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Fuzzy Programming Approach to Solve Multi-Objective Fully Fuzzy Transportation Problem

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Multi-Objective Programming;
Triangular Fuzzy Number;
Fuzzy Transportation Problem;
Fuzzy Decision Variables;
Ranking Function;
Fuzzy Programming Method.

ABSTRACT

The aim this study is presenting the solution methodology of multi-objective fuzzy transportation problem with fuzzy decision variables, where all the input parameters and decisions variables of the programming problems are assumed to be triangular fuzzy number and triangular fuzzy decision variables respectively. Moreover the objectives under considerations are minimization of cost of transportation and minimization of shipping time under fuzzy environment. The fuzziness of the objective functions and the fuzzy constraints of the programming problem are defuzzified using the ranking function and the equality property between two fuzzy numbers, respectively. The consequent crisp multi-objective fuzzy transportation problem is tackled by employing fuzzy mathematical programming approach. Finally fuzzy decision is made after solving the resultant mathematical programming problem using LINGO(Schrage and LINDO Systems (1997)) software. Illustrative numerical example is presented in support of the proposed methodology.

INTRODUCTION

Transportation problem(TP) is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (e.g., factory) to a set of destinations (e.g., warehouse) to meet the specific requirements, Das *et al.* (2016). In other words, transportation problems deal with the transportation of a product manufactured at different plants (supply origins) to a number of different warehouses (demand destinations). Transportation problems were well known as a basic network problem in its classical category. The formulation and discussion of transportation model was introduced by Hitchcock (1941).

Classical TP models and techniques have been effectively applied to problems with a well-defined or accurately known parameters for many years. The coefficient parameters of the majority of TP models are considered

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to be single real number which is accurately known. However, such assumptions are not suitable for dealing with a number of issues that occur in real life because many of the parameters are imprecise and vague as a result of both natural and anthropogenic effects. This motivates to formulate the TP models in uncertain environment. One of the uncertain environments is fuzzy environ-ment. Moreover in most of the literatures, authors assumed the decision parameters as fuzzy numbers while the decision variables as crisp ones. Since the variables are crisp, the solution obtained as the crisp, which is a real number. The solution is exact value in the fuzzy programming problems with fuzzy parameters. The fuzzy aspect of the decision is partly lost in this case so, it is reasonable and important to consider fuzzy mathematical programming problem with fuzzy decision variables. Fuzzy transportation problems involving fuzzy decision variables are considered in this study. Kaur and Kumar (2012) studied a special type of fuzzy TP by assuming that a decision maker is uncertain about the precise values of transportation cost only, where the transportation cost is represented by generalized trapezoidal fuzzy numbers. In their proposed work all the supply and demand of products are crisp parameters that means there is no uncertainty about the supply and demand of the product. According to the explanation of Kumar and Kaur (2011), there may be factors which imposes the occurrences of fuzziness in TP. Some of these are, the decision maker has not enough information about the unit transportation cost of transportation operation and thus the transportation cost uncertain, there may be some sort of vagueness with respect to the demand of a newly introduced product to the market and there exists uncertainty about the product availability at a source or supplier because of time factor. Many authors introduced tools to solve TP. Since transportation problem (TP) is special case of linear programming (LP) problem, one straightforward approach is to apply the existing LP techniques to the fuzzy TP. These techniques are discussed by many researchers namely; Buckley (1988), Buckley (1990), Mitlif (2016), Ebrahimnejad (2013), Ebrahimnejad (2015), etc. However, some these techniques only give crisp solutions, which represent a compromise in terms of fuzzy data.

The traditional view on TP is mainly concerned with distributing any homogeneous product from a group of supply centers, called sources, to any group of receiving centers, called destinations, in such a way as to minimize the single objective total transportation cost, where the transportation cost per unit product is constant regardless of the amount transported, but most of the time in real-life situation, the TPs are not designed as single objective function. The TP that deals with multiple-objective functions is called a multi-objective transportation problem (MOTP). The MOTP is a special type of multi-objective linear programming problem in which objective functions conflict with each other. Furthermore, objective functions are frequently in conflict, thus there is no one best (global optimum) solution, but rather a group of equally good (non-dominated) alternatives known as pareto optimal (PO) solutions. In the framework of multi-objective programming problems, numerous scholars from a wide variety of academic disciplines discussed their work. Recently, multi-objective problems have been proposed by researchers such as Sayyah *et al.* (2019); Sahih *et al.* (2021); Sosa and Dhodiya (2021); Geshniani *et al.* (2020), and others. Researchers namely; Acharya *et al.* (2014) and Dutta *et al.* (2016) introduced MOT problem in stochastic environment. Chakraborty and Chakraborty (2010) discussed cost-time minimization TP, where the demand, supply and transportation cost per unit of the quantities are fuzzy. Nomani *et al.* (2017) introduced a weighted goal programming to solve multi-objective transportation problems with crisp parameters. They used weighted approach based on goal programming to obtain compromise solutions. Roy *et al.* (2018) pro-

posed multi-objective transportation problem (MOTP) under intuitionistic fuzzy environment. They have assumed transportation cost, the supply and the demand parameters as a intuitionistic fuzzy numbers. Jalil *et al.* (2017) proposed a solution approach for obtaining compromise optimal solution of fully fuzzy(all the parameters and decision variables are fuzzy) multi-objective solid transportation problems. In their proposed problem they used ranking function for the defuzzification of fuzzy objective function and the property of equality between fuzzy numbers for the defuzzification of fuzzy constraints. El Sayed and Abo-Sinna (2021); Moges *et al.* (2023); Malik and Gupta (2022); Niksirat (2022), etc. are the works done under fuzzy environment.
As is mentioned above (Paragraph 2), in most of the literature, authors regarded the decision variables as being crisp while the decision parameters are assumed to be fuzzy. This assumptions leads to crisp decisions which is illogical. A fuzzy decision multi-objective fully fuzzy transportation problem proposed in this study. Ranking function and equality between two triangular fuzzy numbers are employed for defuzzification purpose and finally the equivalent crisp multi-objective model is solved fuzzy programming method.
The paper is organized as follows: following the introduction, basic preliminaries are presented in Sect. 2. The mathematical model of multi-objective fuzzy TP is presented in Sect. 3. Solution procedures are provided in 4. Numerical examples are provided in support of the proposed method in Sect. 5. Finally, Conclusion is provided in Sect. 6 followed by supportive references.

BASIC PRELIMINARIES

Definition: [Roy *et al.* (2018)]: A tri-angular fuzzy number $\tilde{a}$ is denoted by (a^p, a, a^o), where a^p, a, a^o are real numbers. The membership function $(\mu_{\tilde{a}}(x))$ of $\tilde{a}$ is given below:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \leq a^p \\ \frac{x-a^p}{a-a^p}, & a^p \leq x \leq a \\ \frac{a^o-x}{a^o-a}, & a \leq x \leq a^o \\ 0, & otherwise \end{cases}$$

Note: The point 'a' is the core value of triangular fuzzy number $\tilde{A}$, where $\mu_{\tilde{a}}(a)$ =1 a^p and a^o are the lower and upper bounds of support of triangular fuzzy number $\tilde{A}$ respectively.

Definition : [Roy *et al.* (2018)]: Let $\tilde{a} = (a^p, a, a^o)$ and $\tilde{b} = (b^p, b, b^o)$ be two triangular fuzzy numbers ,then
(i) $(a^p, a, a^o) \oplus (b^p, b, b^o) = (a^p + a^p, a+b, a^o + b^o)$
(ii) $k(a^p, a, a^o) = (ka^p, ka, ka^o)$, $k \geq 0$
(iii) $(a^p, a, a^o) \otimes (b^p, b, b^o) = (a^p b^p, ab, a^o b^o$, if $a^p \geq 0$ *and* $b^p \geq 0$

Definition : [Ebrahimnejad (2017)]: Let $\tilde{a} = (a^p, a, a^o)$ and $\tilde{b} = (b^p, b, b^o)$ be two triangular fuzzy numbers ,then
(i) $\tilde{a} = \tilde{b}$ iff $a^p = b^p$, a=b and $a^o = b^o$
(ii) $\tilde{a} = (a^p, a, a^o) \geq 0$ iff $a^p \geq 0$

Definition :Ebrahimnejad (2017)]: Fuzzy TP is said to be balanced transportation problem when total supply from all the sources is equal to the total demand in all destinations.

Definition : [Kumar *et al.* (2011)]: A ranking function is a function $\mathbb{R}$:$F(R) \rightarrow R$, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{a} = (a^p, a, a^o)$ be a triangular fuzzy number, then $\mathbb{R}(\tilde{a}) = \frac{a^p + 2a + a^o}{4}$

Definition : [Hasan *et al.* (2015)]: Multi-Objective Optimization Problem (MOOP) involves more than one objective function that are to be minimized or maximized. Answer of MOOP is the set of solutions that define t he b est t radeoff between competing objectives. The following is general mathematical Form of MOOP:

$$\begin{aligned} \max / \min \quad & Z_m(x), m=1,2,3,...,M & (2.1)\\ \text{s.t.} \quad & g_j(x) \geq 0, j=1,2,...,J & (2.2)\\ & h_k(x) \geq 0, k=1,2,...,K & (2.3)\\ & x^L_{\leq} x_i \leq x^U & (2.4) \end{aligned}$$

MATHEMATICAL MODEL

The mathematical model for MOFTP with fuzzy decision variables is represented as:

$$\min : \widetilde{Z}_k \approx \sum_{i=1}^{m}\sum_{j=1}^{n}((c_{ij}^k)^p x_{ij}^p, c_{ij}^k x_{ij}, (c_{ij}^k)^o x_{ij}^o), k \in \{1, 2...K\} \quad (3.1)$$

subject to

$$\sum_{j=1}^{n}(x_{ij}^p, x_{ij}, x_{ij}^o) \approx (a_i^p, a_i, a_i^o), i \in \{1,2,3,...,m\} \quad (3.2)$$

$$\sum_{i=1}^{m}(x_{ij}^p, x_{ij}, x_{ij}^o) \approx (b_j^p, b_j, b_j^o), j \in \{1,2,3,...,n\} \quad (3.3)$$

$$\widetilde{x}_{ij} \succeq 0, i \in \{1,2,3,...,m\}; j \in \{1,2,3,...,n\} \quad (3.4)$$

where,

i. the fuzzy total availability and fuzzy total demand are assumed to be equal(balanced fuzzy transportation problem), $'m'$ is total number of supply points and 'n' is total number of destination points,

ii. $\widetilde{a_i}$=(a_i^p, a_i, a_i^o) is the fuzzy availability of the commodity at i^{th} origin and assumed to be triangular fuzzy number,

iii. $\widetilde{b_j}$=(b_j^p, b_j, b_j^o) is the fuzzy requirement of the commodity at j^{th} destination and assumed to be triangular fuzzy number,

iv. $\widetilde{c}_{ij}$=$(c_i^p j, c_i j, c_i^o j)$ is the fuzzy cost coefficient involved with fuzzy variables in the objective function from i^{th} origin to j^{th} destination, which is also assumed to be triangular fuzzy number,

v. $\widetilde{x}_{ij}$=$(x_{ij}^p, x_{ij}, x_{ij}^o)$ is the fuzzy quantity that should be transported from i^{th} origin to j^{th} destination and assumed to be triangular fuzzy decision variables.

Crisp equivalent of multi-objective fuzzy transportation problem

Since FMP model from (3.1) to (3.4) cannot be solved directly, so ranking function and the property of equality between the fuzzy numbers are respectively applied on the fuzzy objective functions and fuzzy constraints models of MOFTP. The resultant crisp equivalent is obtained as:

$$\min : Z_k = \frac{1}{4}\sum_{i=1}^{m}\sum_{j=1}^{n}((c_{ij}^p)^k x_{ij}^p + 2c_{ij}^k x_{ij} + (c_{ij}^o)^k x_{ij}^o), k = 1, \quad (3.5)$$

subject to

$$\sum_{j=1}^{n} x_{ij}^p = a_i^p, i \in \{1,2,3,...,m\}; \quad (3.6)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, i \in \{1,2,3,...,m\} \quad (3.7)$$

$$\sum_{j=1}^{n} x_{ij}^{o} = a_i^{o}, i \in \{1,2,3,...,m\} \quad (3.8)$$

$$\sum_{i=1}^{m} x_{ij}^{p} = b_j^{p}, j \in \{1,2,3,...,n\} \quad (3.9)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j \in \{1,2,3,...,n\} \quad (3.10)$$

$$\sum_{i=1}^{m} x_{ij}^{o} = b_j^{o}, j \in \{1,2,3,...,n\} \quad (3.11)$$

$$x_{ij}^{o} - x_{ij} \geq 0, i \in \{1,2,3,...,m\}, j \in \{1,2,3,...,n\} \quad (3.12)$$

$$x_{ij} - x_{ij}^{p} \geq 0, i \in \{1,2,3,...,m\}, j \in \{1,2,3,...,n\} \quad (3.13)$$

$$x_{ij}^{p} \geq 0, i \in \{1,2,3,...,m\}, j \in \{1,2,3,...,n\} \quad (3.14)$$

Solution procedure

Now we use the fuzzy programming programming technique to solve the crisp multi-objective programming problem of 3.5 to 3.14. The solution procedures based on the fuzzy programming method is detailed below.

Step 1: Find the ideal solutions $x^1, x^2, ..., x^k$ by picking one objective function at a time and leaving the other objective functions.

Step 2: Construct a pay-matrix with the help of individual best solutions found by the above step. Using table 1 estimate the bounds of z_k (k=1,2,3...,K) from the Payoff matrix.

Step 3: For every objective function z_k (k=1,2,3...,K), we formulate membership function using any one of the following techniques of maximization or minimization:

Table 1: Payoff matrix

	$z_1(x)$	$z_2(x)$	. . .	$z_K(x)$
$x^{(1)}$	$z_1(x^{(1)})$	$z_2(x^{(1)})$	. . .	$z_K(x^{(1)})$
$x^{(2)}$	$z_1(x^{(2)})$	$z_2(x^{(2)})$	. . .	$z_K(x^{(2)})$
.	.	.	. . .	.
.	.	.	. . .	.
.	.	.	. . .	.
$x^{(K)}$	$z_1(x^{(K)})$	$z_2(x^{(K)})$	. . .	$.z_K(x^{(K)})$

Step 4

Case 1: Membership function is formulated in the case of maximization problem as:

$$\mu_{z_k}(x) = \begin{cases} 0, & \text{if } z_k \leq lb_k^- \\ \frac{z_k - lb_k^-}{ub^* - lb_k^-}, & \text{if } lb_k^- \leq z_k \leq ub_k^* \\ 1, & \text{if } z_k \geq ub_k^* \end{cases}$$

where lb_k^- denotes the worst lower bound of z_k and ub_k^* denotes the best upper bound of z_k

Case 2: For minimization problem the membership function is formulated as:

$$\mu_{z_k}(x) = \begin{cases} 0, & \text{if } z_k \geq ub_k^- \\ \frac{ub^- - z_k}{ub^- - lb_k^*}, & \text{if } lb_k^* \leq z_k \leq ub_k^- \\ 1, & \text{if } z_k \leq lb_k^* \end{cases}$$

Where ub_k^- denotes the worst upper bound of z_k and lb_k^* denotes best lower bound of z_k.

Case 1: Apply the augmented variable, λ with max-min operator to formulate a crisp single objective mixed integer programming problem as:

$$\max : \lambda \quad (3.15)$$

subject to

$$\mu_{z_k}(x) \geq \lambda, \ k = 1,2,...K \quad (3.16)$$

$$\sum_{j=1}^{n} x_{ij}^{p} = a_i^{p}, i \in \{1,2,3,...,m\} \quad (3.17)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, i \in \{1,2,3,...,m\} \quad (3.18)$$

$$\sum_{j=1}^{n} x_{ij}^o = a_i^o, i \in \{1,2,3,...,m\} \quad (3.19)$$

$$\sum_{i=1}^{m} x_{ij}^p = b_j^p, j \in \{1,2,3,...,n\} \quad (3.20)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j \in \{1,2,3,...,n\} \quad (3.21)$$

$$\sum_{i=1}^{m} x_{ij}^o = b_j^o, j \in \{1,2,3,...,n\} \quad (3.22)$$

$$x_{ij}^o - x_{ij} \geq 0, i \in \{1,2,3,...,m\}, j \in \{1,2,3,...,n\} \quad (3.23)$$

$$x_{ij} - x_{ij}^p \geq 0, i \in \{1,2,3,...,m\}, j \in \{1,2,3,...,n\} \quad (3.24)$$

$$x_{ij}^p \geq 0, i \in \{1,2,3,...,m\}, j \in \{1,2,3,...,n\} \quad (3.25)$$

$$0 \leq \lambda \leq 1 \quad (3.26)$$

Case 2: Apply the augmented variable, λ with min-max operator to formulate a crisp single objective mixed integer programming problem as:

$$\min : \lambda \quad (3.27)$$

subject to

$$\mu_{z_k}(x) \leq \lambda, \ k \in \{1,2,3,....K\} \quad (3.28)$$

$$\sum_{j=1}^{n} x_{ij}^p = a_i^p, i \in \{1,2,3...m\} \quad (3.29)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, i \in \{1,2,3...m\} \quad (3.30)$$

$$\sum_{j=1}^{n} x_{ij}^o = a_i^o, i \in \{1,2,3...m\} \quad (3.31)$$

$$\sum_{i=1}^{m} x_{ij}^p = b_j^p, j \in \{1,2,3...n\} \quad (3.32)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j \in \{1,2,3...n\} \quad (3.33)$$

$$\sum_{i=1}^{m} x_{ij}^o = b_j^o, j \in \{1,2,3...n\} \quad (3.34)$$

$$x_{ij}^o - x_{ij} \geq 0, i \in \{1,2,3...m\}; j \in \{1,2,3...n\} \quad (3.35)$$

$$x_{ij} - x_{ij}^p \geq 0, i \in \{1,2,3...m\}; j \in \{1,2,3...n\} \quad (3.36)$$

$$x_{ij}^p \geq 0, i \in \{1,2,3...m\}; j \in \{1,2,3...n\} \quad (3.37)$$

$$0 \leq \lambda \leq 1 \quad (3.38)$$

Step 5 At the end, the equivalent single objective MP model is solved by using appropriate techniques or existing software. The obtained PO solutions substituted back to original fuzzy objective function, as the result we can find fuzzy optimal values of each fuzzy objective functions.

Numerical example

In this part, application numerical example is solved using the provided approach, and the conclusions drawn from the results are discussed in further detail.

A firm has two sources O_1 and O_2 and three destinations D_1 ,D_2 and D_3. The fuzzy supply of the commodity from O_1 and O_2 are (75, 95, 125) and (45, 65, 95), respectively. Request of fuzzy demanded product at D_1 ,D_2 and D_3 are (35, 45, 55), (25, 35, 45) and (60, 80, 110), respectively. The company wants to determine the fuzzy quantity of the commodity that should be transported from each origin to each destination so that the total fuzzy transportation cost is minimum with minimum transfer time. For i=1,2, j=1,2,3., let the fuzzy transportation cost for unit quantity of the commodity from i^{th} source to j^{th} destinations be $\tilde{c}_{ij}$, the fuzzy transportation time is also considered as $\tilde{t}_{ij}$ and $\tilde{x}_{ij}$ represents the allocations(or amounts), which is non negative triangular fuzzy real variable. The theoretical data on fuzzy cost of transportation and fuzzy delivery time, is given in the table 2 below.

Table 2: Fuzzy transportation cost per unit(in Rupees) and fuzzy transportation time per unit(in minute)

	D_1	D_2	D_3
$\tilde{c}_{1j}(O_1)$	(15,25,35)	(55, 65,85)	(85,95,105)
$\tilde{c}_{2j}(O_2)$	(65,75,85)	(80,90,110)	(30,40,50)
$\tilde{t}_{1j}(O_1)$	(4, 6, 8)	(6, 8,10)	(7,9,11)
$\tilde{t}_{2j}(O_2)$	(3,5,7)	(5,7,9)	(11,13,15)

From the table 2 mathematical model for MOFTP with fuzzy decision variables becomes:

$$\min : \tilde{z}_1 \approx (15,25,35) \otimes \tilde{x}_{11} \oplus (55,65,85) \otimes \tilde{x}_{12} \oplus (85,95,105) \otimes \tilde{x}_{13} \oplus (65,75,85) \otimes \tilde{x}_{21}$$

$$\oplus (65,75,85) \otimes \tilde{x}_{21} \oplus (80,90,110) \otimes \tilde{x}_{22} \oplus (30,40,50) \otimes \tilde{x}_{23} \tag{3.39}$$

$$\min : \tilde{z}_2 \approx (4,6,8) \otimes \tilde{x}_{11} \oplus (6,8,10) \otimes \tilde{x}_{12} \oplus (7,9,11) \otimes \tilde{x}_{13} \oplus (3,5,7) \otimes \tilde{x}_{21} \oplus (5,7,9) \otimes \tilde{x}_{22}$$

$$\oplus (11,13,15) \otimes \tilde{x}_{23} \tag{3.40}$$

Subject to

$$\tilde{x}_{11} \oplus \tilde{x}_{12} \oplus \tilde{x}_{13} \approx (75,95,125) \tag{3.41}$$

$$\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} \approx (45,65,95) \tag{3.42}$$

$$\tilde{x}_{11} \oplus \tilde{x}_{21} \approx (35,45,65) \tag{3.43}$$

$$\tilde{x}_{12} \oplus \tilde{x}_{22} \approx (25,35,45) \tag{3.44}$$

$$\tilde{x}_{13} \oplus \tilde{x}_{23} \approx (60,80,110) \tag{3.45}$$

$$x_{11}, \tilde{x}_{12}, \tilde{x}_{13} \succeq 0 \tag{3.46}$$

$$\tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23} \succeq 0 \tag{3.47}$$

defuzzification is done using the concept discussed in the subsection 3.1. Then the case(3.27-3.37) of evaluation of membership functions for the minimization (Max-min operator) is applied on the crisp MOTP along with all the procedures discussed above. The fuzzy programming method is applied on the aforementioned crisp equivalent MOTP to find the ideal solutions as detailed below

$x^{(1)} = (x^p_{11}, x_{11}, x^o_{11}, x^p_{12}, x_{12}, x^o_{12}, x^p_{13}, x_{13}, x^o_{13}, x^p_{21}, x_{21}, x^o_{21}, x^p_{22}, x_{22}, x^o_{22} x^p_{23}, x_{23}, x^o_{23})=$
(35,35,55,10,10,10,30,50,60,0,10,10,15,25,35,30,30,50)
$x^{(2)} = (x^p_{11}, x_{11}, x^o_{11}, x^p_{12}, x_{12}, x^o_{12}, x^p_{13}, x_{13}, x^o_{13}, x^p_{21}, x_{21}, x^o_{21}, x^p_{22}, x_{22}, x^o_{22}, x^p_{23}, x_{23}, x^o_{23})=$
(0,10,30,25,35,35,50,50,60,35,35,35,0,0,10,10,30,50)

At the corresponding ideal points the values of crisp objective functions are obtained and given as: z_1=10737.5, z_2=1440.
After constructing a pay-of matrix 2, the bounds of the two objective functions are given by:

$$10737.5 \leq z_1(x) \leq 11825$$

$$1440 \leq z_2(x) \leq 1465.$$

The the single objective crisp problem is obtained by formulating membership function. Thus the programming problem becomes:

$$\max : \lambda \tag{3.48}$$

subject to

$$z_1 + 1387.5\lambda \leq 11825 \quad (3.49)$$
$$z_2 + 25\lambda \leq 1465 \quad (3.50)$$
$$x^p_{11} + x^p_{12} + x^p_{13} = 75 \quad (3.51)$$
$$x_{11} + x_{12} + x_{13} = 95 \quad (3.52)$$
$$x^o_{11} + x^o_{12} + x^o_{13} = 125 \quad (3.53)$$
$$x^p_{21} + x^p_{22} + x^p_{23} = 45 \quad (3.54)$$
$$x_{21} + x_{22} + x_{23} = 65 \quad (3.55)$$
$$x^o_{21} + x^o_{22} + x^o_{23} = 95 \quad (3.56)$$
$$x^p_{11} + x^p_{21} = 35 \quad (3.57)$$
$$x_{11} + x_{21} = 45 \quad (3.58)$$
$$x^o_{11} + x^o_{21} = 65 \quad (3.59)$$
$$x^p_{12} + x^p_{22} = 25 \quad (3.60)$$
$$x_{12} + x_{22} = 35 \quad (3.61)$$
$$x^o_{12} + x^o_{22} = 45 \quad (3.62)$$
$$x^p_{13} + x^p_{23} = 60 \quad (3.63)$$
$$x_{13} + x_{23} = 80 \quad (3.64)$$
$$x^o_{13} + x^o_{23} = 110 \quad (3.65)$$
$$x^o_{ij} - x_{ij} \geq 0 \quad (3.66)$$
$$x_{ij} - x^p_{ij} \geq 0 \quad (3.67)$$
$$x^p_{ij} \geq 0 \quad (3.68)$$
$$0 \leq \lambda \leq 1 \quad (3.69)$$

By employing the LINGO software, the resultant equivalent crisp programming problem (3.48)-(3.69) is solved. The PO solutions and the agumated variable λ obtained are given consecutively as: x^*=(31.66375,41.66375,61.66375,0,3.336248,3.33666248,43.33675,50,60, 3.336248 ,3.336248 ,3.336248 ,25,31.66375,41.66375,16.66375,30,50), λ=0.6668124. The compromising fuzzy objective functions values are $\tilde{z}_1$=(6875.31, 10341.77, 16075.04) and $\tilde{z}_2$=(748.32, 1355.13, 2335).
Figurative description of fuzzy optimal triangular cost and triangular transfer time are detailed in the figure below.

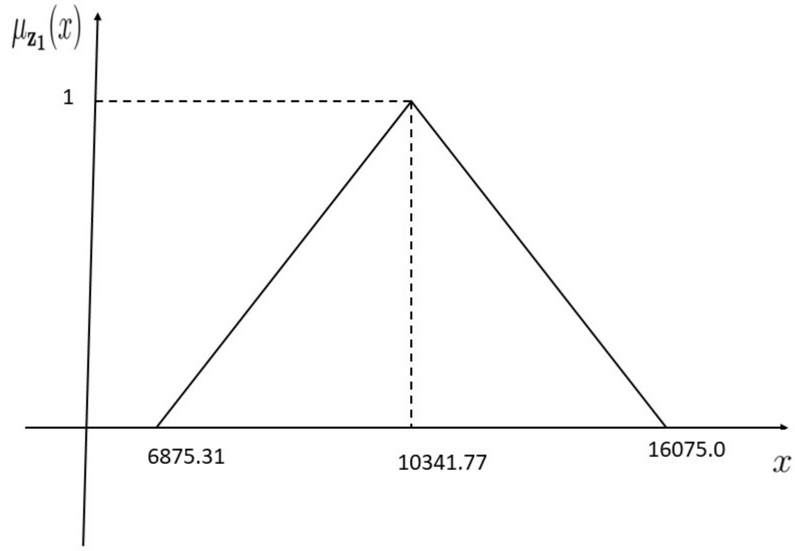

Figure 1: Minimum fuzzy transportation cost

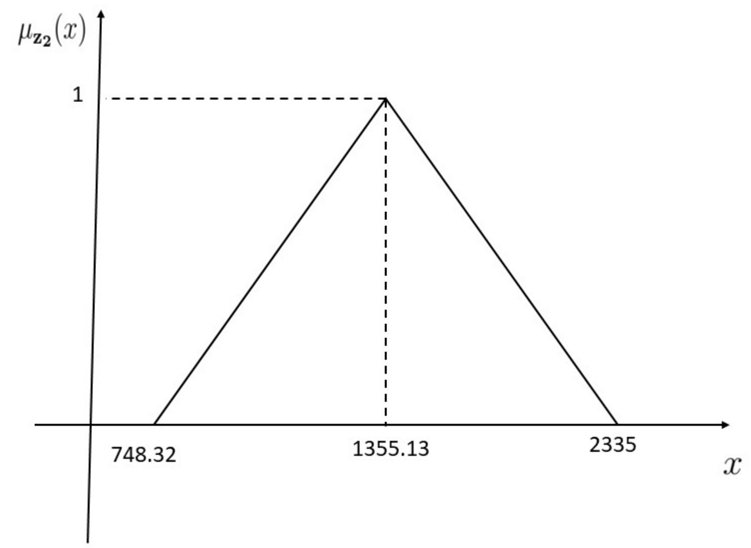

Figure 2: Minimum fuzzy transfer time

DISCUSSION

The defuzzified crisp MOTP is solved by fuzzy programming method. Using Max-min operator, the resultant single objective crisp integer MP problem is coded into LINGO optimization package version 19.0 software and fuzzy PO solutions are obtained. The minimum total fuzzy cost of transportation and the minimum fuzzy transfer time are obtained and interpreted as follows:
Fuzzy transportation cost and fuzzy transfer time $\widetilde{z}_1$=(6875.31, 10341.77, 16075.04) and $\widetilde{z}_2$=(748.32, 1355.13, 2335) respectively are calculated by back substitutions of PO solutions into fuzzy objective functions (3.1) of the the programming problem.
It is found that the least amounts of minimum total transportation cost and transfer time are 6875.31 and 748.32 units respectively. The the most possible amounts of minimum total transportation cost and transfer time are 10341.77 and 1355.13 units respectively. Furthermore, the greatest amounts of minimum total transportation cost and total transfer time are 16075.04 and 2335 units respectively.

CONCLUSION

The classical transportation problem (TP) is primarily concerned with distributing any homogeneous product from a group of supply centers, known as sources, to any group of receiving centers, known as destinations, in such a way that the single objective total transportation cost is minimized, where all parameters are crisp (precisely defined). However, in many circumstances, the decision maker lacks precise knowledge of the TP parameters. and the nature of the TPs are not designed as single objective function. If the nature of the information is vague and the decision maker objectives preference are conflicting, t he c orresponding programming problem is fuzzy multi-objective programming problem, and thus fuzzy MOTP arises. In this paper, we have discussed a solution approach for solving TP, with more than one objective function by considering the presence of vagueness in the real life data of transportation problems, where all the parameters and decision variables are considered as triangular fuzzy numbers. Initially, the fuzzy objective functions are defuzzified by applying the ranking function for triangular fuzzy numbers on each individual objective function. The equality constraints involved in the multi-objective TP model with fuzzy parameters are converted to their crisp equivalent by using the property of equality among the fuzzy numbers. Finally fuzzy programming approach is used to find the compromise solution of the crisp multi-objective programming problem. Optimization solver, LINGO Schrage and LINDO Systems (1997), is used to solve one application example by proposed method and solutions and findings are discussed in detail. As the fuzzy technique is often employed to model many real-world situations, including supply chain modeling, information theory problems, inventory operations problems etc., the suggested method may eventually be extended to address such optimization problems.

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