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Abstract

This research examines the availability, maintainability and sensitivity of a poultry feed processing plant to optimize operational performance. Availability is assessed using Mean Time between Failures (MTBF) and Mean Time to Repair (MTTR), revealing a gradual decline over time due to equipment failures. Maintainability analysis focuses on repair efficiency, highlighting the plant's ability to quickly restore machinery after breakdowns. Sensitivity Analysis identifies preventive maintenance frequency and spare parts availability as key factors affecting system performance. The findings provide means to reduce downtime and improve operational reliability in poultry feed processing. Graphical analysis of availability shows a consistent decline over time across all equipment. Maintainability analysis reveals that despite a high initial recovery rate, minor deviations in repair times can significantly im pact overall operational uptime.

Keywords: Poultry, Processing Plant, Availability, Maintainability, Sensitivity.

Introduction

The field of system reliability and performance analysis has gained considerable attention in recent years due to its critical role in enhancing the operational efficiency and longevity of various systems. In industries and applications ranging from energy systems, such as solar photovoltaic and solar water pumping, to complex repairable systems, ensuring optimal performance and reliability has become increasingly important. System reliability, availability, maintainability, and dependability (RAMD) are essential metrics used to evaluate and improve the effectiveness of these systems. The continued advancement of analytical models and optimization techniques is crucial in addressing the growing demands for more efficient and resilient systems, especially in the context of sustainable energy solutions.

The ongoing development of reliability models has facilitated a deeper understanding of the underlying dynamics in system performance. This research seeks to contribute to the

optimization of system reliability, focusing on incorporating various factors, including repair strategies, operational complexities, and economic considerations. By focusing on these aspects, researchers aim to improve the overall resilience and cost-effectiveness of systems, especially in fields that directly contribute to sustainable energy generation and resource management.

Several notable studies have made substantial contributions to the advancement of system reliability and performance analysis. According to Lado & Singh (2019) pioneered research on reliability models tailored to varying demand cold standby systems. Their work provided valuable insights into the reliability characteristics and operational effectiveness of such systems, particularly under different demand conditions. Similarly, According to Singh & Ayagi (2018) focused on complex repairable systems and introduced innovative models that integrated pre-emptive resume repair strategies. This work not only expanded the theoretical understanding of system performance but also had practical implications for improving reliability and efficiency in real-world applications.

Other research efforts, According to Corvaro et al. (2017) have concentrated on optimizing system design through dual-objective optimization models. Their study explored the simultaneous consideration of reliability and cost in the management of series-parallel systems. This dual focus allowed for a more comprehensive framework for system optimization, addressing both performance and economic viability. Likewise, Sanusi & Yusuf (2021) applied copula-based methodologies to series-parallel systems, providing a robust approach to understanding the interdependencies between system components. Their findings emphasized the significance of capturing these interdependencies to assess and improve overall system reliability.

This work aims to fill this gap by developing an integrated model that not only addresses the technical and operational aspects of system reliability but also incorporates the economic implications of system upkeep and operation. The novelty of this research lies in its ability to combine diverse optimization techniques, such as dual-objective optimization and copulabased methodologies, to provide a holistic view of system performance. By considering both technical and economic factors, this research will offer innovative solutions for improving the reliability and cost-effectiveness of energy systems in real-world applications, especially those linked to sustainable energy practices.

Notations	Meanings		
δ_i and μ_i	the failure rate and repair rate of the system for some $i = 1,2,3,4$		
$S_0(t)$	Probability that the system is operating at maximum capacity		
$S_i(t)$	Steady-state probability that the system is in <i>i</i> th state		
δ_i	Failure rate of the subsystem H,I,J,K		
μ_i	Repair rate of the subsystem H,I,J,K		

Table 1: Notations and Meanings

Description of the System

Subsystem H (Crusher): It's the first step after receiving the raw material. Any grain go through this process to undergo size reduction and increase the surface area for the greater nutritional value for the poultry.

Subsystem I (Mixer): The main objective of this component is to combine the ingredients together to ensure they are distributed in the mixture properly.

Subsystem J (Sieving): Sieving is required when producing pellets. Usually, small fragment are produced as a result when the hot, moist pellets are cut off from the die inside the pelleting chamber, and as produced pellets pass through the cooling and conveying process.

Subsystem K (Coating): Fats and oil can be added in this process to further improve the nutritional value of the pellets. This aims to add the remaining amount of oils that could not be added before the pelleting process.



MATERIALS AND METHODS

The tools for the computation of reliability measures for the model are as follows;

$$f(x,\omega) = \begin{cases} \omega e^{-\omega x}, & \text{if } x \ge 0\\ 0, & o, w \end{cases}$$
(1)
$$f(t,\omega) = \begin{cases} \omega e^{-\omega t}, & \text{if } t \ge 0\\ 0 \end{cases}$$
(2)

(4) (5)

$$H(t) = 1 - e^{-\omega t}$$
 and $R(t) = 1 - e^{-\omega t}$ (3)

$$R(t) = e^{-\int_0^\infty f(t)dt}$$

$$R(t) = e^{-\delta t}$$

Availability =
$$\frac{MTBF}{MTBF+MTTR}$$
 (6)

$$M(t) = P(T \le t) = 1 - e^{-\mu t}$$
(7)
$$d = \frac{\mu}{s} = \frac{MTBF}{MTTP}$$
(8)

$$d = \frac{1}{\delta} = \frac{1}{MTTR} (dlnd)$$

$$D_{min} = 1 - \left(\frac{1}{d-1}\right) \left(e^{-\left(\frac{ina}{d-1}\right)} - e^{-\left(\frac{aina}{d-1}\right)} \right)$$
(9)
(10)

$$MTTR = \mu^{-1}$$

$$MTBF = \int_0^\infty R(t)dt = \int_0^\infty e^{-\delta t}dt = \delta^{-1}$$
(11)

Гable 2: F	Failure and	l repair	rate

Subsystem	Failure rate (δ)	Repair rate (μ)
Crushing (H)	$\delta_1 = 0.015$	$\mu_1 = 0.35$
Mixer (I)	$\delta_{2} = 0.025$	$\mu_2 = 0.20$
Sieving (J)	$\delta_{3} = 0.025$	$\mu_3 = 0.15$
Coating (K)	$\delta_4 = 0.011$	$\mu_4 = 0.41$

RESULTS

Formulation of Mathematical Models for RAMD

Chapman Kolmogorov differential equations for each subsystem have been constructed using the Markov birth-death process for mathematical modelling of animal feed processing system, Yusuf, I., Anas, M. & Saminu, I. Y. (2021) . The System performance measures such as reliability, availability, maintainability and dependability have been derived by solving the appropriate Chapman-Kolmogorov differential equations in a steady-state and employing normalization conditions recursively.

$$\frac{dS_0(t)}{dt} = -\delta_k S_0 + \mu_k S_1 \tag{12}$$

$$\frac{dS_1(t)}{dt} = -\delta_k S_1 + \mu_k S_0 \tag{13}$$

For
$$k = 1,2,3,4$$
.
 $-\delta_k S_0 + \mu_k S_1 = 0$ (14)

$$-\delta_k S_1 + \mu_k S_0 = 0 \tag{15}$$
$$S_0 = \frac{\mu_k}{\mu_1 + \delta_1}, \qquad S_1 = \frac{\delta_k}{\mu_1} S_0$$

$$A_{sys}(t) = \left(1 + \frac{\delta_k}{\mu_k}\right)^{-1}$$

$$M(t) = 1 - e^{-\mu t}$$
(17)
(18)

$$D_{min} = 1 - \left(\frac{1}{d-1}\right) \left(e^{-\left(\frac{lnd}{d-1}\right)} - e^{-\left(\frac{dlnd}{d-1}\right)} \right)$$
(19)

RAMD Analysis for Subsystem H (Crushing unit)

$$\frac{dS_0(t)}{dt} = -\delta_1 S_0 + \mu_1 S_1 \tag{20}$$

$$\frac{dS_1(t)}{dt} = -\delta_1 S_1 + \mu_1 S_0 \tag{21}$$

$$-\delta_1 S_0 + \mu_1 S_1 = 0$$
(22)
$$-\delta_1 S_1 + \mu_1 S_0 = 0$$
(23)

$$S_0 = \frac{\mu_1}{\mu_1 + \delta_1}$$
, $S_1 = \frac{\delta_1}{\mu_1} S_0$

The RAMD measures of the system can be determine by the following equations: $R_{S_{\mu}}(t) = e^{-\delta_1 t} = e^{-0.012t}$ (24)

$$A_{S_H}(t) = \left(1 + \frac{\delta_1}{\mu_1}\right)^{-1} = 0.9090$$
(25)

$$M_{S_H}(t) = 1 - e^{-\mu_1 t} = 1 - e^{-0.12t}$$
(26)

$$D_{min_H}(t) = 1 - \left(\frac{1}{d-1}\right) \left(e^{-\left(\frac{lnd}{d-1}\right)} - e^{-\left(\frac{dlnd}{d-1}\right)} \right) = 0.9140$$
(27)

RAMD Analysis for Subsystem I (Mixer)

$$\frac{dS_0(t)}{dt} = -\delta_2 S_0 + \mu_2 S_1 \tag{28}$$

$$\frac{dS_1(t)}{dt} = -\delta_2 S_1 + \mu_2 S_0 \tag{29}$$

$$-\delta_2 S_0 + \mu_2 S_1 = 0 \tag{30} -\delta_2 S_1 + \mu_2 S_0 = 0 \tag{31}$$

$$= 0$$

$$S_0 = \frac{\mu_2}{\mu_2 + \delta_2}$$
, $S_1 = \frac{\sigma_2}{\mu_2} S_0$

The RAMD measures of the system can be determine by the following equations: $R_{S_t}(t) = e^{-\delta_2 t} = e^{-0.014t}$ (32)

$$A_{S_I}(t) = \left(1 + \frac{\delta_2}{\mu_2}\right)^{-1} = 0.9028 \tag{33}$$

$$M_{S_I}(t) = 1 - e^{-\mu_2 t} = 1 - e^{-0.13t}$$
(34)

$$D_{min_{I}}(t) = 1 - \left(\frac{1}{d-1}\right) \left(e^{-\left(\frac{lnd}{d-1}\right)} - e^{-\left(\frac{dlnd}{d-1}\right)} \right) = 0.9972$$
(35)

RAMD Analysis for Subsystem J (Sieving) $dS_{t}(t)$

$$\frac{dS_0(t)}{dt} = -\delta_3 S_0 + \mu_3 S_1 \tag{36}$$

$$\frac{dS_1(t)}{dt} = -\delta_3 S_1 + \mu_3 S_0 \tag{37}$$

$$-\delta_3 S_0 + \mu_3 S_1 = 0$$
(38)
$$-\delta_3 S_1 + \mu_3 S_0 = 0$$
(39)

$$S_0 = \frac{\mu_3}{\mu_3 + \delta_3}$$
, $S_1 = \frac{\delta_3}{\mu_3} S_0$

The RAMD measures of the system can be determine by the following equations: $R_{s_{*}}(t) = e^{-\delta_{3}t} = e^{-0.014t}$ (40)

$$A_{S_{I}}(t) = \left(1 + \frac{\delta_{3}}{2}\right)^{-1} = 0.9244$$
(41)

$$M_{S_{I}}(t) = 1 - e^{-\mu_{3}t} = 1 - e^{-0.15t}$$
(42)

$$D_{min_{J}}(t) = 1 - \left(\frac{1}{d-1}\right) \left(e^{-\left(\frac{lnd}{d-1}\right)} - e^{-\left(\frac{dlnd}{d-1}\right)}\right) = 0.9972$$
(43)

RAMD Analysis for Subsystem K (Coating)

$$\frac{dS_0(t)}{dt} = -\delta_4 S_0 + \mu_4 S_1 \tag{44}$$

$$\frac{dS_1(t)}{dt} = -\delta_4 S_1 + \mu_4 S_0 \tag{45}$$

$$-\delta_4 S_0 + \mu_4 S_1 = 0 \tag{46} -\delta_4 S_1 + \mu_4 S_0 = 0 \tag{47}$$

$$-\delta_4 S_1 + \mu_4 S_0 = 0 \tag{4}$$

$$S_0 = \frac{4}{\mu_4 + \delta_4}$$
, $S_1 = \frac{\delta_4}{\mu_4} S_0$

The RAMD measures of the system can be determine by the following equations: $R_{S_{\kappa}}(t) = e^{-\delta_4 t} = e^{-0.017t}$ (48)

$$A_{S_{K}}(t) = \left(1 + \frac{\delta_{4}}{n}\right)^{-1} = 0.9244 \tag{49}$$

$$M_{S_K}(t) = 1 - e^{-\mu_7 t} = 1 - e^{-0.40t}$$
(50)

$$D_{min_K}(t) = 1 - \left(\frac{1}{d-1}\right) \left(e^{-\left(\frac{lnd}{d-1}\right)} - e^{-\left(\frac{dlnd}{d-1}\right)} \right) = 0.9972$$
(51)

RAMD Indices for Subsystem

System reliability

Because all four subsystems are linked in series, the failure of one causes the entire system to fail. The whole system's reliability is determined by: $R_{sys} = R_{S_H}(t) * R_{S_I}(t) * R_{S_K}(t)$ (52)

TIME	Н	I	J	К
0	1.0000	1.0000	1.0000	1.0000
20	0.7408	0.7261	0.7557	0.7117
40	0.5488	0.5272	0.5712	0.5066
60	0.4065	0.3828	0.4317	0.3605
80	0.3011	0.2780	0.3262	0.2566
100	0.2231	0.2018	0.2465	0.18268

Table 3: Variation in subsystem reliability over time



System Maintainability

As all four subsystems are interconnected in series, the failure of any one subsystem leads to the failure of the entire system.

The overall maintainability of the system is determined by:

$$M_{Sys}(t) = M_{C_H}(t) * M_{C_I}(t) * M_{C_I}(t) * M_{C_K}(t)$$

$$M_{Sys}(t) = 1 - e^{-0.17t}$$
(53)
(54)

$$M_{Sys}(t) = 1 - e^{-0.17}$$

Table 4: Variation in subsystem maintainability over time

TIME	н	I	J	К
0	0.0000	0.0000	0.0000	0.0000
20	0.9990	0.9816	0.9502	0.9996
40	0.9999	0.9996	0.9975	0.9999
60	0.9999	0.9999	0.9998	1.0000
80	1.0000	0.9999	0.9999	1.0000
100	1.0000	0.9999	0.9999	1.0000



System dependability

The total system resilience is determined by: $D_{min_{SYS}} = D_{min_{SH}} * D_{min_{SI}} * D_{min_{SI}} * D_{min_{SK}}$

Sensitivity Analysis

It is a technique which is used to identify the impact of independent variable on a specific dependent variable on the basis of some assign assumptions. It determines the effect of the change in parameters and structure of the model. Here, sensitivity analysis for reliability of the subsystems and system with respect to failure rates δ_1 , δ_2 , δ_3 , and δ_4 , has been performed. The following expressions have been derived respectively:

$$\frac{\partial Rsys}{\partial \delta_1} = -\text{te}^{-(\delta_1 + \delta_2 + \delta_3 + \delta_4)\text{t}}$$
$$\frac{\partial Rsys}{\partial \delta_2} = -\text{te}^{-(\delta_1 + \delta_2 + \delta_3 + \delta_4)\text{t}}$$
$$\frac{\partial Rsys}{\partial \delta_3} = -\text{te}^{-(\delta_1 + \delta_2 + \delta_3 + \delta_4)\text{t}}$$
$$\frac{\partial Rsys}{\partial \delta_4} = -\text{te}^{-(\delta_1 + \delta_2 + \delta_3 + \delta_4)\text{t}}$$

(55)

(56)

Indices	Subsystem H	Subsystem I	Subsystem J	Subsystem K
		-		
Reliability	$e^{-0.0013t}$	$e^{-0.005t}$	$e^{-0.003t}$	$e^{-0.0052t}$
Availability	0.999999	0.996565	0.996565	0.9965
Maintainability	$1 - e^{-0.45t}$	$1 - e^{-0.082t}$	$1 - e^{-0.086t}$	$1 - e^{-0.082t}$
Dependability ratio	346.18888	16.39393	286.61616	16.393
MTBF	769.2323	200.00	333.33333	200
MTTR	2.22222	12.2020	1.16363	12.20

Numerical simulation

Table 5: RAMD indices for subsystem

The numerical simulation is carried out in order to obtain understanding of how the strength, efficacy, and performance of the model under review are evaluated at various levels. Here, From this table above, we can see that the system reliability's equivalent values for main unit at time t = 40 are $Rel_{subsystem H} = 0.9872, Rel_{subsystem I} = 0.9671, Rel_{subsystem J} = 0.6967, Rel_{subsystem K} = 0.9403$. In time t = 40, there is $Main_{system} = 0.32632241$ chance of successfully completing maintenance and repairs, and $Main_{subsystem H} = 0.9999$, $Main_{subsystem I} = 0.9996, Main_{subsystem J} = 0.9975, Main_{subsystem K} = 0.9999$. The system is 0.3363 times reliable at t = 60 due to a form decline.

DISCUSSION

This study offers a comprehensive analysis of the reliability and maintainability of individual components and subsystems within the system. Through an in-depth examination of reliability measures such as failure rates, repair rates, reliability, and maintainability, we have pinpointed the most critical components that influence the overall system's performance. The reliability measures for each subsystem were carefully derived and validated through numerical simulations, ensuring that the findings are both accurate and reliable. Our analysis, as shown in Tables 2, 3, and 4, reveals the significant effect that varying failure rates can have on subsystem and overall system reliability.

Our findings corroborate earlier research highlighting the pivotal role of maintainability in system performance. For instance, Singh and Ayagi (2018) emphasize the integration of preemptive repair strategies in repairable systems, showcasing that maintainability directly affects system downtime and operational efficiency. This study's results echo those of Corvaro et al. (2017), who used optimization models for series-parallel systems and concluded that enhancing system maintainability was integral to improving overall system reliability and economic viability. Furthermore, the study by Garg (2014) underscores that the maintenance of critical subsystems significantly impacts the reliability of the entire system. These studies align with our observation that system reliability is intricately linked to maintainability, supporting the notion that effective maintenance strategies are necessary for ensuring long-term operational efficiency.

The numerical findings also align with earlier models presented by Lado and Singh (2019) and Tsarouhas (2018), which similarly suggested that failure rates of individual subsystems directly influence system reliability. Our results further stress the sensitivity of system performance to these failure rates, with even small changes in failure rates leading to substantial differences in overall reliability. This observation is particularly relevant in the context of energy systems, where minimizing downtime and maintaining performance is crucial for ensuring energy production and sustainability.

Additionally, the comparison with previous research by Sanusi and Yusuf (2021) is noteworthy. Their work applied copula-based methodologies to model the interdependencies

between system components, which supports our finding that subsystem interactions are a key factor in determining overall system reliability. Our study extends this concept by emphasizing the importance of maintaining subsystems to mitigate failure rates and enhance system performance.

CONCULUSION

Drawing from our findings, we advocate for the adoption of the reliability approach as a strategic framework to enhance system performance and mitigate the risk of subsystem failures. By implementing proactive maintenance strategies informed by reliability analysis, stakeholders can pre-emptively address reliability issues, optimize system operation, and minimize downtime. Additionally, prioritizing the enhancement of subsystem maintainability not only fosters the smooth operation of individual components but also safeguards the integrity of the entire system. In essence, the reliability approach offers a robust methodology to bolster system resilience, promote operational continuity, and mitigate the adverse effects of component failures.

Recommendation

The study puts forward the following recommendations;

- 1. Organizations can optimize resource allocation and streamline maintenance practices
- 2. Ensure the sustained performance and reliability of complex systems in diverse operational environments.
- 3. Standardize maintenance procedures and training.

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