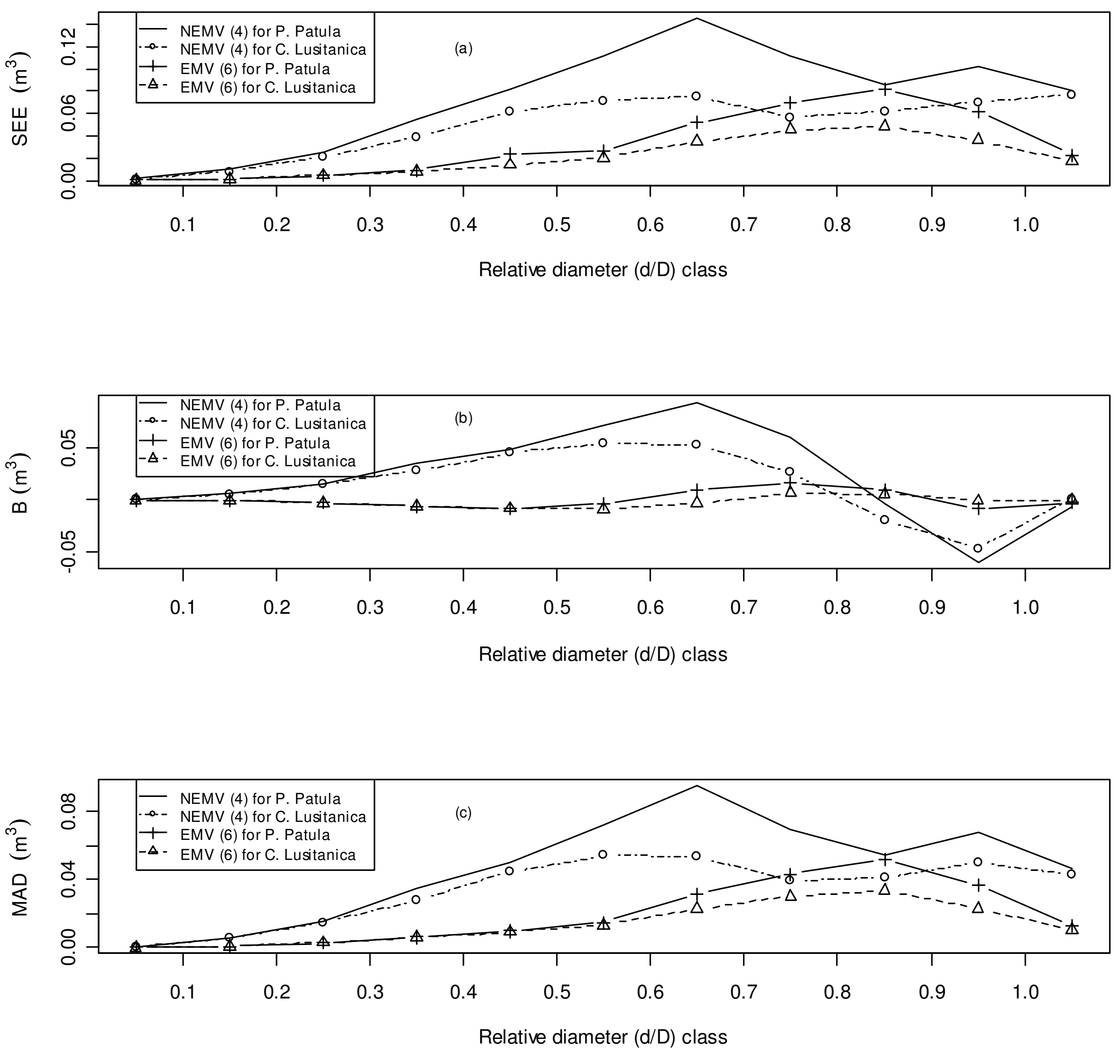

Figure 1: Performance comparison of the NEMV model (4) and EMV model (6) by relative diameter sections.

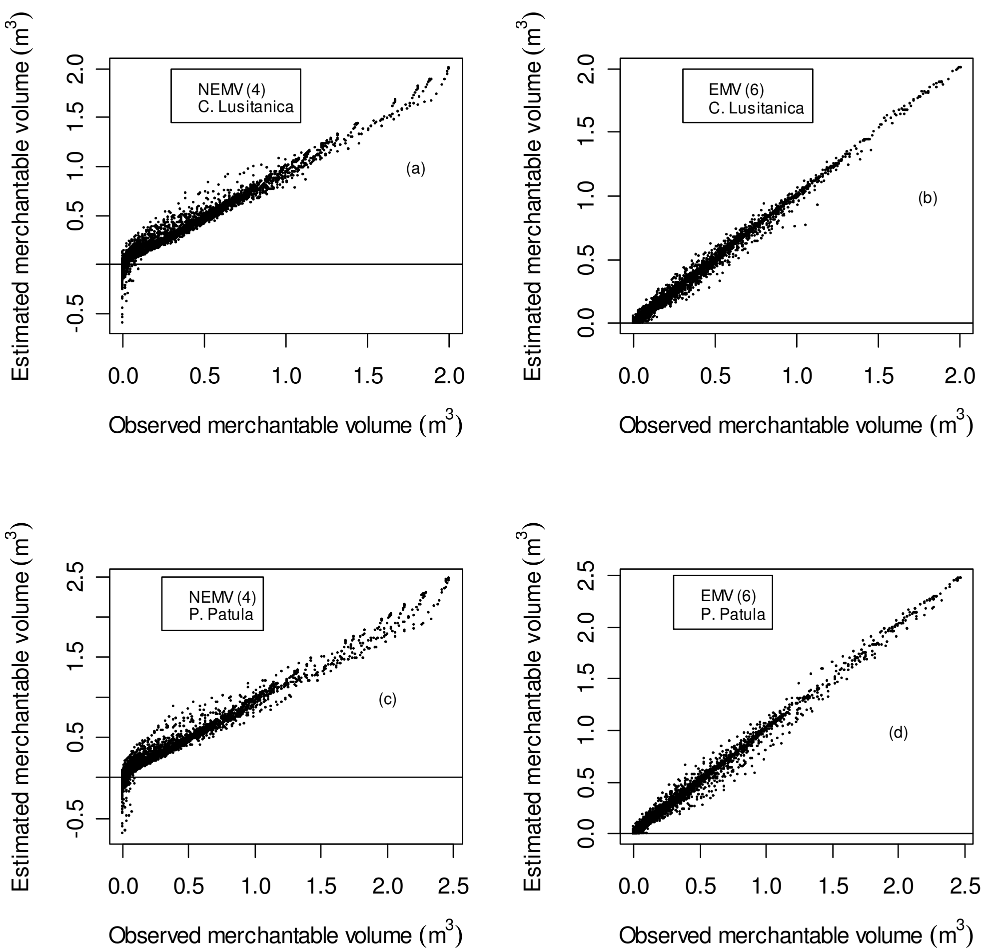

Figure 2: Plots of estimates of EMV and NEMV models versus observed merchantable volumes for *C. lusitanica* and *P. patula* trees.

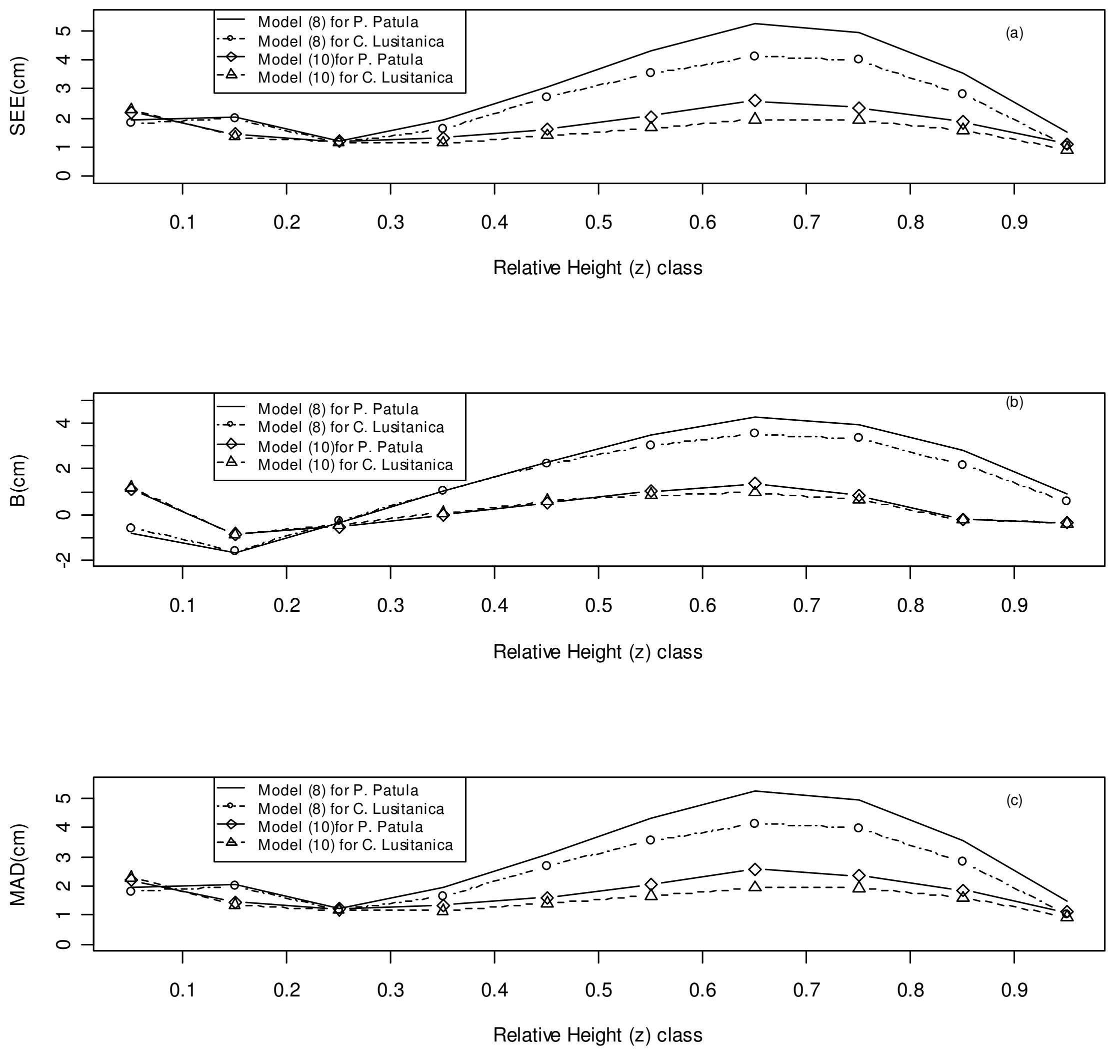

Figure 3: Performance statistics plots by relative height class for the diameter predicting taper models (8) and (10).

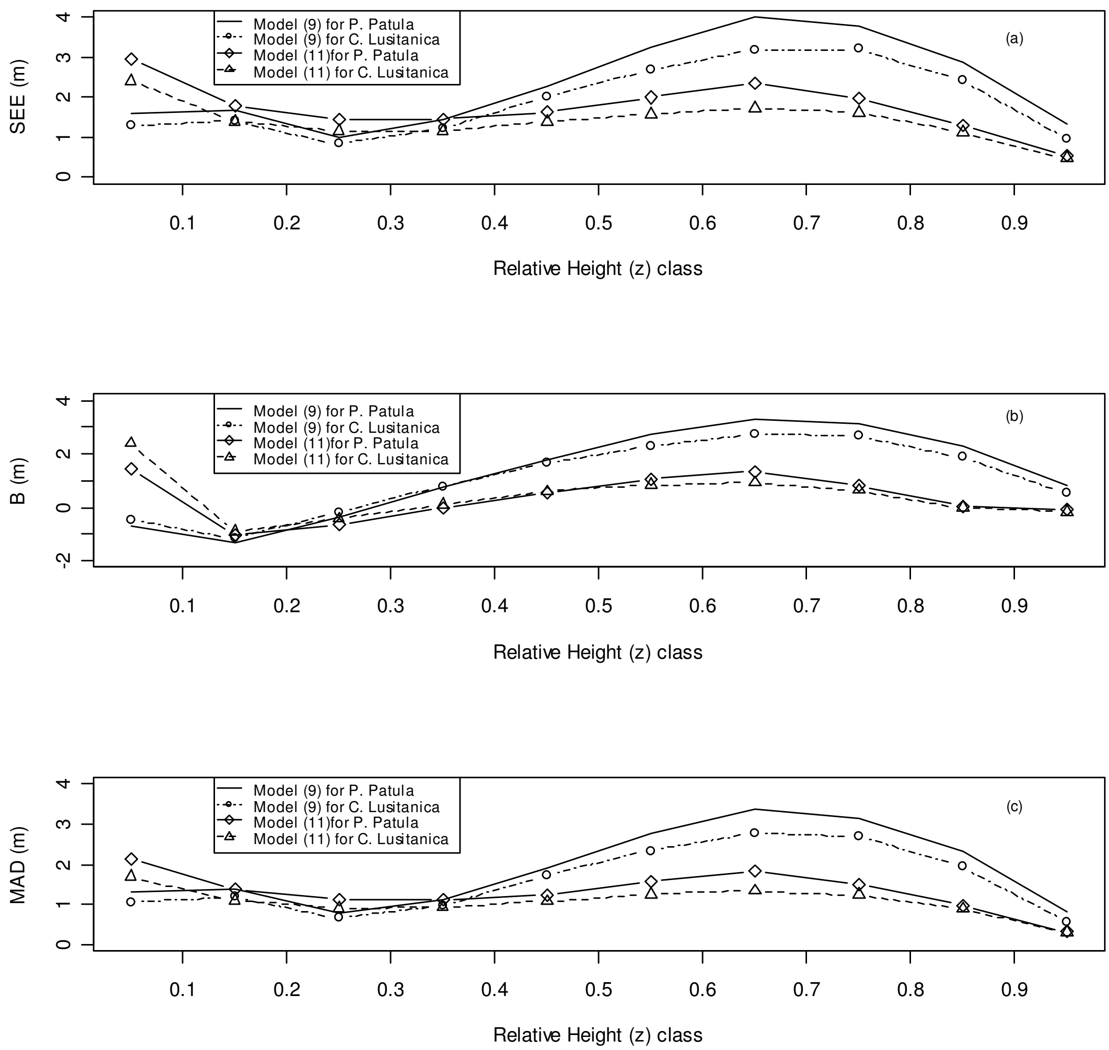

Figure 4: performance statistics plots by relative height class for the height predicting taper models (9) and (11).

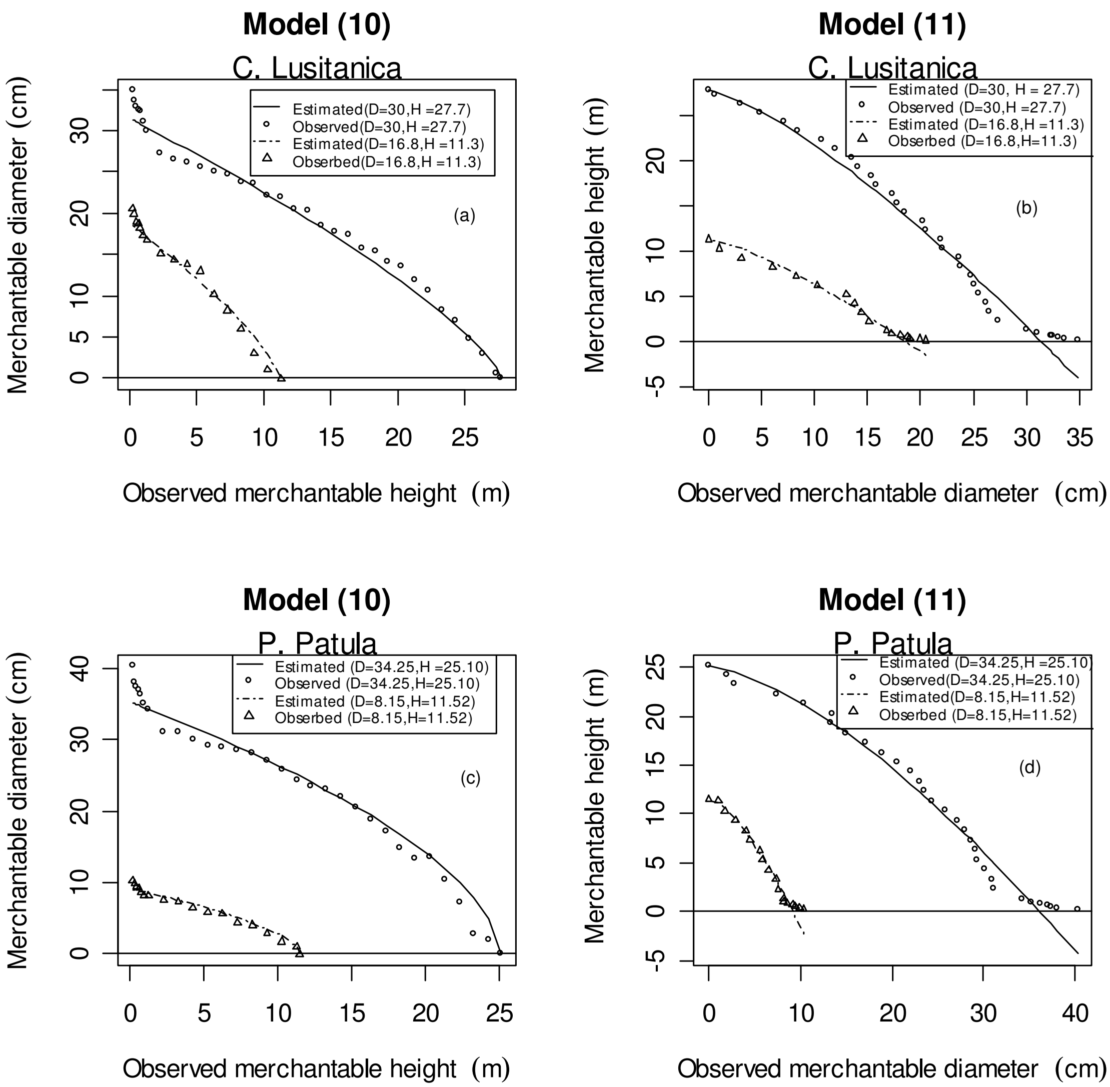

Figure 5: Estimated (from models (10) and (11)) versus observed taper pro¯les of selected *C. lusitanica* and *P. patula* trees.