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Locations of the out-of-plane equilibrium points for different binaries

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Abstract

This paper locates the Lagrangian points found on the out-of-orbital plane of motion which are called out-of-plane libration points(OLPs). The study is carried out using the model of the circular restricted three-body problem (CR3BP) when the main bodies are assumed as oblate-radiating spheroids and radiating bodies. Two pairs of OEPs $L_{6,7}$ *and* $L_{8,9}$ *have been obtained using semi-analytical and numerical means, respectively. A comparison was made between the two methods and it is seen that the locations of the OEPs deviate slightly when applied to five binary systems (Lalande 21258, BD+195116, Ross 614, 70 Ophiuchi and 61 Cygni).*

Keywords: Binary systems, OEPs, CRTBP

INTRODUCTION

The restricted three-body problem (R3BP) describes the dynamics of a test particle having infinitesimal mass and moving under the gravitational effects of two primaries, where these primaries move in circular orbits around their center of mass on account of their mutual attraction and the test particle not impelling their motion. This formulation has had important implications in several scientific fields, including celestial mechanics, galactic dynamics, molecular physics, and chaos theory. Also, the motion of artificial satellites forms some of the areas where the R3BP is applied. The classical R3BP as discussed by Szebehely (1967) has five equilibrium points (EPs). Three of these points lie on the line joining the primaries while the other two form a triangle, with the primaries.

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In particular, the plane perpendicular to the plane of motion of the primaries has been shown to result in out-of-plane equilibrium points (OEPs) when the photo gravitational effects of one or both primary bodies are taken into consideration or when one or both primaries are sufficiently oblate in shape. The solutions of the classical R3BP do not comprise the out-of-plane EPs. These points are very important in celestial mechanics within the frame of the R3BP and they serve as crucial loci located outside the orbital plane of the host bodies. Several researchers under different characterizations of the perturbing forces, have engaged in studying motion around these OEPs.

Todoran (1993) claimed that the OEPs as implied by Radzevskii (1950) and computed by many researchers, do not exist. However, Ragos and Zagouras (1993) made a counterclaim in their paper where they verified that these points do exist. Such considerations concerning the OEPs can be seen in Roman (2001), who reviewed the role of the radiation pressure in the photogravitational R3BP and analyzed the existence of the OEPs. By utilizing a numerical simulation for the binary RW-Monocerotis system, they obtained a pair of OEPs $L_6(-0.055;0;+1.07)$ and $L_7(-0.055;0;-1.07)$. Douskos and Markellos (2006) reported the existence of OEPs when they considered the case of one or both primaries radiating and the case of the oblateness of one primary and radiation of the other. They obtained numerical evidence indicating that the OEPs are unstable. On the other hand, Wu et. Al., (2018) commented that the presentation by Douskos and Markellos (2006) deviated from the intuitive physical point of view where the gravitational force is the only force acting in the z-direction.

Other researchers who have made contributions to the study of OEPs include Das et al. (2009), Singh and Umar (2012), Huda et al. (2015) Abouelmagd and Mostafa (2015), Suraj et al (2018), Idrisi and Ullah (2021, 2022, 2024) and, Leke and Singh (2023).

In this paper, we search for the existence of the OEPs and locate their positions under the framework of the CR3BP when the primaries are both radiating and oblate as well as giving practical applications by using the binary Lalande 21258, BD+19 5116, Ross 614, 70 Ophiuchi and 61 Cygni systems. The work improves the understanding of gravitational field stability by linking theoretical results to real astronomical events through the analysis of these concrete examples. This work is an important addition to the academic and applied fields of celestial mechanics, with important ramifications for astrodynamics, as understanding OEPs can guide spacecraft navigation and mission design.

Equations of Motion

Let m_1, m_2 and m_3 be the masses of the bigger primary, smaller primary, and the infinitesimal body, respectively. Here, the primary bodies are moving in circular orbits about their common barycentre, while the infinitesimal body is moving and exerting no influence in the plane of motion of the primaries.

The mass parameter is given by $\mu = \dfrac{m_2}{m_1 + m_2}$. Let the unit of distance be taken as the distance between the primaries, such that the gravitational constant $G = 1$. The unit of mass has been chosen so that $m_1 + m_2 = 1$. We take the dimensionless masses of the primary bodies as $m_1 = 1 - \mu$ and $m_2 = \mu$. We let $Oxyz$ be the synodic coordinate system with the position of the infinitesimal body as $P(x, y, z)$ and the primary and secondary bodies as $P_1(\mu, 0, 0)$ and $P_2(-(1-\mu), 0, 0)$ respectively (Fig. 1)

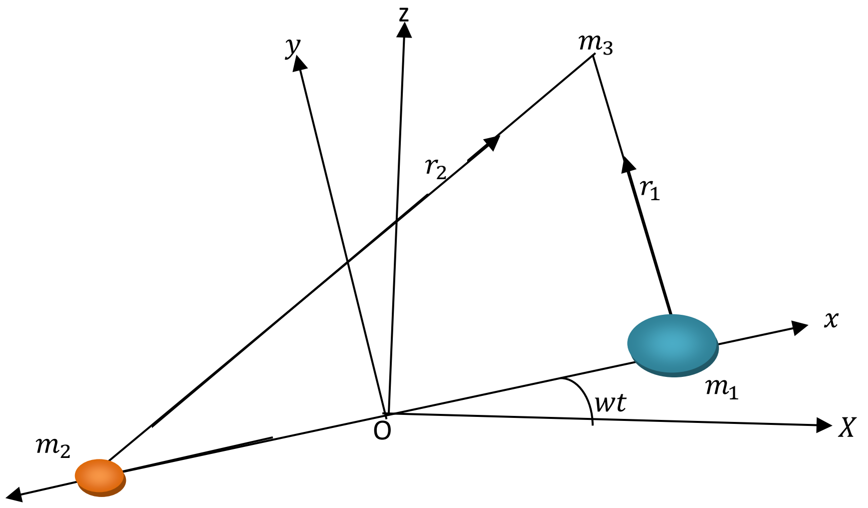

Fig. 1 Model Description of the Problem.

Thus, the equations of motion of the infinitesimal body in the dimensionless synodic coordinate system with radiation pressure parameters q_1 and $q_2 (q_i \le 1, i = 1, 2)$ and oblateness parameters A_1 and $A_2 \left(A_i << 1,\ i = 1,2\right)$ (Singh & Ishwar (1999)) are

$$\ddot{x} - 2n\dot{y} = \Omega_x,$$
$$\ddot{y} + 2n\dot{x} = \Omega_y, \tag{1}$$
$$\dot{z} = \Omega_z,$$

where

$$\Omega = \frac{1}{2}n^2(x^2 + y^2) + \frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)A_1 q_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3} - \frac{3(1-\mu)z^2 A_1 q_1}{2r_1^5} - \frac{3\mu A_2 q_2 z^2}{2r_2^5}, \tag{2}$$

$$\Omega_x = n^2 x + \frac{15A_1(1-\mu)q_1(x-\mu)z^2}{2r_1^7} - \frac{3A_1(1-\mu)q_1(x-\mu)}{2r_1^5} - \frac{(1-\mu)q_1(x-\mu)}{r_1^3} + \frac{15A_2\mu q_2(x+1-\mu)z^2}{2r_2^7} - \frac{3A_2\mu q_2(x+1-\mu)}{2r_2^5} - \frac{\mu q_2(x+1-\mu)}{r_2^3}, \tag{3}$$

$$\Omega_y = n^2 y + \frac{15A_1(1-\mu)q_1 yz^2}{2r_1^7} - \frac{3A_1(1-\mu)q_1 y}{2r_1^5} - \frac{(1-\mu)q_1 y}{r_1^3} + \frac{15A_2\mu q_2 yz^2}{2r_2^7} - \frac{3A_2\mu q_2 y}{2r_2^5} - \frac{\mu q_2 y}{r_2^3}, \tag{4}$$

$$\Omega_z = \frac{15A_1(1-\mu)q_1 z^3}{2r_1^7} - \frac{9A_1(1-\mu)q_1 z}{2r_1^5} - \frac{(1-\mu)q_1 z}{r_1^3} + \frac{15A_2\mu q_2 z^3}{2r_2^7} - \frac{9A_2\mu q_2 z}{2r_2^5} - \frac{\mu q_2 z}{r_2^3}, \tag{5}$$

and the distances between the infinitesimal body and primary and secondary bodies are given as

$r_1^2 = (x-\mu)^2 + y^2 + z^2$, and $r_2^2 = (x-\mu+1)^2 + y^2 + z^2$, respectively.

The mean motion for each of the systems is given by

$$n = \sqrt{1 + \frac{3}{2}(A_1 + A_2)} \;,$$

and the Jacobian integral of motion is represented as

$$C = n^2(x^2 + y^2) + \frac{2q_1(1-\mu)}{r_1} + \frac{2q_2\mu}{r_2} + \frac{A_1q_1(1-\mu)}{r_1^3} + \frac{\mu q_2 A_2}{r_2^3} - \frac{3A_1q_1(1-\mu)z^2}{r_1^5} - \frac{3\mu A_2 q_2 z^2}{r_2^5}$$
$$-\dot{x}^2 - \dot{y}^2 - \dot{z}^2.$$

In Table 1, we present the physical parameters of the binary systems. The parameters M_A and M_B are the masses of the more massive and less massive stars in each binary system as compared to the mass of the Sun. The symbol μ as shown earlier is the mass parameter. The luminosity of the binary systems denoted by L_A and L_B respectively are obtained from the relation (Mia and Kushvah, (2016))

$$\frac{L}{L_S} \approx \left(\frac{M}{M_S}\right)^{3.9},$$

where L_S and M_S are the luminosity and mass of the Sun.

Radiation pressure has had a key effect on the formation of stars and the shaping of clouds of dust and gases on a wide range of scales. The mass reduction factor is represented as $q_i = 1 - \frac{F_p}{F_g},\ i = 1,2$ (F_p and F_g are the radiation pressure and the gravitational attraction forces being exerted by the binary systems on objects around them) or $q_i = 1-\beta,\ i = 1,2$ or based on the Stefan Boltzmann's law (Xuetang and Lizhong, (1993)) as

$$q_i = 1 - \frac{A\kappa L}{a\rho M},\ i = 1,2,$$

where M, L, and κ are the mass, luminosity, and radiation pressure efficiency factor of a star. Also, a and ρ are the radius and density of the dust grain particles moving in the binary systems while $A = \frac{3}{16\pi c G}$ is a constant with c and G as the speed of light and Gravitational constant.

The values of the luminosity and mass reduction factor $q_i, i = 1,2$ have been obtained by computing in the $C.G.S.$ system of unit, using $L_s = 3.846\times10^{33} erg/s$, $c = 3\times10^{10} cm/s$, $G = 6.67384\times10^{-8} cm^3 g^{-1} s^{-2}$, $M_s = 1.989\times10^{33} g$ and $\kappa = 1$. Also, we have assumed the values for the radius and density of the dust grain particles as $a = 2\times10^{-2} cm$ and $\rho = 1.4 g/cm^3$ ((Xuetang and Lizhong (1993)). Arbitrary values are been used for the oblateness coefficients A_1 and A_2 as shown in Table 1

Table 1. Physical Parameters of the Five Binary Systems

	Lalande 21258	*BD* +19 5116	*Ross* 614	70 Ophiuchi	61 Cygni
$M_A(M_S)$	0.480000	0.330000	0.170000	1.02000	0.70000
$M_B(M_S)$	0.100000	0.16000	0.100000	0.64000	0.63000
μ	0.172400	0.326500	0.370400	0.38550	0.47390
$L_A(L_S)$	0.006370	0.002650	0.000492	0.47000	0.08870
$L_B(L_S)$	0.0000344	0.000370	0.000029	0.08950	0.04140
q_1	0.9726920	0.9834750	0.994045	0.05181	0.73925
q_2	0.9992920	0.9952410	0.999407	0.71223	0.86477
A_1	0.1000000	0.1200000	0.140000	0.16000	0.18000
A_2	0.1100000	0.1300000	0.150000	0.17000	0.19000

Computations of the Out-of-plane Equilibrium Points

The positions of the OEPs can be obtained when $\Omega_x = \Omega_y = \Omega_z = 0$, $\dot{x} = \dot{y} = \dot{z} = \ddot{x} = \ddot{y} = \ddot{z} = 0$ and when $y = 0$ and $z \neq 0$. As such, by applying these conditions in equations (3) and (5) respectively, we obtain

$$n^2 x + \frac{15A_1(1-\mu)q_1(x-\mu)z^2}{2r_{10}^7} - \frac{3A_1(1-\mu)q_1(x-\mu)}{2r_{10}^5} - \frac{(1-\mu)q_1(x-\mu)}{r_{10}^3} + \frac{15A_2\mu q_2(x+1-\mu)z^2}{2r_{20}^7} - \frac{3A_2\mu q_2(x+1-\mu)}{2r_{20}^5} - \frac{\mu q_2(x+1-\mu)}{r_{20}^3} = 0 \quad (6)$$

and

$$\frac{15A_1(1-\mu)q_1z^2}{2r_{10}^7} - \frac{9A_1(1-\mu)q_1}{2r_{10}^5} - \frac{(1-\mu)q_1}{r_{10}^3} + \frac{15A_2\mu q_2z^2}{2r_{20}^7} - \frac{9A_2\mu q_2}{2r_{20}^5} - \frac{\mu q_2}{r_{20}^3} = 0\,, \quad (7)$$

where

$r_{10} = \sqrt{(x-\mu)^2 + z^2}$ and $r_{20} = \sqrt{(x+1-\mu)^2 + z^2}$.

Equation (6) can also be written as

$$x[n^2 + \frac{15A_1(1-\mu)q_1z^2}{2r_{10}^7} - \frac{3A_1(1-\mu)q_1}{2r_{10}^5} - \frac{(1-\mu)q_1}{r_{10}^3} + \frac{15A_2\mu q_2z^2}{2r_{20}^7} - \frac{3A_2\mu q_2}{2r_{20}^5} - \frac{\mu q_2}{r_{20}^3}]$$
$$= \frac{15A_1(1-\mu)\mu q_1z^2}{2r_{10}^7} - \frac{3A_1(1-\mu)\mu q_1}{2r_{10}^5} - \frac{(1-\mu)\mu q_1}{r_{10}^3} - \frac{15A_2\mu(1-\mu)q_2z^2}{2r_{20}^7} + \frac{3A_2\mu(1-\mu)q_2}{2r_{20}^5} + \frac{\mu(1-\mu)q_2}{r_{20}^3}.$$

By making x the subject of the last equation, we have

$$x = \frac{\frac{15A_1(1-\mu)q_1\mu z^2}{2r_{10}^7} - \frac{3A_1(1-\mu)q_1\mu}{2r_{10}^5} - \frac{(1-\mu)q_1\mu}{r_{10}^3} - \frac{15A_2\mu(1-\mu)q_2z^2}{2r_{20}^7} + \frac{3A_2\mu(1-\mu)q_2}{2r_{20}^5} + \frac{\mu(1-\mu)q_2}{r_{20}^3}}{[n^2 + \frac{15A_1(1-\mu)q_1z^2}{2r_{10}^7} - \frac{3A_1(1-\mu)q_1}{2r_{10}^5} - \frac{(1-\mu)q_1}{r_{10}^3} + \frac{15A_2\mu q_2z^2}{2r_{20}^7} - \frac{3A_2\mu q_2}{2r_{20}^5} - \frac{\mu q_2}{r_{20}^3}]}. \quad (8)$$

Also, from Eqn. (7) we make z the subject and get

$$z = \pm \left[\frac{\frac{9A_1(1-\mu)q_1}{2r_{10}^5} + \frac{(1-\mu)q_1}{r_{10}^3} + \frac{9A_2\mu q_2}{2r_{20}^5} + \frac{\mu q_2}{r_{20}^3}}{[\frac{15A_1(1-\mu)q_1}{2r_{10}^7} + \frac{15A_2\mu q_2}{2r_{20}^7}]} \right]^{\frac{1}{2}}. \quad (9)$$

To obtain an analytical approximation for the coordinates $(x, 0, \pm z)$ in the form of power series to third-order terms in A_1 (Douskos and Markellos (2006)), we set $x = \mu$ and $z = \sqrt{3}\sqrt{A_1}$ in Eqns. (8) and (9) and get

$$x_0 = \mu - \frac{3\sqrt{3}\,\mu\,(2+3A_2)(1-q_2)}{2q_1(1-\mu)} A_1^{3/2} - \frac{9\sqrt{3}\,\mu(2+6q_2+45A_2q_2)}{4q_1(1-\mu)} A_1^{5/2} + O(A_1^3), \quad (10)$$

and

$$z_0 = \pm\sqrt{3}\,A_1^{1/2} - \frac{9\,\mu\, q_2(2+9A_2)}{4q_1(1-\mu)} A_1^2 - \frac{63\sqrt{3}\,\mu^2(2+3A_2)^2(1-q_2)^2}{16q_1^2(1-\mu)^2} A_1^{5/2} + O(A_1^3). \quad (11)$$

Also, we set $x = -1 + \mu$ and $z = \sqrt{3}\sqrt{A_2}$ in Eqns. (8) and (9) and get

$$x_0 = (\mu-1) - \frac{3\sqrt{3}(\mu-1)\,(2+3A_1)(1-q_1)}{2q_2\mu} A_2^{3/2} - \frac{9\sqrt{3}(\mu-1)(2+6q_1+45A_1q_1)}{4q_2\mu} A_2^{5/2} + O(A_2^3), \quad (12)$$

and $$z_0 = \pm\sqrt{3}\,A_2^{1/2} + \frac{9(\mu-1)q_1(2+9A_1)}{4q_2\mu} A_2^2 - \frac{63\sqrt{3}(\mu-1)^2(2+3A_1)^2(1-q_1)^2}{16q_2^2\mu^2} A_2^{5/2} + O(A_2^3). \quad (13)$$

In Fig. 2, we illustrate the positions of the four OEPs as well as the fixed location of the primaries of the Lalande 21258 binary system ($\mu = 0.1724, q_1 = 0.972692, q_2 = 0.999292, A_1 = 0.10$ and $A_2 = 0.11$).

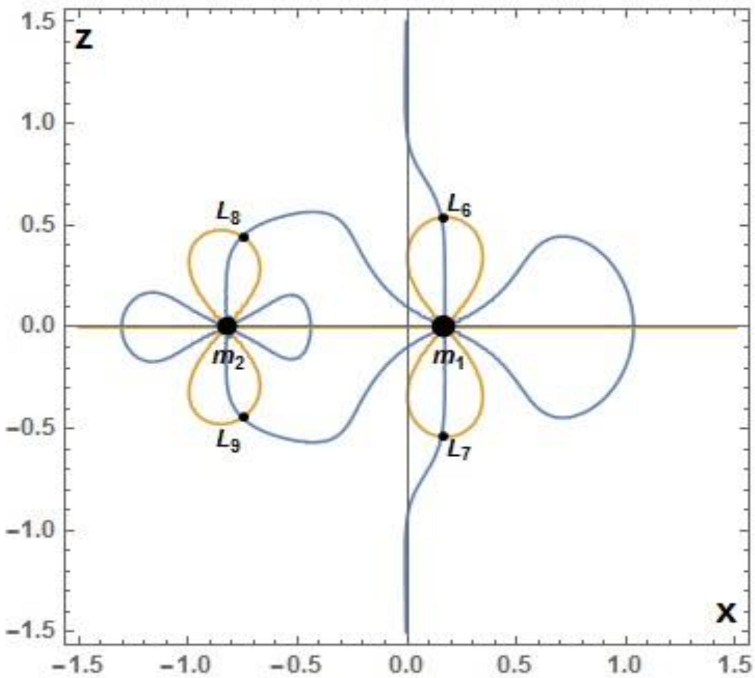

Fig. 2 Positions (small dots) and numbering of the equilibrium points ($L_i, i = 6,...,9$) through the intersection of $\Omega_x = 0$ (blue) and $\Omega_z = 0$ (Purple) of the binary Lalande 21258 system.

Since we have determined the number of equilibrium points, we make use of the values of the physical parameters in Table 1 to determine the locations of the OEPs for the five binary systems under consideration and then make comparisons between the coordinates obtained by using analytical methods (Huda et al. 2015) and the coordinates obtained by using numerical methods (Douskos and Markellos, 2006) as shown in Table 2.

Table 2. The out-of-plane coordinates $L_{6,7}(x,0,\pm z)$ (almost above and below of the larger primary) of the five binary systems showing the results for both the semi-analytical and numerical methods

Binary Systems	$(x,\pm z)$ (Semi-Analytical)	$(x,\pm z)$ (Numerical)
Lalande 21258	0.17237097, ±0.53332504	0.15627609, ±0.53896914
BD +19 5116	0.32589449, ±0.54961343	0.28414059, ±0.57076183
Ross 614	0.37028298, ±0.56069126	0.31038058, ±0.60170202
70 Ophiuchi	0.27741571, ±0.42142905	0.27741541, ±0.42142964
61 Cygni	0.38988094, ±0.44985560	0.35848845, ±0.61167169

Table 3. The out-of-plane coordinates $L_{8,9}(x,0,\pm z)$ (almost above and below of the smaller primary) of the five binary systems showing the results for both the semi-analytical and numerical methods

Binary Systems	$(x,\pm z)$ (Semi–Analytical)	$(x,\pm z)$ (Numerical)
Lalande 21258	–0.82760000, ±0.57445665	–0.75339854, ±0.44497997
BD +19 5116	–0.67350000, ±0.62449980	–0.59223015, ±0.51974285
Ross 614	–0.62960000, ±0.67082039	–0.53788566, ±0.55540617
70 Ophiuchi	–0.61450000, ±0.71414284	–0.39996234, ±0.51405513
61 Cygni	–0.39034559, ±0.75498344	–0.39034545, ±0.61716921

DISCUSSION

By modeling the primaries as oblate and radiating bodies in the CRTBP, we investigated the existence and locations of OEPs. The theory has been applied to five binary systems; 21258, BD+19 5116, Ross 614, 70 Ophiuchi, and 61 Cygni.

Using physical parameters for these binary systems we obtained the realistic values for the radiation components and then assumed the values for the oblateness parameters. Two pairs of OEPs $L_{6,7}(x_0,0,\pm z_0)$ and $L_{8,9}(x_0,0,\pm z_0)$ for each of the binary systems were obtained by solving the equations (3) and (5) using analytical and numerical methods. These points are plotted for real system values in Fig 2, where the equilibria above and below m_1 are referred to as L_6 and L_7 while those around m_2 are referred to as L_8 and L_9. It should be pointed out that L_6 is symmetrical with L_7 with respect to the axis Ox where L_8 and L_9 are the same. Moreover, L_6 is not symmetrical with L_8 w.r.t the axis Oz and L_7 and L_9 are not the same. Further, a comparison was made between the results of the two methods and it can be seen that both methods produced slightly different values.

CONCLUSION

The CRTBP modeling employed in this research successfully illustrated the dynamics of oblate and radiating bodies, providing insights into their influence on the out-of-plane equilibrium points (OEPs) in binary systems. Our investigation yielded valuable insights into the spatial configurations of the equilibrium points within the selected binary systems. The differences observed between the analytical and numerical results underscore the importance of choosing the correct methodology for astrophysical modeling, suggesting that enhancements in the refinement process may result in more precise predictions.

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