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Response Surface Methodology on Optimization of Kinematic Viscosity of a Lubricant, Function of Temperature and Pressure

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Abstract

Effect of kinematic viscosity that is a function of Temperature and Pressure was studied here using Response Surface Methodology (RSM). A secondary data was used that came from a research on the kinematic viscosity of a lubricant in strokes as a function of temperature (0_c), x_1 *and pressure in atmospheres* (atm), x_2*. From the analyses, it was found that the cubic regression models with F-value of 6222.91 suggests that the model is significant. Some other Statistic measures were used to justify the adequacy of the model. There are Coeeficeint of Determination* R^2*, its Adjusted and predicted values. Any* R^2 *close to 1 suggests a good fit of the model. Here,* R^2 *is* 0.9992 *and the Adjusted* R^2 *is* 0.9991 *which indicates adequate signal of the model. It was also found that 0.6798 (atm) of pressure and -0.23559* (0_c) *of temperature will give optimum movement of kinematic viscosity of 13.068.*

Keywords: Kinematic Viscosity, Model Adequacy, Optimization; Response Surface Methodology.

INTRODUCTION

The kinematic viscosity (or, more properly, the coefficient of kinematic viscosity) is a convenient form in which the viscosity of a fluid may be expressed (Houghton *et.al* 2013).

(Britannica & Editors of Encyclopeadia, 2024) defined viscosity as a concept where fluid shows struggle against a flowing, which is being distorted due to extensional stress forces or shear stress. There are two related measures of fluid viscosity; Dynamic and Kinematic. The viscosity of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress. The shear resistance in a fluid is caused by inter-molecular friction exerted when layers of fluid attempt to slide by one another (ToolBox, 2003).

Kinematic viscosity is the type which is computed by calculating the ratio of the fluid mass density to the dynamic fluid, viscosity or absolute fluid viscosity. It is from time to time known as momentum diffusivity. (Toppr, 2021).

Kinematic viscosity is a critical performance parameter for aviation jet fuels. The kinematic viscosity of jet fuel is related to pumpability at the operating temperature (often below −20°C), and also the ability to lubricate the pump (Lapuerta and Canoira, 2016).

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Atabami, et.al (2012) explained kinematic viscosity as fluid resistance. Owing to inadequate fuel atomization, high viscosity can result in the development of soot and engine deposits. Biodiesel has 10–15 times higher kinematic viscosity than fossil diesel fuels. This is due to large molecular weight and its large chemical structure.

Applying statistical and mathematical modelling to the occurrence of the kinematic viscosity with the functions of temperature and pressure cannot be overemphasized. Bodunwa and Adewole (2022) used the dataset on kinematic viscosity and one other data to compare the performance of A and D-Optimal designs using Imperialist comparative algorithm. It was reported that D-optimal design gives the best optimal design than A-optimal using their variance -covariance function.

Experimentation is the process of planning a study to meet specified objectives which constitutes a foundation of the empirical sciences (Zhu, 2012). One major advantage of experiment is its ability to control the experimental conditions; as well as to determine the variables to include in a study (Fackle, 2008). Since the introduction of experimental design principle in the first half of the 1930, optimal experimental designs have been gaining attention and had become useful tools among researchers in various fields (Atkinson and Donev, 1992; Atkinson, 1996; Atkinson, Donev and Tobias, 2007; Berger and Wong, 2009). Response Surface Methodology (RSM) has gained more interest in research methodology almost in every area of study in science and industry Lamidi *et.al* (2022).

It is a widely used mathematical and statistical method for modelling and analyzing a process in which the response of interest is affected by various variables (Braimah, Anozie, & Odejobi, 2016) (Braimah et.al., 2016). In recent literatures, (Yolmeh and Jafari, 2017) used RSM to get the optimization of different food processes such as extraction, drying, blanching, enzymatic hydrolysis and clarification, production of microbial metabolites and formulation. Also, (Sushanta *et.al*., 2018) developed the efficient technique for the production of clean coal by optimizing the operating parameters with the help of RSM. (Kumar *et.al* 2019) used RSM to optimize the temperature and time for maximum bio-oil yield. (Bodunwa, *et.al* 2023) worked on application of response surface methodology on the impact of ozone and Sulphu dioxide on the yield of soybean, the adequacy of the model was found and the quantity of these factors that gives optimum yield was established.

From the reviewed literatures, the relationship that exist among the kinematic viscosity, temperature and pressure with the use of experimental design especially the response surface methodology has not been emphasized. Application of this methodology was used in this study. Response surface methodology is a collection of statistical and mathematical methods that are useful for designing experiments, building models, evaluating the effect of factors, and searching for optimum conditions for desirable responses (Box and Wilson, 1951).

METHODOLOGY

A research carried out and discussed in (Linssen, 1975) was used in this study. The data is on the kinematic viscosity of a lubricant in strokes as a function of temperature (0_c) x_1 and pressure in atmospheres (atm) x_2 . The independent variables were coded between -1 and +1 as used in Bodunwa *et.al* (2023).

Table 1: Data on Kinematic Viscosity, functions of Temperature and Pressure

Y	x_1	x_2	$x_1$code	x_2 code	Y	x_1	x_2	$x_1$code	x_2 code
5.10595	1	0	-1	-1	6.23206	1375.82	37.8	-0.63183	-0.23559
6.38705	740.803	0	-0.80188	-1	4.6606	191.084	37.8	-0.9491	-0.23559
7.38511	1407.47	0	-0.62335	-1	4.29865	1	37.8	-1	-0.23559
5.79057	363.166	0	-0.90301	-1	7.96731	2922.94	37.8	-0.21751	-0.23559
5.10716	1	0	-1	-1	9.34225	4044.6	37.8	0.082863	-0.23559
6.36113	805.5	0	-0.78456	-1	10.5109	4849.8	37.8	0.298493	-0.23559
7.97329	1868.09	0	-0.5	-1	11.8215	5605.78	37.8	0.500942	-0.23559
10.4725	3285.1	0	-0.12053	-1	13.068	6273.85	37.8	0.679849	-0.23559
11.9272	3907.47	0	0.04614	-1	8.80445	3636.72	37.8	-0.02637	-0.23559
12.4262	4125.47	0	0.10452	-1	6.8553	1948.96	37.8	-0.47834	-0.23559
9.1563	2572.03	0	-0.31149	-1	6.11898	1298.47	37.8	-0.65254	-0.23559
4.54223	1	25	-1	-0.49444	3.38099	1	98.9	-1	1
5.82452	805.5	25	-0.78456	-0.49444	4.45783	685.95	98.9	-0.81657	1
6.70515	1505.92	25	-0.59699	-0.49444	5.20675	1423.64	98.9	-0.61902	1
7.71659	2339.96	25	-0.37363	-0.49444	6.29101	2791.43	98.9	-0.25273	1
5.29782	422.941	25	-0.88701	-0.49444	7.32719	4213.37	98.9	0.128059	1
6.22654	1168.37	25	-0.68738	-0.49444	5.76988	2103.67	98.9	-0.43691	1
7.57338	2237.29	25	-0.40113	-0.49444	4.08766	402.195	98.9	-0.89256	1
10.354	4216.89	25	0.129002	-0.49444	3.37417	1	98.9	-1	1
11.9844	5064.29	25	0.355933	-0.49444	5.83919	2219.7	98.9	-0.40584	1
12.4435	5280.88	25	0.413935	-0.49444	6.72635	3534.75	98.9	-0.05367	1
9.52333	3647.27	25	-0.02354	-0.49444	7.76883	4937.71	98.9	0.322035	1
8.34496	2813.94	25	-0.2467	-0.49444	8.91362	6344.17	98.9	0.69868	1
5.17275	516.822	37.8	-0.86186	-0.23559	9.98334	7469.35	98.9	1	1
6.64963	1737.99	37.8	-0.53484	-0.23559	8.32329	5640.94	98.9	0.510358	1
5.80754	1008.73	37.8	-0.73013	-0.23559	7.1321	4107.89	98.9	0.099812	1

Establishing the relationship that exist between response (Y) and the set of independent variables (X_i) is the most RSM problem. In order to get this done, the first step is to model this by linear function of the independent variables, then the approximation function is the first-order model. A model that incorporates curvature is usually required to approximate the response in the region close to optimum, and in most cases, a second order model is adequate (Montgomery, 2001). Here, we used cubic model since it performed well compared to any other models. Estimation of parameters in the model (1) using Design-Expert Software (version 8.0.1.0, Stat-Ease, Inc., Minneapolis, USA) was utilized in this study. This also used to get the 3D surface contour plot of the model.
Model (1) is used in this study

$$y_i = \beta_0 + \sum_{i=1}^{k} X_i\beta_i + \sum_{i=1}^{k} X_i^2\beta_{ii} + \sum_{i=1}^{k} X_i^3\beta_{ii} + \sum_{i=1}^{k} X_iX_j\beta_i + \epsilon_i \qquad (1)$$

Where $y_i,\ i = 1, 2, \ldots r$ is the response variable, here the kinematic viscosity, the independent (control) variables denoted by $x_1\ and x_2$, (temperature and pressure), β_i are the coefficient parameters and the ϵ_i is the error term

RESULTS AND DISCUSSION

Results on the Model adequacy

Table 2: Regression coefficients of the predicted model for the responses

		Intercept	Temperature (A)	Pressure (B)	(AB)	A^2	B^2	A^2B	AB^2	A^3	A^3
Y	Estimate	8.39	4.16	-2.17	-2.65	1.01	0.8375	-1.23	0.8514	0.9494	-0.0898
	Standard Error	0.05331	0.0534	0.1394	0.0654	0.0681	0.0578	0.0723	0.0664	0.0844	0.1415
	P value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	0.5290

From (1), the cubic model describing the relationship between the response variable and the independent variables is given in equation (2)

$$Y = 8.39 + 4.16X_1 - 2.17X_2 - 2.65X_1X_2 + 1.01X_1^2 + 0.8375X_2^2 - 1.23X_1^2X_2 + 0.8514X_1X_2^2 + 0.9494X_1^3 - 0.080X_2^3 \quad (2)$$

The Model in (2) with F-value of 6222.91 suggest the model is fitted the data set. There is only a 0.01% chance that an F-value this large could occur due to noise. P-values less than 0.0500 indicate model terms are significant. In this case the temperature (A), pressure (B), their interactions (AB), the squares (A^2, B^2), and these A^2B, AB^2, A^3 are significant model terms. The Lack of Fit F-value of 500.76 implies the model is significant.
To show the accuracy of the model in (2), some statistic measures were used to check the suitability of the model, the Goodness of Fit and the statistical significance of the terms in the model.

Akaike Information Criterion (AIC) and Bayesian Information Criteria (BIC) are used to measure the goodness of fit of the model. The values of the cubic model used in this study and the quadratic model were compared. It was noticed that the cubic model has the minimum values of -94.58 and -109.05 respectively for AIC and BIC compared to 3.48 and -6.52 for the quadratic model.

Table 3: ANOVA for Cubic model for the Response variable: Kinematic Viscosity

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	320.77	9	35.64	6222.91	< 0.0001	significant
A-A	34.78	1	34.78	6073.11	< 0.0001	
B-B	1.39	1	1.39	242.58	< 0.0001	
AB	9.38	1	9.38	1637.06	< 0.0001	
A^2	1.25	1	1.25	218.11	< 0.0001	
B^2	1.20	1	1.20	210.15	< 0.0001	
A^2B	1.67	1	1.67	291.49	< 0.0001	
AB^2	0.9423	1	0.9423	164.52	< 0.0001	
A^3	0.7249	1	0.7249	126.57	< 0.0001	
B^3	0.0023	1	0.0023	0.4029	0.5290	
Residual	0.2463	43	0.0057			
Lack of Fit	0.2463	41	0.0060	500.76	0.0020	significant
Pure Error	0.0000	2	0.0000			
Cor Total	321.01	52				

Std.Dev = 0.0757, Mean = 7.44 R^2= 0.9992 $Adj\ R^2 = 0.9991$ Ad Pre = 295.5

Some Statistic used here to justify the adequacy of the model are the Coeeficeint of Determination R^2, its Adjusted and predicted values. Any R^2 close to 1 suggests a good fit of the model. Here, R^2 *is* 0.9992 and the Adjusted R^2 *is* 0.9991 which is indicates adequate signal of the model. Using Normality Plot residual showing below in Figure 1 is another way of checking the adequacy of the model.

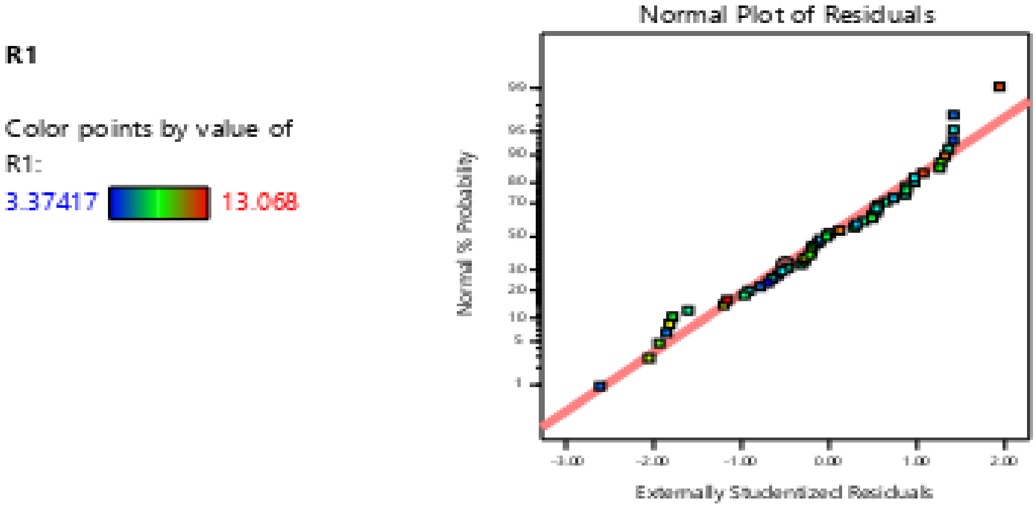
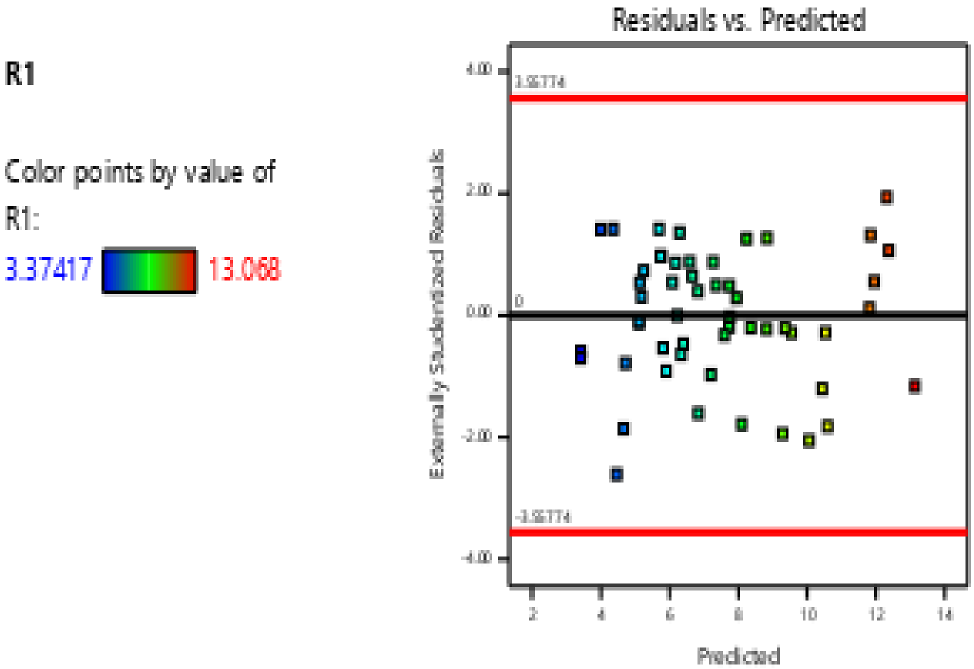


Fig 1: Normality Plot residual

The above Figure 1 of Normality Plot residual indicates that there is no strong indication of nonnormality, neither there is any evidence showing that there is possibility of outliers in the kinematic viscosity which is response variable in this study.



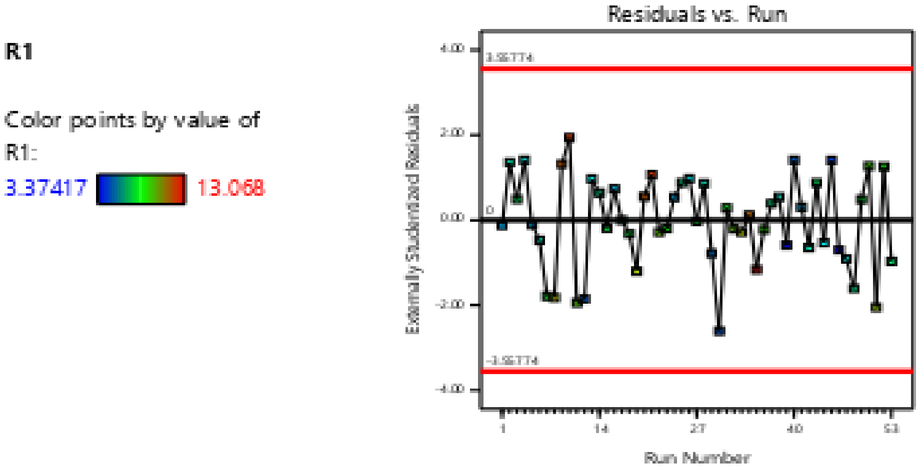


Figure 2: Plot of Residual versus predicted and Run

Figure 2 above show the relationship that exist between the residual and the predicted. It was shown that the predicted values invloves round the residuals which shows that model is accurate.

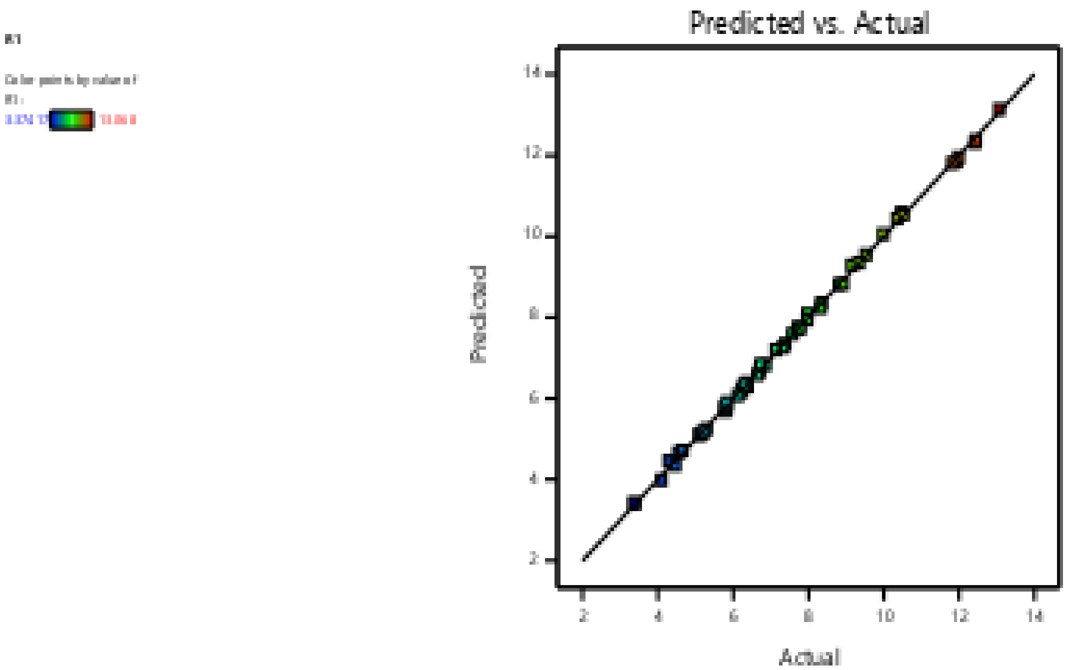


Figure 3: Plot of predicted versus Actual

The above Figure 3 display the plots of forecast and actual values of the kinematic viscosity. Since these are sensibly close which specifies that they are very alike, this approve the positive run of the experiment.

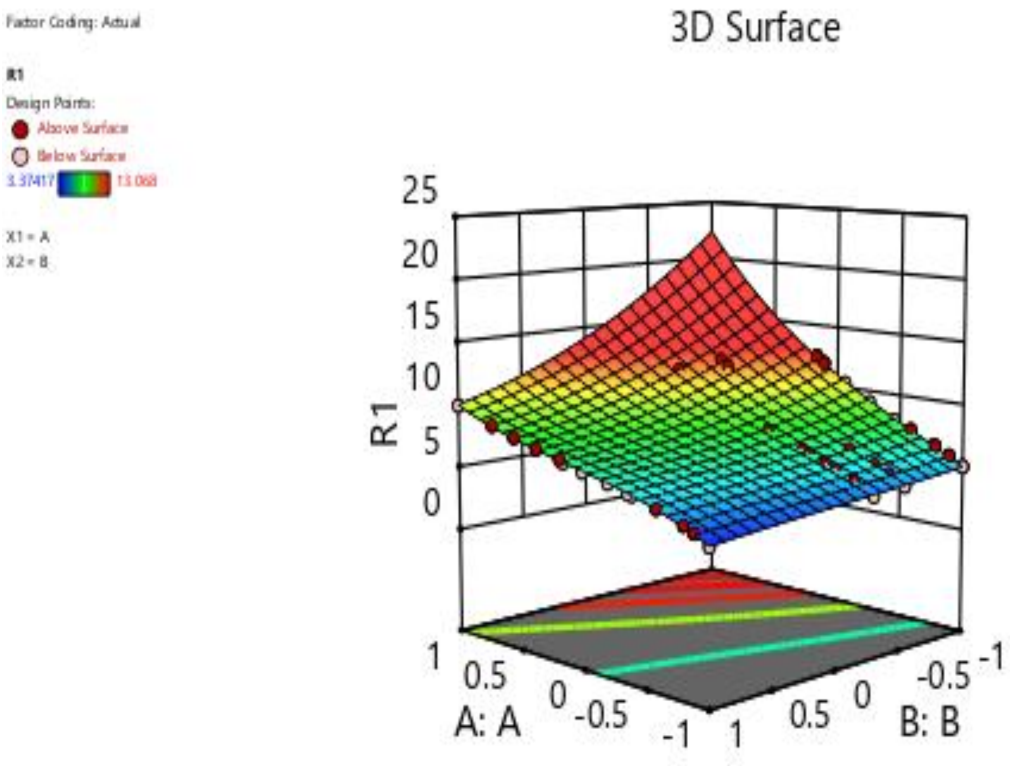


Figure 4: 3D Response Surface plots

The 3D plots are the graphical representation of the regression equations in order to determine the optimum value of the variables within the design space (Khuri & Cornell, 1996). The worth of forecasting maximum on the surface area is narrowed in the smallest ellipse in the contour diagram. It was cleared in Figure 4 the combinations of those variables that gives the optimal values of the function of Temperature and Pressure on the Kinematic Viscosity. The 0.6798 (atm) of pressure and -0.23559 (0_c) of temperature will give optimum movement of kinematic viscosity of 13.068

CONCLUSION

Applying a statistical model and optimizing the experimental design has been established to be a suitable device to predict and examine the interaction effects among the independent factors. Effect of kinematic viscosity of lubricant that is functions of Temperature and Pressure was looked into in this study. It was found that 0.6798 (atm) of pressure and -0.23559 (0_c) of temperature will give optimum movement of kinematic viscosity of 13.068. it was found the cubic regression models in (4) with F-value of 6222.91 suggests that the model is significant.

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