

Evaluation of the Effect of Viscosity on Blood Flow Using Viscosity Models

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Abstract

The study of blood hemodynamics and rheological properties, particularly blood viscosity, is essential for understanding and treating certain cardiovascular conditions. Blood is a non-Newtonian fluid, meaning its viscosity varies with shear rate, a behavior driven by the aggregation and deformation of red blood cells (RBCs). Accurate modeling of blood viscosity is therefore critical for simulating blood flow in both physiological and pathological states. In this work, various viscosity models were assessed to identify the most suitable that represents the blood's behavior in relation to shear rate. Although the Newtonian model is simple, it fails to capture the non-Newtonian characteristics of blood, particularly at varying shear rates. In comparison, the Power Law, Walburn-Schneck, and Carreau-Yasuda models offer more detailed and complex approaches to modeling blood viscosity. The Effect of blood viscosity on the overall blood flow was evaluated based on the four viscosity models. The work was carried out in a MATLAB Simulation Environment. The Volumetric blood flow rate was evaluated from the Poiseuille's equation using three viscosity models whose values were defined by shear rates ranging from $(217.5 - 226.5)s^{-1}$. The percentage change in the blood volume resulting from changes in the viscosity was examined for each model. The two viscosity models (Power and Carreau models) concurrently reported equal percentage change of blood flux variation with the corresponding variation of the viscosity due to change in shear rate. This shows that any of the two models can be used to study the variation of blood flow to the viscosity with respect to cardiovascular complications.

Keywords: Blood flow, Blood viscosity models, Power model, Carreau model, Walburn-Schneck model

INTRODUCTION

Blood circulation as we all know ensures proper functioning of the body by supplying oxygen and nutrients to all organs in the body. Various diseases lead to the impairment of this supply. Therefore, understanding the hemodynamics and rheological characteristics of blood can improve our understanding of the physiology of certain diseases. Among these rheological properties includes the blood viscosity, which has been found to vary with shear rate, hematocrit, temperature and disease conditions (Connes *et al.*, 2015, Lipowsky, 2016).

The blood viscosity in particular has been a key indicator for understanding the implication and treatment of certain medical conditions. The apparent viscosity is given as the ratio of the shear stress to the shear strain. In Newtonian fluids, the apparent viscosity is constant while in non-Newtonian fluids, the viscosity is dependent on the shear rate. It is well known that

blood is a non-Newtonian fluid since it exhibits shear thinning and shear thickening behaviors particularly because of the red blood cells (RBCs) (Galdi *et al.*, 2008). At low shear rates, the RBC aggregates which increases blood viscosity whereas at high shear rates, blood viscosity decreases and RBC deforms (Baskurt *et al.*, 2003, El-Aragi, 2013). On the other hand, some models assume blood flow to be Newtonian at shear rates above 100s^{-1} but the actual shear rate in large arterial vessels fluctuates dramatically between 0 to 1000s^{-1} throughout a cardiac cycle (Siebert & Fodor, 2009). The viscosity of blood as a non-Newtonian fluid has been extensively studied by numerous researchers in relation to its shear rate dependence (Cho & Kensey, 1991) and therefore, in order to account for non-Newtonian viscosity of blood in a numerical simulation of hemodynamic flow, one needs a constitutive equation which defines the relation between viscosity and shear rate.

The computational analysis of fluid flow shows notable differences when using Newtonian and non-Newtonian viscosity models, stressing the importance of selecting the appropriate viscosity model (Johnston *et al.*, 2004). For the sake of simplicity, blood was modeled as a Newtonian fluid in some studies, which revealed that as flow velocity and shear rate increases, blood flows uniformly and its viscosity approaches a constant value (Boyd *et al.*, 2007). Mahmood *et al.*, 2019 analyzed the flow of power law fluid to represent the features of shear thinning, shear thickening and Newtonian materials. The Power law model is one of the viscosity models that is the simplest and most widely used (Wajihah & Sankar, 2023). This is because the Power law model gives positive results that emulates the shear rate dependence on blood viscosity especially at low shear rates. However, the observed decrease in viscosity at high shear rates does not accurately represent the true behavior of blood (Siebert & Fodor, 2009). The same limitation applies to the Walburn Schneck model even though it incorporates the effects of hematocrit on the blood viscosity (Johnston *et al.*, 2004). The Carreau Yasuda model on the other hand, uses viscosity functions that are defined for all shear rates, from very low to very high (Siebert & Fodor, 2009). Computational fluid dynamics (CFD) modelling has also been carried out in some of these models. Liu *et al.* (2021) constructed CFD models using Newtonian and non-Newtonian models to investigate pressure and wall shear stress. Also, Lynch *et al.*, 2022 investigated the differences between the Newtonian and non-Newtonian on properties such as volume, vorticity and wall shear stress. Now even with these researches conducted on the viscosity models, the issue of selecting the most suitable one amongst them remains a challenge. The primary questions one needs to ask is which among these viscosity models is the most suitable that will account for the blood flow?

The aim of this work was to find out which of the viscosity models will be the most suitable model to account for blood flow considering the variation of the blood viscosity with shear rate. The study was conducted using a MATLAB 2015a Simulation Package.

METHODOLOGY

The dynamics of blood flow is formulated by the work of Jean Poiseuille in the 19th century where he found the relationship between volume flow rate, pressure difference and viscous resistance.

The law states that “the volume flow rate is directly proportional to the pressure difference and fourth power radius of the tube, and inversely proportional to the viscosity and length of the tube”.

The resistance R to laminar flow of an incompressible fluid having a viscosity η through a horizontal tube of uniform radius r and length l is given by;

$$R = \frac{8\eta l}{\pi r^4} \quad (1)$$

The flow rate Q is given by:

$$Q = \frac{(P_2 - P_1)}{8\eta l} \pi r^4 \tag{2}$$

To consider the effects of the viscosity on various models three non-Newtonian models are considered including the Newtonian model. These models were given by (Cho & Kensey, 1991) and (Siebert & Fodor, 2009).

i. Newtonian Model

$$\eta = 0.0035 \text{ Pa.s} \tag{3}$$

This viscosity value corresponds to 40% hematocrit count

ii. Power law Model

$$\eta = \eta_0 \dot{\gamma}^{n-1} \tag{4}$$

where $\eta_0 = 0.035 \text{ Pa.s}$ and $n = 0.6$

iii. Walburn Schneck Model

$$\eta = C_1 e^{C_2 H} \cdot e^{C_4 \cdot TMPA/H^2} \cdot \dot{\gamma}^{-C_3 H} \tag{5}$$

where $C_1 = 0.00797, C_2 = 0.0608, C_3 = 0.00499, C_4 = 14.585 \text{ g}^{-1}, H = 40\%$
and $TMPA = 25.9 \text{ g/l}$

iv. Carreau -Yasuda Model

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty) [1 + \lambda \dot{\gamma}^2]^{-\frac{n-1}{2}} \tag{6}$$

where λ and n are empirically determined constant parameters. At high shear rates, the fluid acts like a Newtonian fluid with viscosity η_∞ , whereas at low shear rates, the fluid acts like a Newtonian fluid with viscosity η_0 . The parameters λ and n control how the fluid behaves in the non-Newtonian regime between these two asymptotic viscosities (Boyd *et al.*, 2017).

The Simulations were carried out using a MATLAB 2015a Software. The volumetric flow rate was evaluated using equation (2) and the viscosity used were proposed by four models (equations 3,4,5&6) whose values were dictated by shear rate ranging from $(217.5 - 226.5) \text{ s}^{-1}$. The overall volumetric flow rate of each model was evaluated at different shear rates. During the result analysis, 30%, 60% and 90% changes in the viscosity values were introduced. The corresponding blood volumetric flow rate for each model was calculated. Finally, the percentage change in the blood volume due to the changes in the viscosity was also evaluated for each model.

RESULTS

The viscosity variations of the four models is presented in Table 1. And the corresponding volumetric flow rate of each model is shown in Table 2. The percentage analysis of the overall flow rate is shown in Table 3.

Table 1: Viscosity variations of four models in Pas

S/N	Newtonian	Carreau	Power law	Walburn Schneck
1	0.003450	0.004212	0.004065	0.003912
2	0.003450	0.004210	0.004057	0.003909
3	0.003450	0.004208	0.004050	0.003905
4	0.003450	0.004206	0.004043	0.003902
5	0.003450	0.004204	0.004035	0.003898
6	0.003450	0.004201	0.004028	0.003895
7	0.003450	0.004199	0.004021	0.003891
8	0.003450	0.004197	0.004014	0.003888
9	0.003450	0.004195	0.004006	0.003884
10	0.003450	0.004193	0.003999	0.003881

Table 2: Volumetric flow rate variations of four models in m^3/s

S/N	Newtonian	Carreau	Power law	Walburn Schneck
1	289.855072	237.361769	245.98898	255.59359
2	289.855072	237.488480	246.44076	255.82819
3	289.855072	237.614376	246.89129	256.06193
4	289.855072	237.739465	247.34059	256.29482
5	289.855072	237.863755	247.78867	256.52687
6	289.855072	237.987255	248.23554	256.75808
7	289.855072	238.109973	248.68121	256.98846
8	289.855072	238.231918	249.12568	257.21801
9	289.855072	238.353097	249.56896	257.44675
10	289.855072	238.473518	250.01107	257.67468

Table 3: Comparison of Percentage Change in Viscosity with corresponding changes in Volumetric flow rate of the four models

Percentage change	Carreau		Power law		Walburn Schneck	
	η (Pas)	Q (m^3/s)	η (Pas)	Q (m^3/s)	η (Pas)	Q (m^3/s)
30%	0.22800407	0.2272155	0.227171254	0.22433859	0.225806452	0.225045529
60%	0.56374745	0.5626142	0.67293578	0.558555378	0.548387097	0.559557732
90%	0.89215886	0.8916831	0.891651376	0.890079536	0.903225806	0.777746277

DISCUSSION

Table 1 presents viscosity values for four models measured across ten shear rates. All models show decreasing viscosity with higher shear rates, but the patterns differ. In the Newtonian Model, the viscosity remains constant at 0.003450 Pas, regardless of shear rate. While in the Carreau model, the viscosity is high and decreases gradually and approaches the Newtonian viscosity at high shear rates. This is in agreement with what Liu et al, 2021 and Lynch et al, 2022 obtained. The Power Law model on the other hand, exhibits a more significant decrease in viscosity with increasing shear rate, indicating shear-thinning behavior. The same scenario was also reported by Mahmood et al, 2019. The Walburn Schneck model also shows a decrease in viscosity with shear rate, similar to the Power Law model but with slightly different values.

Table 2 provides the volumetric flow rate for each model under similar conditions. The flow rate increases as the shear rate increases, with the Power Law and Walburn Schneck models showing a greater increase compared to the Carreau model. For the **Newtonian model**, the flow rate remains constant across measurements due to the constant viscosity. While for the **Carreau model**, the volumetric flow rate increases at a moderate rate, which is consistent with its decreasing viscosity but more gradual compared to the Power law and Walburn Schneck models. The **Power law model** shows a more significant increase in flow rate with shear rate. This is consistent with its larger decrease in viscosity, leading to higher flow rates as shear rate increases. And the **Walburn Schneck model** shows a substantial increase in flow rate similar to the Power Law model but with slightly different values.

Table 3 shows the percentage analysis of three models. It can be seen that a 30% decrease in viscosity resulted in a 22% increase in volumetric flow rate in Carreau, Power and Walburn

Schneck models. Also, a 60% decrease in viscosity resulted in a 56% increase in volumetric flow rate in Carreau, Power and Walburn Schneck models. And finally, a 90% decrease in viscosity resulted in an 89% increase in volumetric flow rate in Carreau and Power models and a 78% increase in Walburn Schneck model. This shows that the Carreau and Power models are in agreement and hence any of the two models can therefore be used to evaluate the changes in blood flow with respect to certain cardiovascular diseases.

CONCLUSION

This study compared four viscosity models to assess their effects on blood flow rates within a shear rate range of 217.5 to 226.5 s⁻¹, using MATLAB simulations. It highlighted the importance of selecting the right viscosity model based on fluid behavior. The Carreau model was effective for fluids behaving more Newtonian at high shear rates, while the Power Law and Walburn Schneck models better captured shear-thinning behavior. Results showed that the Carreau and Power models produced identical changes in blood flow with viscosity variations, indicating their equal effectiveness. Future research should validate these models under various conditions and explore their clinical applications to improve understanding and management of blood flow disorders.

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