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Recent Advances in Understanding the Chandrasekhar Maximum Mass of White Dwarf Stars

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Abstract

The maximum mass of white dwarf stars, the final products of stellar evolution, is a crucial parameter in astrophysics. The Chandrasekhar limit, approximately 1.4 $M_{\odot}$, is widely recognized as the upper limit. However, recent observations have revealed the existence of super-Chandrasekhar white dwarfs, which are believed to be precursors of bright type Ia supernovae. The origin and fundamental nature of these super-Chandrasekhar white dwarfs remain poorly understood, leading to extensive debate within the scientific community. This research paper reviews current understanding of the maximum mass limit of white dwarf stars, examining various models, including strong magnetic fields, rapid rotation, non-commutative geometry and modified theories of gravity. Furthermore, the criticisms and challenges encountered by these Models, such as electron captures, pycnonuclear reactions, general relativity, and observational limitations were analyzed. This analysis yielded a conclusion with the proposition of potential avenues for future exploration in this area of study.

Keywords: White, Dwarfs, Supernovae, Models, Mass

INTRODUCTION

White dwarf stars, are remnants of low to intermediate mass stellar objects, and are formed when stars deplete their nuclear fuel and shed their outer layers. These stars gradually cool and dim, eventually becoming "cold" or "black" dwarfs (Caiazzo *et al.*, 2023). They are sustained by electron degeneracy pressure and they possess a mass comparable to the Sun but condensed into a smaller volume (Low, 2023). Understanding white dwarf physics is essential for comprehending stellar evolution and nucleosynthesis.

The mass distribution of white dwarfs provides insights into the initial masses and evolutionary paths of precursor stars. The Chandrasekhar mass limit, at 1.4 $M_{\odot}$, denotes the maximum mass for stability of the star before its collapse. Highly magnetized white dwarfs can exceed this limit, reaching up to 2.5 $M_{\odot}$, challenging conventional degeneracy pressure understanding (Das & Mukhopadhyay, 2013; Mathew *et al.*, 2020). Mechanisms like accretion, fusion, and effects of modified gravity contribute to exceeding this mass limit (Gvaramadze *et al*., 2019; Astashenok *et al*., 2022).

Exceeding the Chandrasekhar limit triggers collapse of the star under the influence of gravitational force. Ultimately, this collapse may trigger carbon fusion reactions within its core, resulting in a thermonuclear event known as a Type Ia supernova (Das & Mukhopadhyay, 2013). These occurrences, which are regarded as some of the most luminous and forceful explosions in the cosmos, bear significant importance in the field of cosmology as they provide a means to measure the rate at which the universe is expanding (Das & Mukhopadhyay, 2013). The Chandrasekhar limit guides the understanding of fate of stars, determining which stars become white dwarfs, supernovae, neutron stars, or black holes.

DEGENERATE EQUATION OF STATE LEADING TO CHANDRASEKHAR MASS LIMIT

The degenerate equation of state for electron gas is based on the following assumptions:

i. The electrons are non-interacting and obey the Fermi-Dirac statistics.
ii. The ions are fully ionized and form a uniform background of positive charge.
iii. The electrons are relativistic, meaning their kinetic energy is comparable to or greater than their rest mass energy. (Rowell, 2008)

The derivation of the equation of state is as follows (Rowell, 2008), (Konovalov & Son, 2022):
The number density of electrons is given by

$$n_e = \frac{g}{h^3}\int_0^\infty \frac{P^2 dp}{e^{(P_C-\mu)K_B T}+1} \quad (1)$$

where g is the degeneracy factor, h is the Planck constant, p is the momentum, c is the speed of light, μ is the chemical potential, k_B is the Boltzmann constant, and T is the temperature.
The pressure of electrons is given by

$$P_C = \frac{g}{3h^3}\int_0^\infty \frac{P^4 dp}{\sqrt{P^2C^2+M_e^2C^4}\ \left(e^{(P_C-\mu)K_B T}+1\right)} \quad (2)$$

where m_e is the electron mass.
The energy density of electrons is given by

$$\in_e = \frac{g}{h^3}\int_0^\infty \frac{P^2\sqrt{P^2C^2+M_e^2C^4}\,dp}{e^{(P_C-\mu)K_B T}+1} \quad (3)$$

In the limit of zero temperature and high density, the integrals can be evaluated analytically by using the Fermi momentum,

$$P_f = \left(3\pi^2 n_e\right)^{1/3} \quad (4)$$

and the Fermi energy,

$$\in_f = \sqrt{P_f^2C^2+M_e^2C^4} \quad (5)$$

The equation of state then becomes

$$P_e = K_1 n_e^{1/3} f_1(x) \quad (6)$$

where

$$K_1 = \frac{\left(\frac{\hbar c}{8\pi}\right)^{4/3}\left(3\pi^2\right)^{2/3}}{M_e^{4/3}} \quad (7)$$

$$x_1 = \frac{P_f}{M_e C} \tag{8}$$

and

$$f_1(x) = \frac{x\left(1+2x^2\right)\sqrt{1+x^2} - lin\left(x+\sqrt{1+x^2}\right)}{8x^3} \tag{9}$$

Similarly, the energy density becomes

$$\in_e = K_2 n_e^{4/3} f_2(x) \tag{10}$$

where,

$$K_2 = \left(\frac{\hbar c}{8\pi}\right)^{4/3} \tag{11}$$

and ,

$$f_2(x) = x^3\sqrt{1+x^2} \tag{12}$$

where g= Degeneracy factor, h= Planck constant , c =Speed of light, m_e = Electron mass, k_B=Boltzmann constant, n(x)=Number density of electrons , p(x)=Momentum of an electron, μ= Chemical potential ,T= Temperature, P=Pressure of the electron gas ,ε= Energy density of the electron gas ,P_F=Fermi momentum, εF: Fermi energy and x= Compressibility factor.

The Chandrasekhar equation of state leads to the classical mass limit of 1.44 solar masses for white dwarfs by considering the hydrostatic equilibrium and mass-radius relation of a spherical star (Rowell, 2008; Celebonovic, 1998):

The hydrostatic equilibrium condition is given by;

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} \tag{13}$$

where G is the gravitational constant, M(r) is the mass enclosed within radius r, and ρ is the mass density.

Assuming that the mass density is proportional to the number density of electrons, $= An_e$, where A is a constant depending on the composition of the star, we can use the equation of state to eliminate n_e and obtain a differential equation for P(r).Integrating this equation from the center to the surface of the star, we obtain

$$\frac{K_1 A^{4/3}}{GM} f_1(x_c) = R \tag{14}$$

where x_c is the value of x at the center, and R is the radius of the star.

Using this relation, we can express the mass enclosed within radius r as,

$$M(r) = 4\pi R^3 A n_c f_3(\xi) \tag{15}$$

where n_c is the number density at the center, $\xi = \frac{r}{R}$, and

$$f_3(\xi) = \frac{-\xi}{f_1(x_c)} \int_0^{\xi} f_1(x(\eta))\,d\eta \tag{16}$$

The total mass of the star is then given by

$$M = 4\pi R^3 A n_c f_3(1) \tag{17}$$

The maximum mass is obtained by maximizing $f_3(1)$ with respect to x_c, which yields x_c, ≈2.54 and $f_3(1)$ ≈0.0196.

The maximum mass is then given by

$$M_{max} = 5.76\pi A^{2/3} K_2^{1/2} f_3(1) \quad (18)$$

Using the values of the constants and assuming a carbon-oxygen composition with A=2m_p, where m_p is the proton mass, we obtain,

$$M_{max} \approx 1.44 M_\odot \quad (19)$$

$M_\odot$ is the solar mass.
This is the basic derivation of the Chandrasekhar equation of state and the mass limit for white dwarfs.

SUPER CHANDRASEKHAR WHITE DWARF STARS AS THE PROGENITOR OF OVER-LUMINOUS, PECULIAR TYPE IA SUPERNOVAE

Recent studies suggest that observations of over luminous peculiar type Ia supernovae indicate the existence of super-Chandrasekhar mass white dwarfs, which can lead to a reconsideration of the expansion history of the Universe and support the viability of single-degenerate supernova progenitors.

The observation of the variety of Type Ia supernova explosions, concentrating on the excessively bright SNe Ia 2003fg, 2006gz, and the moderately bright SN 1991T with 1D spherical radiation transport calculations reveals that SN 2006gz is an explosion of a super-Chandrasekhar-mass white dwarf (WD) while SN 1991T represents a marginal case, which could either be a Chandrasekhar or a super-Chandrasekhar-mass WD explosion (Maeda & Iwamoto, 2009). SNLS-03D3bb, a type Ia supernova that is bright and intense was the outcome of a super-Chandrasekhar mass explosion, which might have involved a rotating white dwarf or the merger of the two. This result was obtained using Supernova Legacy Survey (SNLS) to discover and observe SNLS-03D3bb at a redshift of z = 0.2440 *(Howell et al.*, 2006). Another over luminous type Ia supernovae SN 2007if, defy the legacy of Chandrasekhar mass limit as 2.4$M_\odot$, where obtained by calculating the bolometric light curve and Si II velocity development of SN 2007if using photometric and spectroscopic measurements (Scalzo *et al.*, 2010). The supernova SN 2009dc, which is thought to be a super-Chandrasekhar mass (SC) Type Ia supernova with a super-Chandrasekhar white dwarf progenitor, is presented and its optical photometry and spectra are examined. This resulted to a suggestion that a super-Chandrasekhar mass white dwarf progenitor was created as a result of a gravitational interaction between the host galaxy of SN 2009dc and a nearby galaxy (Silverman *et al.*, 2010). Theoretical models investigating Super Chandrasekhar white dwarf stars as progenitors of over-luminous type Ia supernovae have also been explored in this study. Tomaschitz (2018) examined ultra-relativistic electron dispersion effects, supporting the existence of super-Chandrasekhar mass white dwarfs. Hachisu *et al*. (2012) proposed a model based on accreting white dwarfs, reaching masses of 2.3-2.7 $M_\odot$, confirming the high luminosity of SNe Ia. Das & Mukhopadhyay (2013) investigated accretion onto magnetized white dwarfs, showing varying accretion rates lead to different explosions. Deb *et al*. (2021) studied anisotropically magnetized white dwarfs, emphasizing caution when using them as standard candles due to mass-radius variations.

Some papers challenge the validity and applicability of the models that invoke super-Chandrasekhar white dwarfs as the origin of over luminous supernovae (Lobato *et al*., 2023; Chamel *et al*., 2013). The study by (Chamel *et al*., 2013) demonstrates that the stability of hypothetical super-Chandrasekhar white dwarfs as the progenitors of over luminous type Ia supernovae like SN 2006gz and SN 2009dc may be severely restricted by electron captures and pycnonuclear reactions there by limiting their stability and reducing their mass below Chandrasekhar limit.

THEORETICAL MODELS FOR SUPER-CHANDRASEKHAR WHITE DWARFS

Evidence from various studies suggests that theoretical models explaining white dwarfs exceeding the Chandrasekhar limit include strongly magnetized white dwarfs, magnetostatic equilibrium, non-commutative geometry and modified gravity models.

Non-Commutative Geometry: This model assumes that the space-time coordinates of the white dwarf are non-commutative, meaning that they do not commute with each other, leading to modifications in their equation of state and allowing for higher masses and densities. Using this model, up to 2.58 $M_{\odot}$ was obtained as maximum mass of white dwarfs (Kalita *et al.*, 2021). Studies on non-commutative dispersion relations and mass-radius relations suggest that modified non-commutative formulations can support white dwarfs with masses surpassing the Chandrasekhar limit, potentially up to 4.68$M_{\odot}$, which could explain over-luminous type Ia supernovae (Mathew *et al.*, 2020; Pal & Nandi, 2019). Projection based on non-commutativity among position and momentum variables implies a new mass limit of approximately 2.6 $M_{\odot}$ (Surajit *et al.*, 2019). Additionally, using non-commutative geometry-inspired models, it is shown that the maximum mass of white dwarfs can exceed the Chandrasekhar limit (Astashenok *et al.*, 2022). Investigation on large white dwarfs exceeding the Chandrasekhar limit suggests that they may result from non-commutative squashed fuzzy spheres, based on non-commutative geometry (Kalita *et al.*, 2019).
However, the non-commutative model has some limitations and obstacles from both a theoretical and observational standpoint. A number of these limitations are as follows: The concept of non-commutativity, which represents a crucial element of the model, has not been directly observed and remains a hypothesis (Kalita *et al.*, 2019). The model introduces an ad hoc length scale to determine the non-commutativity effect intensity, which lacks derivation from fundamental theory and may vary across spin fields, thereby rendering the model more intricate and less inherent (Kalita *et al.*, 2021). These limitations highlight the need for additional observational evidence and refinement of the non-commutative model.

Modified Theories of Gravity: This model explores the impact of alternative theories of gravity, such as f(R), f(T), f(G), and f(R,T) gravity, on the structure and stability of white dwarfs. These theories modify the Einstein-Hilbert action by adding functions of the Ricci scalar, the torsion scalar, the Gauss-Bonnet invariant, or both. Depending on the chosen function and parameters, this model predicts white dwarfs with masses ranging from 0.5 to 3.5 $M_{\odot}$. Wojnar, (2021), have computed the super Chandrasekhar limit by utilizing Palatini's f(R) gravity in conjunction with a basic Lane-Emden model, In contrast to this, the Chandrasekhar mass limit of white dwarfs was studied using various models of f (R) gravity, including the simple relativistic polytropic equation and the realistic Chandrasekhar equation of state. The results show a decrease in white dwarf maximum mass compared to General Relativity predictions (Astashenok *et al.*, 2022). Research utilizing modified gravity frameworks explores cooling mechanisms, age determination, and Chandrasekhar mass limits of white dwarfs (Panah *et al.*, 2019; Liu *et al.*, 2019; Kalita & Sarmah, 2022). These studies employ various equations of state and solve modified Tolman-Oppenheimer-Volkoff equations to derive mass-radius relations, suggesting that in modified gravity contexts, white dwarfs can exceed the Chandrasekhar limit significantly.

Various studies have employed modified Einstein's gravity to explore sub- and super-Chandrasekhar limiting mass white dwarfs under different gravitational theories. Surajit Kalita *et al,*. (2018) utilized center density to explain the occurrence of these white dwarfs, establishing a limiting mass of approximately 1$M_{\odot}$. Rocha *et al.*, (2020) investigated charged white dwarfs within f(R, T) gravity, uncovering super-Chandrasekhar limiting mass white dwarfs associated with over-luminous SNeIa. Li *et al.,* (2023) analyzed white dwarfs under Rastall-Rainbow gravity, revealing a marked divergence from general relativity's predictions, suggesting potential for surpassing the Chandrasekhar limit. Critics of the Modified Gravity model point out its lack of a clear physical mechanism to explain gravity modification at high densities and its implications on other astrophysical phenomena (Mathew & Nandy, 2020). Additionally, challenges in observational consistency arise, particularly in explaining spectra, light curves, and host galaxies of over-luminous type Ia supernovae (Das & Mukhopadhyay, 2015).

Strong Magnetic field: This model explores the implications of strong and non-uniform magnetic fields on white dwarf properties, suggesting that they can increase maximum mass by providing additional pressure support and reducing electron capture rates (*Das et al.*, 2017). Mukhopadhyay *et al*., (2021) investigate magnetized white dwarfs, revealing masses greater than the Chandrasekhar mass limit, his results aligned with observations of peculiar type Ia supernovae. Similarly, Das & Mukhopadhyay (2012) modify the equation of state using Landau quantization to propose white dwarfs with masses around 2.3 $M_\odot$, significantly exceeding the Chandrasekhar limit. Numerical solutions of the Einstein-Maxwell equations indicate static magnetized white dwarfs with maximum masses of 2.13 $M_\odot$ (Newtonian) and 2.09 $M_\odot$ (general relativistic effects) by Franzon *et al.,* (2015). Other studies explore the impact of general relativity on strongly magnetized white dwarfs (Wen *et al.,* 2014) and examine stability and mass-radius relationships (Kundu *et al.,* 2012) there by finally proposing the possibility of white dwarfs with a mass exceeding the Chandrasekhar limit. Coelho et al. (2014) suggest a maximum mass limit of 2.58 $M_\odot$ for ultra-magnetized white dwarfs, potentially explaining super luminous Type Ia supernovae.

Bhattacharya et al. (2022) and Gupta et al. (2020) employ numerical analysis to investigate the structural characteristics and evolution of super-Chandrasekhar white dwarfs under strong magnetic fields. They explore variations in radiative opacity, magnetostatic equilibrium, and the degenerate core equation of state to assess the structural properties of these white dwarfs. Their findings suggest that super-Chandrasekhar white dwarfs with masses exceeding 2 $M_\odot$ can be produced by strong central magnetic fields of roughly 10^{14} G, maintaining their properties despite reduced luminosities. Additionally, Das *et al.,* (2014) present stable solutions for magnetized white dwarfs, exceeding 3 $M_\odot$, utilizing magnetostatic equilibrium models. They demonstrate that the maximum stable mass of these white dwarfs surpasses the conventional Chandrasekhar mass limit, suggesting the influence of magnetic pressure gradients within the framework of general relativity. In contrast, Bera & Bhattacharya (2017) focus on examining the instability and perturbation of strongly magnetized degenerate stars, specifically super-Chandrasekhar mass white dwarfs, through numerical simulations. They find that axisymmetric configurations with poloidal or toroidal magnetic fields are unstable, suggesting that long-lived super-Chandrasekhar mass white dwarfs supported by a magnetic field are unlikely to exist in nature due to their instability.

While there are some evidences for the existence of strong magnetic fields in white dwarfs the direct observation of super-Chandrasekhar mass white dwarfs with strong magnetic fields is still limited, the internal magnetic fields of these stars cannot be directly investigated by observations,. This lack of observational evidence raises questions about the validity of the strong magnetic fields model (Franzon *et al.*, 2017). The formation mechanisms for super - Chandrasekhar mass white dwarfs with strong magnetic fields are still not well understood. The strong magnetic fields model proposes that these objects are the result of mergers between two white dwarfs but the details of this process are not fully established (Ferrario & Wickramasinghe, 2005).

Rapid Rotation: This model proposes that rapid rotation increases the maximum mass of a white dwarf by providing centrifugal support and reducing effective gravity, potentially explaining white dwarfs with masses up to 2.2 $M_{\odot}$ (Sharma *et al.,* 2017). Terrero *et al.,* (2018) investigate the effects of rotation on the structure of magnetized white dwarfs using Hartle's formalism within general relativity, finding that rotation has a greater impact on white dwarf structure than magnetic fields. Both magnetized and non-magnetized stable configurations show an increase in maximal mass due to rotational effects, reaching approximately 1.5 $M_{\odot}$. Caiazzo *et al.,* (2021) employ photometric color selection and GPU implementation to study highly magnetized, rapidly rotating white dwarfs, and finding that their maximum mass exceeds the Chandrasekhar limit.
Critiques of the rotation model regarding the super-Chandrasekhar mass of white dwarfs include challenges in achieving very high rotation rates required to significantly increase the maximum mass, as observed white dwarfs have rotation periods of only a few hours (Kawaler, 2003). Additionally, there are concerns about dynamic stability, as centrifugal forces exceeding gravitational forces can lead to instability and potential collapse or fragmentation (Otoniel *et al.,* 2022). Some studies suggest that rotation may actually decrease the maximum mass of white dwarfs by causing heating and reducing electron degeneracy, which weakens the star's support against gravity (Boshkayev *et al.,* 2011).

Relativistic Framework
The Chandrasekhar mass limit of white dwarf stars is explored through various approaches. Low (2023) combines introductory quantum mechanics and Einstein's special theory of relativity to determine electron degeneracy pressure, approximating the Chandrasekhar mass to be 1.4 $M_{\odot}$. Gregoris & Ong (2022) utilize the uncertainty relation within Doubly Special Relativity to calculate the Chandrasekhar mass, yielding a star with zero radius in the generalized uncertainty principle (GUP) framework. Mathew *et al.,* (2017) investigate the maximum mass of white dwarf stars in general relativity, finding it to be around 1.4166 $M_{\odot}$ for helium and carbon white dwarfs and 1.2230 $M_{\odot}$ for iron white dwarfs. They confirm Chandrasekhar's limit and stable mass-radius curves consistent with astronomical observations.

Althaus *et al.,* (2023) employ the La Plata stellar evolution code to calculate rest-mass carbon-oxygen ultra-massive white dwarf evolutionary sequences, considering general relativity's impact.

Critically, Astashenok *et al.,* (2022) confirm Chandrasekhar's limit in GUP white dwarfs, supporting stable mass-radius curves. Nunes *et al.,* (2021) explore the structure and stability of massive hot white dwarfs, uncovering insights into their properties. Pei (2018) predicts a white dwarf mass limit of 1.44 $M_{\odot}$ using statistical mechanics. Tomaschitz (2018) studies high-mass white dwarfs using the Chandrasekhar model based on the Dirac equation, supporting the existence of super-Chandrasekhar mass white dwarfs. Moussa (2015) proposes modifications to the Lane-Emden equation in quantum gravity.

The relativistic framework exploring super Chandrasekhar maximum mass of white dwarf stars faces criticisms, notably for its reliance on the generalized uncertainty principle (GUP), an unfully developed theory of quantum gravity. GUP predicts limited precision in particle position and momentum, impacting the equation of state for degenerate matter used in mass-radius calculations (Kempf, 1994; Mazur & Mottola, 2023). Additionally, the framework did not explain certain properties of super Chandrasekhar white dwarfs, such as their high magnetic fields (Chamel *et al.,* 2013; Felipe *et al.,* 2005).

Table 1: Comparison of maximum mass of white dwarfs from Different Methods

S/N	Source	Maximum Mass Of White Dwarfs (in solar masses)	Method or Model Used
1.	(Bhattacharya *et al.*, 2022)	2.00	Strongly magnetized White dwarf
2.	(Franzon *et al*., 2015)	2.09	Pseudo-spectral technique to numerically solve the Einstein-Maxwell equations (under general relativistic effects)
3.	(Franzon *et al*., 2015)	2.13	Pseudo-spectral technique to numerically solve the Einstein-Maxwell equations (under Newtonian conditions)
4.	(Sharma *et al.,* 2017)	2.20	Rapid rotation
5.	(Das & Mukhopadhyay, 2012)	2.30	Model of a relativistic electron gas in a strong magnetic field
6.	(Hachisu et al., 2012).	2.30-2.70	Model for Type Ia supernovae (SN Ia), based on three binary evolution processes:
7.	(Das & Mukhopadhyay, 2013)	2.33	Observational evidence and the Lane-Emden formalism to solve the magnetostatic equilibrium condition
8.	(Scalzo et al., 2010)	2.40	Photometric and spectroscopic measurements.

CONCLUSION

The discovery of over-luminous, peculiar Type Ia supernovae (SNeIa) has challenged the understanding of the Chandrasekhar mass limit (CML) and the possibility of super-Chandrasekhar white dwarf stars (SCPWDs). Theoretical models have been proposed to explain the formation of SCPWDs, but each of these models faces challenges. Despite these challenges, the theoretical models for SCPWDs provide a promising explanation for the most luminous SNeIa. Further theoretical and observational studies are needed to better understand the formation, explosion, and implications of SCPWDs, including investigating the feasibility of the strong magnetic fields model, developing more sophisticated models of non-commutative geometry and modified gravity, and searching for observational evidence of rotating white dwarfs with high spin rates enough to significantly increase their maximum mass. Addressing these and other challenges would pave a way for better understanding of SCPWDs and their role in the universe. This knowledge could help to improve the use of SNeIa as standard candles and to better understand the evolution of massive stars and the formation of neutron stars.

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