REVIEW ARTICLE

Mathematics Curriculum, the Philosophy of Mathematics and its Implications on Ethiopian Schools Mathematics Curriculum

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Abstract

It is my observation that the current school mathematics curriculum in Ethiopia is not producing competent mathematics students. Many mathematicians in Ethiopia and other part of the world have often expressed grief that the majority of students do not understand mathematical concepts, or do not see why mathematical procedures work, or do not know when to use a given mathematical technique (Cuoco, A.1995). According to Cuoco, A.A, et.al. (1996) for generations, school students have studied something in school that has been called mathematics but has very little to do with the way mathematics is created. Much of the failure in school mathematics is due to the tradition of the curriculum design and inappropriate teaching to the way student learns (National Research Council 1989).

The mathematics curriculum has a great influence on how teachers teach in a classroom. In a traditional curriculum where a traditional teaching model is being employed a teacher demonstrates an algorithm or technique, assigns a set of problems for students to do on their own, and tests a student a week or two weeks later on accumulation of their skills.

*On the other hand Interactive Mathematics Curriculum (IMC) is designed around the process aspect of mathematics in contrary to the curriculum we have at hand nowadays in schools. According to Cuoco, A.A. et.al. (1996:377) the organizing principle of IMC is the "**Habit of Mind**" the students are expected to develop where as in the traditional curriculum the organizing principle is the "**content.**" A curriculum designed around habits of mind comprises both the content and the process*

The existing mathematics Curriculum that is underway in Ethiopia can be labeled as traditional for its main organizing principle is the content that needs to be covered for a given grade level in a given academic year rather than the habits of mind that the students need to develop. Part of the solution to this problem could be adapting IMC to all school levels.

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1 **INTRODUCTION AND ORIENTATION**

To gain in depth understanding of the concept "Curriculum" requires examining of its philosophical foundations. Consequently the purpose of this paper is to discuss on the philosophical foundations of mathematics curriculum, the influence of philosophy of mathematics on designing mathematics curriculum and its implication for school mathematics in Ethiopia based on the literature.

The philosophy of mathematics influences the content and organization of the mathematics curriculum. It defines what constitutes valid source of information from which come accepted theories, principles and ideas relevant to mathematics curriculum.

According to Lacatos in S. Lerman (1990) the two philosophical schools of mathematics namely, absolutist and fallibilist philosophy of mathematics make influence on the content, organization, methods and general structure of mathematics curriculum. In this article curriculum organization based on the principle "Habits of Mind" will be discussed meticulously.

It is also tried to make all the way through discussion of mathematics Curriculum called "Interactive Mathematics Curriculum" (IMC) as compared to a traditional mathematics Curriculum. In IMC the organization standard is the *habit of mind* that the students are expected to develop, whereas the organizational principle of a traditional mathematics Curriculum is the *content* that the students are expected to cover at a certain grade level.

The role of the teacher and how students learn in IMC is also well thought-out. In IMC when a student is asked a question like "what is Mathematics?" s/he is expected to give a short answer like "it is about problem solving" as a substitute of "it is about triangles" which is the case in traditional curriculum organized around content.

Finally a conclusion is made on which philosophy of mathematics to follow in designing a school mathematics Curriculum.

1.2 The Philosophy of Mathematics and Mathematics Curriculum

It is known that, philosophical Foundation of Mathematics Curriculum refers to the "why?" of students' activity in schools concerning mathematics. The Philosophy of mathematics helps especially a curriculum designer of mathematics to answer questions like: What schools are for? What subjects of mathematics are of value? What materials and methods are to be used? And how students learn mathematics? The Philosophy of Mathematics also helps to determine: The goals of Mathematics education, the contents and general organization of mathematics curriculum, the process of teaching and learning of mathematics.

According to Lacatos in S. Lerman (1990) there are two main currents in the philosophy of mathematics, namely the absolutist and the fallibilist philosophy of mathematics. The absolutist philosophy of mathematics includes the schools Platonism, Logicism, Intitutionism and Formalism.

1.2.1 The Absolutist Philosophy of Mathematics and its Curriculum.

For the absolutists, mathematical knowledge or mathematical objects have independent realm of world, they have independent existence from the society. (Ernest, P. 1991). The following are some of the properties of mathematics Curriculum for the absolutist Philosophy of Mathematics. (Ernest, P. 1991, Lacatos in S. Lerman 1990, Grouws DA 1992)

- It is organized around the content (content-centered)
- The teacher is an "explainer" to help students to understand, relate ideas and concepts. The teacher is an Authority and his knowledge is unquestionable.
- There is a fixed Curriculum model in that changing the curriculum means changing one subject for another. For instance instead of taking plane Geometry, they are made to take statistics etc. Since for the absolutists mathematical objects are discovered and static the corresponding curriculum is fixed.
- Learning is through abstraction, relating mathematical ideas and concepts with no real counter part.
- Mathematics is seen as an isolated and discreet discipline and in the corresponding Curriculum the subjects are treated separate and no subject integration.

In general, from the above points it can be inferred that mathematics Curriculum in the view of the absolutist philosophy of mathematics focuses on the content (focus upon a fixed body of knowledge), the basic principle of organizing the Curriculum is the subject matter or on what to think and not on the "process" or not on "How to think"

1.2.2 The Fallibilist Philosophy of Mathematics and Mathematics Curriculum.

According to Popper in Ernest (1991), the Fallibilist believe that mathematical knowledge or objects of mathematics are results of human activity (social and cultural results). Popper continued that Fallibilist is the only philosophy of mathematics that believes history is germane to mathematics education and to mathematics itself. Moreover, for the fallibilist philosophy of mathematics the focus of teaching is not the content. It is the process aspect that is more important. But this does not mean that the content has no place, but it must be combined with the method.

Content in schools, according to National Council of Teachers of Mathematics (NCTM, 2000) refers to Numbers, Algebra, Geometry, Measurement, Data analysis and Probability and Process Refers to Problem Solving, Communication, Reasoning, Interconnection and Representation.

The view of fallibilist philosophy on mathematics Curriculum has resemblance in many ways with the views of the pragmatist's school of thought. For the fallibilists just like the pragmatists the focus is not on the subject matter rather on the process aspect of mathematics in which reality is constantly changing, knowledge is not static which in turn shows that mathematical objects are not static and absolute.

Some of the properties of mathematics Curriculum for the fallibilist philosophy of mathematics are summarized by Lakatos (1978), Davis and Hersh (1980) and Ernest (1991) as follow:

- Children are liberated from traditional emphasis on rote learning, lesson recitation and textbook authority.
- Learning is possible as the person actively engages in problem solving which is transferable to variety of situations and subjects.
- The role of the teacher is helping students to identify their problems and seek solution to the problem
- Teaching and learning is child-centered unlike the traditional philosophies.
- Learning is an integral part of life and not a preparation for a future life.
- The Curriculum is problem-centered that help students develop how to think. - that is the organizing principle of such a curriculum.

2 Organizational Principles of Mathematics Curriculum

2.1 The Need for Change of Mathematics Curriculum. Mathematicians in Ethiopia and many other parts of the world have often lamented that the majority of students do not understand mathematical concepts, or don't see why mathematical procedures work, or don't know when to use a given mathematical technique. According to the National Research Council's (1989:30) "much of the failure in school mathematics is due to a tradition of teaching that is in appropriate to the way students learn."

In the traditional teaching model that has dominated our schools for many years, a teacher demonstrates an algorithm or technique, assigns a set of problems for students to do on their own, and then tests the students a week later on the accumulation their skills.

Students in such a situation often do not understand what they are doing because they are simply following instructions. They typically see no need for the mathematics, other than to pass the test. The result is a system in which students "view mathematics as a rigid system of externally dictated rules, governed by standards of accuracy, speed, and memory". (National Research Council, NRC, 1989:40)

The solution to this dilemma lies in active student engagement in learning as indicated by NRC (1989:6).

> *Research in learning shows that students actually construct their own understanding based on new experiences that enlarge the intellectual framework in which ideas can be created* . . . . *Mathematics becomes useful to a student only when it has been developed through a personal intellectual engagement that creates new understanding*.

This process of "personal intellectual engagement" lies at the heart of what we call Interactive Mathematics Curriculum (IMC) view of learning and are shared by educators around the world. NCTM Curriculum and Evaluation Standards elaborate on what this means in terms of what should happen in the classroom:

> *Students should be exposed to numerous and various interrelated experiences that encourage them to value the mathematics enterprise, to develop mathematical* ***habits of mind****, and to understand the role of mathematics in human affairs;* . . . *they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems;* . . . *they should read, write, and discuss mathematics; and . . . they should*

conjecture, test, and build arguments about a conjecture's validity. (NCTM 1989:5)

2.2 Organizing an Interactive Mathematics Curriculum.

Cuoco, A.A, et.al. (1996) asserted that for many years school students have studied what is called mathematics. But this mathematics has very little connection with the way mathematics is created, applied outside of schools. Cuoco continued one of the reasons for this is a view of curriculum in which mathematics is seen as device for communicating established results and methods. That is the objective is preparing students for life after school by giving them a lot of facts for the students and mathematics was not considered as part of life. For instance students learn solving equations; find areas and calculate interest on a loan. In this view Curriculum reform simply means replacing one set of established results by another one. For instance, in steady of studying Algebra, students study Statistics. In this type of reform in curriculum the methods used are the same in which they learn some properties, work some problems in which they apply the properties and move on.

The properties discussed in the above paragraph reflect the characteristics of traditional curriculum. There is another way of looking at mathematics Curriculum. In this method the priorities are turned around. Much more important are the habits of mind by the people who create those results. Cuoco A.A and Goldberg E.P (1996). In this new approach instead of asking what content is good to be included in the Curriculum the question now turns to "what habits of mind are core? What are good mathematical habits of mind? In this type of Curriculum organization, the method by which mathematics is created and the techniques used by researches is highly emphasized. The goal of mathematics teaching is not to train school students for to be University mathematicians but it is to help students learn and adopt some of the ways that mathematicians think about problems. The objective is to give students the tools they will need in order to use, understand and even make mathematics that does not yet exist and to close the gap between the habits of mind of inventors of mathematics and the students.

In general, according to Cuoco, A.A and Goldberg, E.P (1996) a curriculum organized around habits of mind do the following:

- Close the gap between what the users and makers of mathematics do and what they say.
- Lets students in the process of creating, inventing, conjecturing, and experimenting
- Help students develop the habit of reducing things to Lemmas for which they have no proofs.
- Helps students to look for logical and heuristic connections between new ideas and old ones
- Gives the students a genuine research experience.

This approach to Curriculum extends beyond mathematics, and reflection shows that certain general habits of mind cut across every discipline. There are also more mathematical habits and finally, there are ways of thinking that are typical of specific content areas.

For instance, in high school, students need to acquire some useful habits of mind and some mathematical approaches that have shown themselves worthwhile over the years. (Cuoco A.A and Goldberg, E.P 1996) In addition to this the authors indicated that, there are content specific habits that high school graduates should have-some geometrical habits of mind that support the mathematical approaches and some algebraic ways of thinking that complement the geometric approach.

By emphasizing the ways of thinking that are essential in mathematics, one can design mathematics courses that simultaneously serve the needs of students who will go on to advanced mathematical study and students who will not.. Cuoco AA and Goldenberg EP(1996) address a series of mathematical "habits of mind," arguing that students should be **pattern sniffers**, **experimenters**, **describers, tinkerers**, **inventors**, **visualizers**, **conjecturers**, and **guessers**. The authors' discussed that the materials for teaching and learning school mathematics must provide students with problems and activities that develop these habits of mind and put them into practice. The authors' further pointed that every problem in a mathematics curriculum, organized around the habits of mind needs to:

- Contain important, useful mathematics.
- Have multiple ways, using different solution strategies.
- Have various solutions or allows different decisions or positions to be taken and defended.
- Engage students and encourages discourse.
- Requires higher-level thinking and problem solving.
- Contribute to students' conceptual development.
- Connect to other important mathematical ideas.
- Promotes the skillful use of mathematics.
- Provides an opportunity to practice important skills.
- Creates an opportunity for the teacher to assess what students are learning and where they may be expert

The habit of mind approach seems to be gaining acceptance among other mathematics educators too. For instance *Everybody Counts* (NRC, 1989) describes it this way:

> *"Mathematics offers distinctive modes of thought which are both versatile and powerful . . . Experience with mathematical modes of thought builds mathematical power--a capacity of mind of increasing value in this technological age . . ."*

According to Kleiner, (1986) a Curriculum that uses workplace and everyday tasks to support the goal of developing mathematical thinking is less likely to use the tasks *as* the Curriculum; it is less likely to let the message "high school graduates should be able to solve problems like these" evolve into "high school graduates should be able to solve these problems." Conversely, a Curriculum firmly rooted in concrete problems is less likely to turn the goal of developing mathematical habits of mind into a "mathematics appreciation" Curriculum that studies little more than lists of mathematical ways of thinking. The author also pointed that the dialectic between problem solving and theory-building is the fuel for progress in mathematics, and mathematics education should exploit its power. Kleiner further discussed that, problems can be both sources for and applications of methods, theories, and approaches that are characteristically mathematical. For example, through the work of Descartes, Euler, Lagrange, Galois, and many others, techniques for solving algebraic equations developed alongside theory about their solutions.

One important question is, "What does it mean to organize a curriculum around mathematical ways of thinking?" One way to think about it is to imagine a common core curriculum for all students lasting through, say, grade 10. Students would work on problems, long-term investigations, and exercises very much as they do now, except the activities would be aimed at developing specific mathematical approaches. In contrast to other kinds of organizers currently in use (applications, everyday situations, whimsy, even computational skill), the benchmark for deciding whether or not to include an activity in a Curriculum would be the extent to which it provides an arena in which students can develop specific mathematical ways of thinking, Cuoco, A.(1995) such as:

- Algorithmic thinking: Constructing and using mechanical processes to model situations.
- Reasoning by continuity: Thinking about continuously varying systems.
- Combinatorial reasoning: Developing ways to "count without counting."
- Thought experiment: Learning to imagine complex interactions.
- Proportional reasoning: Thinking about scaling, area, measure, and probability.
- Reasoning about calculations: Developing algebraic thinking about properties of operations in various symbol systems.
- Topological thinking: Generalizing notions of closeness and approximation to non-metric situations.

These themes would run throughout the K-10 experience. They would be discussed explicitly in class, in diverse contexts, while students were working on problems. After a decade of this core curriculum, students could choose from a set of electives that would vary from school to school and from year to year. Courses in probability, geometry, physics, history, algebra, cryptography, linear algebra, art, data-analysis, accounting, calculus, computer graphics, trigonometry, and whatever else interests teachers and students are all candidates. If students have a solid foundation in mathematical thinking, they will be prepared for a wide array of high-powered courses designed to meet the interests and needs of the entire spectrum of students. This is a genuine alternative to the tracking system: it would give students a choice and a chance to pursue their interests. But no matter what choices they made, students would be assured of a substantial mathematics program that built on a core curriculum centering on mathematical habits of mind.

Cuoco, A. (1995) contend that such a curriculum would help students develop general strategies for doing mathematics, establish underlying mathematical (not just contextual) connections among the tasks, and help students develop the intellectual prowess necessary to deal with the kinds of problems they'll face after graduation.

3 Teacher's Role and Students in an Interactive Mathematics Curriculum

An approach to learning that maximizes student involvement in **thinking** through important mathematical issues leads to a different role for the teacher. It de-emphasizes the teacher's role as creator of concepts and disseminator of algorithms, and sees the teacher more as a facilitator of learning. The teacher provides learning opportunities, asks thought-provoking questions, and allows students to develop their own mathematical frameworks. The teacher uses his or her expertise to provide the "glue" needed to help students tie ideas together and to clarify any misconceptions that may arise. The teacher no longer dictates which steps students will take to solve a problem. An approach, which promotes lifelong learning, combines the mathematics at hand with thinking and reasoning skills, and encourages risk-taking and perseverance.

The Interactive Mathematics Curriculum lets students be active, engaged learners, using what they already know, making conjectures and learning from errors. The IMC curriculum presents students with rich mathematical contexts and gives them well-designed opportunities to discover and develop mathematical concepts as well as to prove important results. This approach offers students meaning for abstract concepts, gives them ownership of mathematical ideas, and heightens this interest in mathematics.

Many teachers, including me, are most likely taught mathematics in a system where mastery of skills was the focus. We were the ones who succeeded in that system. IMC is not based on a mastery approach to learning. While the IMC curriculum seeks to attain the same goal of long-term mathematical understanding as mastery approaches, IMC promotes that understanding through a series of spiraled mathematics experiences, which result over time in mathematical proficiency. The teacher has to remind himself that he/she is trying to build independent thinkers and reasoners. As the teacher see his/her students struggling with an awkward approach to a problem, the teacher may have to work hard to keep from giving them the most elegant way. In this type of curriculum the teacher must be patient and be confident about the curriculum and must believe that his/her students are creative and capable!

Developing knowledge experientially with an activity-oriented Curriculum takes more time than "delivering" knowledge through lecture. A student developing his or her own mathematical ideas to solve challenging problems has to do many things-see a need for the math, try a problem with tools already at hand, look for a pattern, investigate a conjecture, and convince himself or herself of the findings. Of course that takes more time-it is a much longer process than listening to a lecture and practicing an algorithm. The process is what gives the student ownership of the end.

In IMC important mathematical ideas are embedded in the context of interesting problems. As students explore a series of connected problems, they develop understanding of embedded ideas and, with the aid of the teacher, abstract powerful mathematical ideas, problem-solving strategies, and ways of thinking Cuoco, A. (1995). Some problems involve real-world applications or whimsical situations, while others are purely mathematical. A problem's context provides a vehicle for understanding and remembering the mathematical concepts.

Over the past three to four decades, a growing body of knowledge from the cognitive sciences has supported the notion that students develop their own understanding from their experiences with mathematics. In order to give students this kind of experience, a curriculum must be organized so that students continually solve problems that contain important mathematical concepts and skills. This kind of organization is quite different from the traditional philosophy; requiring students to learn by first observing a teacher demonstrate how to solve a problem and then practicing that method on similar problems.

Students' perceptions about a discipline come from the tasks or problems in which they are asked to engage. For example, if students in a geometry course are asked to memorize definitions, they think geometry is about memorizing definitions. If students spend a majority of their mathematics time practicing paper-and-pencil computations, they come to believe that mathematics is about calculating answers to arithmetic problems as quickly as possible. They may become skillful at quickly performing specific types of computations, but they may not be able to apply these skills to other situations or to recognize problems that call for these skills.

On the other hand, if the purpose of studying mathematics is to understand concepts and procedures and be able to solve a variety of problems, then students should spend most of their mathematics time solving problems. If time is spent solving problems, reflecting on solution methods, examining why the methods work, comparing methods, and relating methods to those used in previous situations, then students are likely to build more robust understandings and strategies and to view mathematics as a valuable tool for making sense of and solving interesting problems.

4 Implications to Ethiopian Schools

The important characteristics of IMC discussed elsewhere in this article is that it comprises both the **process** and the **content** of mathematics though the process aspect is given more consideration. The process aspect refers to Problem Solving, Communication, Reasoning, Interconnection and Representation (NCTM, 2000). I contend that the process aspect of mathematics is overlooked in the mathematics curriculum currently underway in schools. Indeed two of the evidence for my argument could be that of problem solving approach as a process and the history of mathematics are missed in the curriculum.

Problem solving approach refers to teaching mathematical topics through problem solving contexts and enquiry-oriented environments which are characterized by the teacher 'helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying' (Lester et al., 1994:154). According to Anderson and White (2004) and Anderson (2000) teachers agree that problem solving is an important life skill for students to develop and that students need to develop a range of problem solving skills. Therefore, they support the focus on problem solving in syllabus documents. However, there is little evidence of the use of challenging, unfamiliar and open-ended problems in mathematics curriculum and mathematics classrooms in Ethiopia. In almost all school mathematics textbooks exercises are used as compared to problems (Open-ended and unfamiliar problems)

The other characteristic of IMC that is missing from the current curriculum is the history of mathematics. The objective of mathematics many believes is to remember facts, repeat the required procedures and get the right answer and every one knows that there is always one “right” answer. But on the other hand mathematics didn't arrive on this planet in a vacuum and that many of those responsible for much of the disciple, took months and decades to get it right. Many of the highly regarded mathematicians struggled, failed, and tried again in the hope to solve a problem, prove theorem or to make new mathematical extension. Hence incorporating the history of mathematics throughout the curriculum would be a useful starting point to show students the developmental nature of mathematics, enliven the teaching of the subject and create an interest in a further study of mathematics. (Swetz, 1984)

Critics declare that studying history has no relevance and is not useful to creating problem solvers. The way a student learns mathematics is by doing mathematics and everything else is secondary. But when we look at early mathematicians who developed the science they didn't use only mechanical manipulations to learn the mathematics. Hence these narrow experiences shortchange students and give them inaccurate view of real mathematical experience.

In general the above criticisms are short-cited and didn't accurately reflect what is possible when expanding students' experience. They fail to see what teachers and students can gain if well-rounded curriculum is adopted. Regarding the importance of history of mathematics Babin et.al (1991:12-13) states:

> *Reading original texts allow the teacher or students to study the nature of mathematical activity and gain access to the philosophical concepts permeating mathematics texts. This process changes the image of mathematics and enables learners to see it as an activity*.

The above quote entails that, by reading or getting experience of original texts of earlier mathematicians or philosophers we can learn the nature of mathematics and in general the philosophy of mathematics and then after one could be able to see mathematics as an activity of human beings and not as an isolated body of from the culture of the society. Original sources in mathematics learning can help us to bring students close to the experiences of mathematical creation, and to initiate them into the way mathematics is practiced.

5 Conclusions

In any discipline if one wants to design a Curriculum he /she needs the philosophy of Curriculum in general and of that particular discipline in particular. It is the philosophy of Curriculum that presents points of view, which guide the development of the curriculum at a particular time.

It is not possible to say that, to design a Curriculum for instance a mathematics Curriculum one needs to depend only on the philosophy of mathematics. That is not enough. There needs to look at the psychological, Historical and sociological foundations as well.

The absolutist and the fallibilst philosophy of mathematics are the ones that shape the design of mathematics Curriculum. Based on these philosophical points of view one can think of two types of mathematics Curriculum that can be called Traditional and Interactive Mathematics Curriculum.

The absolutist philosophy of mathematics doesn't see mathematics as a human endeavor. Absolutists perceive mathematics as it is discovered and no invention of new mathematics is possible. For this philosophical school the objects of mathematics have their own independent existence. Mathematics education has no relation with the history of mathematics. Hence, the organizing principle of mathematics Curriculum for this school is the **content** or the subject matter (the content is more important than the method). Teaching is more of explanatory and prescribing. The focus is on teaching what to think and not how to think. Learning is more of memorization of fact and principles and recitation of the lesson.

Contrary to these assertions the fallibilist philosophy of mathematics considers mathematics as a human endeavor. They believe that history is germane to mathematics education. Mathematics is imbued with human culture and the objects of mathematics are not free of revision. Human beings invented mathematics and they will also invent new mathematical results in the future in order to solve **their own problem** and invention can also be from **the existing mathematical objects**.

A Mathematics Curriculum designed based on this philosophical school will help the child in liberating him/her from traditional emphasis on rote learning, Lesson recitation and textbook authority. Learning is possible as the person actively engages in problem solving which is transferable to variety of situations and subjects and the role of the teacher is helping students to identify their problems and seek solution to the problem. Moreover, teaching learning is child-centered unlike in the traditional curriculum.

Too much emphasis on any single philosophy to design a mathematics Curriculum in our country is not acceptable, (that is exclusively on the subject matter or on the method). I think for designing mathematics Curriculum we should look for a middle road, a highly elusive and abstract concept, where there is no extreme emphasis on subject matter or student; cognitive development. What we need is a prudent school of philosophy that is feasible and that serves the needs of the students and society.

An organizing principle of a mathematics Curriculum called "habits of mind" (students should be pattern sniffers, experimenters, describers, tinkerers, inventors, visualizers, conjecturers, and guessers**)** can do the purpose. Here students are expected to develop good habits of mind that can be transferred to solving different problems in life and that can also help them to learn higher mathematics for those interested. In this type of Curriculum the slogan "mathematics for all is!" applicable.

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