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Means-Based Class of modified Weerakoon-Fernando method for determining the zero of nonlinear equation

O. Ogbereyivwe[1] and S.S. Umar[2]
[1]Department of Mathematics,
Delta State University of Science and Technology,
Ozoro

[2]Department of Statistics,
Federal Polytechnic, Auchi,
Edo State.

Email: ogho2015@gmail.com

Abstract

This paper introduces a generalized class of modified Weerakoon and Fernando iterative method (WFM) for determining the zero of equations that are nonlinear. The idea applied in the modification, involves the composition of a perturbed Newton method with a corrector iterative function that is based on parametric bi-variate rational function of degree one and a power means generating function. The convergence analysis on the developed class of methods, shows that it is of convergence order four with efficiency index of 1.5874. Some existing modified WFM were established to be concrete members of the developed class of methods. Numerical results obtained when the new methods were implemented are provided to compare their effectiveness and efficiency.

Keywords: Nonlinear equations, power-means generator, Iterative method, Rational weight function

Introduction

The Weerakoon-Fernando iterative method (WFM) in Weerakoon and Fernando (2000) is one of the late 1990s modifications of the Newton method (NM), $w_{i+1} = s_i - \frac{\mu(s_i)}{\mu'(s_i)},\ i = 0,1,\ldots$ obtained by the composition of the NM with a mean-based iterative function as corrector function that is presented as:

$$s_{i+1} = s_i - \frac{\mu(s_i)}{M_A[\mu'(s_i), \mu'(w_i)]} \qquad (1)$$

where $M_A[\mu'(s_i), \mu'(w_i)]$ is the arithmetic mean (A.M) of the evaluated values of $\mu'(\cdot)$ at the points s_i and w_i. The fundamental benefit of this modification was that, the WFM has convergence order (CO) 3 and better efficiency in computation compared with the NM that has CO 2 and lower in efficiency.

Since the emergence of the WFM, several authors have taken further steps to enhance its CO and efficiency. Among these steps, includes the composition of the Jarratt-type perturbed NM : $y_{i+1} = s_i - \frac{2}{3}\frac{\mu(s_i)}{\mu'(s_i)}$, with a corrector function that is based on weight function(s). In most cases, the weight functions are obtained by subjecting them to satisfy some n-times derivative

*Author for Correspondence

evaluations at some points. For instance, Lofti (2020), Chand *et al.* (2020) and Chicharo *et al.* (2019) presented a modified WFM with CO 4 as:

$$s_{i+1} = s_i - G\left(\frac{\mu'(y_i)}{\mu'(s_i)}\right)\frac{\mu(s_i)}{M_A[\mu'(s_i),\mu'(y_i)]} \tag{2}$$

In Soleymani *et al.* (2005) another improved WFM with the idea of bi-weight functions was constructed as:

$$\mu_{i+1} = s_i - G\left(\frac{\mu'(y_i)}{\mu'(s_i)}\right)H\left(\frac{\mu(s_i)}{\mu'(s_i)}\right)\frac{\mu(s_i)}{M_A[\mu'(s_i),\mu'(y_i)]} \tag{3}$$

In Ogbereyivwe and Ojo-Orobosa (2021), the authors modified the work of Soleymani *et al.* (2005), by replacing the arithmetic mean $M_A[\cdot,\cdot]$ in (3) with means generating function (MGF) transformation as $\Phi\left(\frac{\mu'(s_i)}{\mu'(y_i)},\frac{\mu'(y_i)}{\mu'(s_i)}\right)$. They put forward their modification as:

$$\mu_{i+1} = s_i - G\left(\frac{\mu'(y_i)}{\mu'(s_i)}\right)H\left(\frac{\mu(s_i)}{\mu'(s_i)}\right)\Phi\left(\frac{\mu'(s_i)}{\mu'(y_i)},\frac{\mu'(y_i)}{\mu'(s_i)}\right). \tag{4}$$

Deriving motivation from the use of the MGF utilized in Ogbereyivwe and Ojo-Orobosa (2021) in constructing new iterative method, this article offers a wide class of enhanced WFM in Weerakoon and Fernando (2000), for deciding the zero of nonlinear equations. The new class of methods were developed based on the use of the rational weight function and power-means generating functions.

Materials and Methods

Basic Definitions

Definition 1: An iterative method efficiency is measured using the numerical value obtained by $\rho^{\frac{1}{T}}$, where T is the sum of all different functions computation in a complete iteration cycle, see Traub, 1964; Ogbereyivwe and Ojo-Orobosa, 2021).

Definition 2: Let s_* a simple solution of $\mu(s) = 0$ and $\eta_i = |s_i - s_*|$ the iteration error at ith iteration of an iterative method. If an equation in the form $\eta_{i+1} = \beta\eta_i^{\rho} + O(\eta_i^{\rho+1})$ can be obtained from the iteration function of the method by the use of Taylor's expansions of $\mu(\cdot)$ and $\mu'(\cdot)$ as contained in the method, then η_{i+1} is referred to as the iterative method error equation, β is error equation constant and ρ represents its theoretical CO, see (Traub, 1964; Ogbereyivwe and Ojo-Orobosa, 2021).

Definition 3: Consider $p, q \in \mathbb{R}$. If $m \neq 0$ is real number that is finite, then the m −power mean generator of p and q is denoted as $\Omega_m(p,q)$ and expressed as:

$$\Omega_m(p,q) = (M[p^m, q^m])^{\frac{1}{m}}. \tag{6}$$

Consequently, several means-types that are prototype of (6) can be generated. For instance:

$m = -1$, gives $\Omega_{-1}(p,q) = \left(\frac{p^{-1}+q^{-1}}{2}\right)^{-1}$ the harmonic mean.

$m = 2$, provides $\Omega_2(p,q) = \left(\frac{p^2+q^2}{2}\right)^{\frac{1}{2}}$ the root mean square.

$m = -2$, generates $\Omega_{-2}(p,q) = \left(\frac{p^{-2}+q^{-2}}{2}\right)^{-\frac{1}{2}}$ the inverse-root, inverse-square mean.

$m = \frac{1}{2}$, obtains $\Omega_{\frac{1}{2}}(p,q) = \left(\frac{\sqrt{p}+\sqrt{q}}{2}\right)^2$ the square mean-root.

$m = 3$, returns $\Omega_3(p,q) = \left(\frac{p^3+q^3}{2}\right)^{\frac{1}{3}}$ the cube-root mean cube and so on.

The Class of methods and its convergence analysis

We consider the following perturbation to the WFM. The NM in the first step of the WFM is replaced with the Jarratt's method first step in Jarratt (1966), known as weighted NM. Then, the AM weight function in the WFM second step, was replaced with an iterative MGF, say $\Omega_m[\mu'(s_i),\mu'(y_i)]$, and an iterative parametric bi-variate rational approximation function, say $R_{2,2}\left[\frac{\mu'(s_i)}{\mu'(y_i)},\frac{\mu'(y_i)}{\mu'(s_i)}\right]$, as following:

$$s_{i+1}=s_i-\ R_{2,2}\left[\frac{\mu'(s_i)}{\mu'(y_i)},\frac{\mu'(y_i)}{\mu'(s_i)}\right]\frac{\mu(s_i)}{\Omega_m[\mu'(s_i),\mu'(y_i)]}.$$

(7)

where

$$R_{2,2}\left[\frac{\mu'(s_i)}{\mu'(y_i)},\frac{\mu'(y_i)}{\mu'(s_i)}\right]=\frac{a_1+a_2\frac{\mu'(s_i)}{\mu'(y_i)}+a_3\frac{\mu'(y_i)}{\mu'(s_i)}}{a_4+a_5\frac{\mu'(s_i)}{\mu'(y_i)}+a_6\frac{\mu'(y_i)}{\mu'(s_i)}}, \tag{8}$$

$a_i\in\mathbb{R}$. Next, we investigate the convergence of the method in (7). Firstly, the following theorem is put forward.

Theorem 2.1: *Consider the function $\mu:\Delta\subset\mathbb{R}\longrightarrow\mathbb{R}$ in real space. Suppose μ is sufficiently differentiable and $\mu'(\cdot)\neq 0$ in Δ. For s_0 an initial guess close to s_* and every mean-type generated by the MGF $\Omega_m[\mu'(s_i),\mu'(y_i)]$ used in (7), will produce a sequence of approximations s_i such that $\lim\limits_{i\longrightarrow n} s_i=s_*$.*

Proof: The substitution of s by s_i in the Taylor's expansion of $\mu(s)$ and $\mu'(s)$ around s_* produces:

$$\mu(s_i)=\mu'(s_*)\left[\eta_i+\sum_{n=2}^{4}c_n\eta_i^n+O(\eta_i^5)\right],\ \ \text{i}=0,1,\ldots \tag{9}$$

and

$$\mu'(s_i)=\mu'(s_*)\left[\eta_i+\sum_{n=2}^{4}c_n\eta_i^{n-1}+O(\eta_i^5)\right], \tag{10}$$

where $c_n=\frac{1}{n!}\frac{\mu^{(n)}(s_*)}{\mu'(s_*)}$, $n=1,2\ldots$ and $O(\eta_i^5)$ represents truncated higher order expansions of the series.

Using (9) and (10) in the expansion of the first step of equation (7), we have

$$y_i=s_i-\frac{2}{3}\frac{\mu(s_i)}{\mu'(s_i)}=\frac{1}{3}\eta_i+\frac{2}{3}c_2\eta_i^2+\frac{4}{3}(c_3-c_2^2)\eta_i^3+O(\eta_i^4) \tag{11}$$

The Taylor's expansion of $\mu(y_i)$ and $\mu'(y_i)$ about $\mu'(s_*)$ are:

$$\mu(y_i)=\mu'(s_*)\left[\frac{1}{3}\eta_i+\frac{7}{9}c_2\eta_i^2+\left(\frac{37}{27}c_2-\frac{8}{9}c_2^2\right)\eta_i^3+O(\eta_i^4)\right] \tag{12}$$

and

$$\mu'(y_i)=\mu'(s_*)\left[1+\frac{2}{3}c_2\eta_i+\left(\frac{c_2+4c_2^2}{3}\right)\eta_i^2+\left(4c_2c_3-\frac{8}{3}c_2^3\right)\eta_i^3+O(\eta_i^4)\right] \tag{13}$$

respectively. Applying the expressions in (10) and (13) in the expansion of the MGF $\Omega_m[\mu'(s_i),\mu'(y_i)]$, the next expression was obtained:

$$\Omega_m=1+\frac{4}{3}c_2\eta_i+\frac{1}{9}(15c_3+2c_2^2(2+m))\eta_i^2 \\ +\frac{1}{27}\left(54c_4+6c_2c_3(5+4m)-4c_2^3(4+5m)\right)\eta_i^3+O(\eta_i^4) \tag{14}$$

Using (9), (10), (12) and (13) in (7) the error equation next is obtained.

$$s_{i+1}=s_*+\left(1-\frac{a_1+a_2+a_3}{a_4+a_5+a_6}\right)\eta_i+\sum_{n=2}^{4}P_n\eta_i^n \tag{15}$$

where $P_n=P_n(a_1,a_2,a_3,a_4,a_3,a_6,c_1,c_2,c_3,c_4,m)$ is a polynomial with the indicated variables. For the term with η_i, in (15) to be annihilated, require that

$$a_6=a_1+a_2+a_3-a_4-a_5. \tag{16}$$

When the relation in (16) is substituted the expression in (15) with $P_2=0$, we have

$$a_5=\frac{5a_1+9a_2+a_3-4a_4}{8}. \tag{17}$$

Again, by substituting a_5 expression (see (17)) in Equation (15) with $P_3 = 0$, gives

$$a_4 = \frac{1}{8}(11a_3 + 2ma_3 + a_2(3 + 2m) + a_1(15 + 2m)). \quad (18)$$

Using the expression for a_4 in equation (18) with $P_4 = 0$, we have

$$s_{i+1} = s_* + \left(\frac{a_1\left(27c_3 + 2c_2^2(m-17) + a_3\Lambda\right)}{27(a_1 + a_2 + a_3)}\right)\eta_i^4 + O\left(\eta_i^4\right). \quad (19)$$

where $\Lambda = 27c_3 - 2c_2^2(35 + 3m) + a_2\left(27c_3 + 2c_2^2(5m - 7)\right)$.

By Definition 2, the expression in (19) is the general class of method in (7) asymptotic error equation and its convergence order is four. Consequently, the proof is ended.

Remark 2.1: The backward substitution of the expression in (18) into the one in (17), and then applied in (16) with some simplifications, yielded a new expressions for a_6 and a_5 as

$$a_6 = \frac{-a_1(9+2m) - a_2(5+2m) + a_3(3-2m)}{16} \quad (20)$$

and

$$a_5 = \frac{-a_1(5+2m) + a_2(15-2m) - a_3(9+2m)}{16} \quad (21)$$

respectively. When the equations in (18), (20) and (21) are substituted in (8), the proposed class of methods in (7) becomes a means-based tri-parametric class of CO four methods.

Remark 2.2: The derived class of methods in (7) require the assessment of a function at a point s and its derivatives at two points s and y in one iteration circle. By Definition 1, its general efficiency index is 1.5874. Furthermore, the value m in the error equation in (19), defines the types of means used and is responsible for the perturbation in performance among the members of the generalized class of iterative method in (7).

Remark 2.3: A major setback of the methods developed in Lofti(2020), Chand *et al.* (2020), Chicharro *et al.* (2019) and Ogbereyivwe and Ojo-Orobosa (2021) is that, their corrector iterative functions require the use of a suitable weight functions that must have high derivatives and evaluated at some points. The process of getting these weight functions are usually through trials or certain heuristic means. Again, obtaining derivatives of some functions can be difficult. The proposed class of methods in (7) is able to circumvent these setbacks as it does not require the use of differentiation in determining the weight functions, rather it simply require arbitrary assignment of values to the parameters a_1, a_2 and a_3.

Some special classes of the new class of methods

Next, we offer some classes of the developed class of methods.

Class 1: For $a_1 = -\frac{m}{4}$, $a_2 = \frac{m+3}{8}$ and $a_3 = \frac{m+5}{8}$ a class of CO 4 methods is offered as:

$$s_{i+1} = s_i - \left[-\frac{m}{4} + \left(\frac{m+3}{8}\right)\frac{\mu'(y_i)}{\mu'(s_i)} + \left(\frac{m+5}{8}\right)\frac{\mu'(s_i)}{\mu'(y_i)}\right]\frac{\mu(s_i)}{\Omega_m} \quad (22)$$

The class of iterative methods in equation (22) has some established existing members. For instance, $m = 1$ will produce the CO 4 method offered in Chand *et al.* (2020) and Chicharo *et al.* (2019). Many other variants of (22) can be generated by varying the value of m.

Class 2: When $a_1 = -\frac{m+2}{4}$, $a_2 = -\frac{m+9}{8}$ and $a_3 = -\frac{m+3}{8}$, another new class of CO 4 methods is put forward as:

$$s_{i+1} = s_i - \left[\frac{m+9}{8} - \left(\frac{m+2}{4}\right)\frac{\mu'(s_i)}{\mu'(y_i)} + \left(\frac{m+3}{8}\right)\left(\frac{\mu'(s_i)}{\mu'(y_i)}\right)^2\right]\frac{\mu(s_i)}{\Omega_m} \quad (23)$$

Class 3: Consider $a_1 = -\frac{m+6}{4}$, $a_2 = -\frac{m+5}{8}$ and $a_3 = -\frac{m+15}{8}$, then a new class of CO 4 methods is obtained as:

$$s_{i+1} = s_i - \left[\frac{m+15}{8} - \left(\frac{m+6}{4}\right)\frac{\mu'(y_i)}{\mu'(s_i)} + \frac{m+5}{8}\left(\frac{\mu'(y_i)}{\mu'(s_i)}\right)^2\right]\frac{\mu(s_i)}{\Omega_m} \qquad (24)$$

The case $m = 1$ in equation (24), will result to the order 4 method developed in Lofti (2020).
Class 4: When $a_1 = 1,\ a_2 = a_3 = 0$, a new class of CO 4 methods is presented as:

$$s_{i+1} = s_i + \left[\frac{16\mu'(s_i)\mu'(y_i)}{(9+2m)\mu'(s_i)^2 - 2\mu'(s_i)\mu'(y_i) + (5+2m)\mu'(y_i)^2}\right]\frac{\mu(s_i)}{\Omega_m}. \qquad (25)$$

Results and Discussion

The applicability of the developed methods was illustrated in this section. Some concrete methods obtained from the class (7), and its subclasses (22), (23) and (24) were implemented on some nonlinear problems using MAPLE 2017 programming software. For all program execution, $|\mu(s)| \le 10^{-500}$ was adopted as halting criterion. To minimize error of truncation, computation outputs were made to be 1000 digits precision. The methods used for comparison includes Weerakoon and Fernando (2000) (WFM), Chand *et al.* (2020) and Chicharo *et al.* (2019) (i.e. Class 1 with $p = 1$) and Lofti (2020) (i.e. Class 3 with $p = 1$). We note that the compared methods have been established literature.

The nonlinear equations used for computation test are both theoretical and real-life equation given next:
$i.)$ $\mu_1(s) = s^3 - 9s + 1$, $s_* = 2.9428 \ldots$ $ii.)$ $\mu_2(s) = s + 2s^3\sin(s) - 1$, $s_* = 0.6558 \ldots$
The real-life models were taken from Sivakumar and Jayaraman (2019).

iii.) Plank's Constant: $\mu_3(s) = \exp(-s) - 1 + \frac{s}{5}$, $s_* = 4.9651 \ldots$

iv.) Projectile model: $\mu_4(s) = \frac{\pi}{4} + s - 0.5\cos(s)$, $s_* = -0.3094 \ldots$

v.) Chemical engineering fractional conversion model:
$\mu_5(s) = s^4 - 7.7075s^3 + 14.7445s^2 + 2.511s - 1.674$, $s_* = 0.2777 \ldots$

Discussion

In Table 4.1, the computation results obtained in terms of number of iterations (N) and residual errors ($|\mu(s_{i+1})|$) from each methods implementation on the test problems, are presented in the format $(N)A.Be-C$ (to represent $(N)A.B \times 10^{-C}$, where $N, A, B, C \in \mathbb{R}$) for comparison. Observe that, the specified members derived from the developed class of methods successfully solved the problems used for the test and computationally outperformed the WFM. The case $p = 1$ in Class 1 and Class 3, represents the methods put forward in Chand *et al.* (2020) or Chicharo *et al.* (2019) and Lofti (2020) respectively, are not keeping up the results of other variant of p values in terms of precision for most of the tested problems. In particular, the cases of $p = -1, -2$ and -3.

Table 4.1: Class Concrete methods results comparison

Class 1								
$\mu_i(s)$	s_0	WFM	$m=1$	$m=-1$	$m=2$	$m=-2$	$m=3$	$m=-3$
$\mu_1(s)$	2.5	(6)3.6e-385	(5)1.5e-468	(5)1.2e-476	(5)6.1e-468	(5)1.3e-489	(5)8.4e-455	(5)2.9e-517
$\mu_2(s)$	1.38	(7)2.0e-854	(6)1.3e-833	(6)9.6e-866	(6)1.1e-819	(6)1.2e-877	(6)2.1e-810	(6)1.3e-882
$\mu_3(s)$	2.0	(7)2.0e-776	(5)3.3e-249	(5)8.2e-243	(5)2.3e-250	(5)5.5e-250	(5)6.8e-247	(5)1.7e-271
$\mu_4(s)$	0.0	(6)1.4e-776	(4)5.5e-265	(4)7.2e-269	(4)1.7e-263	(4)1.2e-270	(4)4.7e-262	(4)3.6e-272
$\mu_5(s)$	0.5	(6)6.6e-748	(5)1.2e-698	(5)5.6e-704	(5)4.1e-696	(5)2.2e-706	(5)6.7e-694	(5)2.2e-708
Class 2								
$\mu_1(s)$	2.5	(6)3.6e-385	(4)3.3e-287	(5)7.3e-585	(5)4.0e-721	(5)2.9e-540	(5)1.3e-737	(5)2.9e-517
$\mu_2(s)$	1.38	(7)2.0e-854	(5)3.5e295	(5)1.5e-251	(5)7.4e-328	(5)4.5e-236	(5)6.6e-337	(5)5.9e-221
$\mu_3(s)$	2.0	(7)2.0e-776	(5)5.2e-349	(5)3.6e-286	(5)1.5e-383	(5)9.2e-274	(5)1.3e-402	(5)1.7e-271
$\mu_4(s)$	0.0	(6)1.4e-776	(4)4.0e-313	(4)1.6e-287	(4)8.7e-350	(4)3.1e-279	(4)5.4e-350	(4)3.6e-272
$\mu_5(s)$	0.5	(6)6.6e-748	(5)1.9e-745	(5)3.5e-726	(5)1.5e-756	(5)3.1e-717	(5)2.0e-769	(5)2.2e-708
Class 3								
$\mu_1(s)$	2.5	(6)3.6e-385	(5)3.5e-307	(5)6.1e-342	(5)2.4e-293	(5)1.4e-368	(5)5.1e-280	(5)2.6e-409
$\mu_2(s)$	1.38	(7)2.0e-854	(6)2.0e-654	(6)5.4e-724	(6)1.6e-625	(6)2.1e-761	(6)7.9e-602	(6)4.6e-798
$\mu_3(s)$	2.0	(7)2.0e-776	(6)2.0e-707	(6)3.9e-713	(6)7.0e-705	(6)3.6e-730	(6)3.4e-700	(6)1.8e-777
$\mu_4(s)$	0.0	(6)1.4e-776	(4)9.3e-240	(4)1.4e-248	(4)4.7e-236	(4)8.4e-254	(4)9.1e-233	(4)1.1e259
$\mu_5(s)$	0.5	(6)6.6e-748	(5)1.2e-653	(5)4.4e-671	(5)9.1e-646	(5)1.7e-680	(5)1.5e-638	(5)3.3e-690
Class 4								
$\mu_1(s)$	2.5	(6)3.6e-385	(5)3.4e-529	(5)3.4e-527	(5)4.0e-520	(5)8.2e-528	(5)9.3e-502	(5)7.3e-537
$\mu_2(s)$	1.38	(7)2.0e-854	(5)7.8e-260	(5)7.3e-243	(5)5.0e-272	(5)2.2e-235	(5)3.0e-289	(5)2.2e-227
$\mu_3(s)$	2.0	(7)2.0e-776	(5)1.5e-226	(5)5.7e-254	(6)7.5e-802	(5)2.4e-265	(6)3.1e-701	(5)3.2e-281
$\mu_4(s)$	0.0	(6)1.4e-776	(4)1.4e-287	(4)4.0e-281	(4)1.1e-291	(4)3.0e-279	(4)6.1e-297	(4)2.3e-275
$\mu_5(s)$	0.5	(6)6.6e-748	(5)1.9e-726	(5)1.7e-719	(5)1.7e-730	(5)4.3e-716	(5)3.1e-735	(5)1.7e-712

Conclusion

This work put forward a wide class of modified WFM that have some well-established modified WFM as particular members. Unlike most of the existing modifications of the WFM that require weight function that must be subjected to high derivatives and evaluated at some points which is a cumbersome process, the developed class of methods only require free assignment of values to the parameters contained in it. Again, the idea of varying the arithmetic mean used in the WFM by using power-means generating function, enabled the modifications of some existing modifications of the WFM. This is another contribution of this work.

For future work, convergence and chaotic behavior of the developed class of methods can be investigated. Also, the extension of the methods to obtaining multiple roots of single nonlinear equations and system of nonlinear equations can be embarked upon.

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