

# Effects of Restricting Wastage to Only Staff Who Reach the Peak of Their Career in Manpower Planning Models

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## Abstract

*Wastage is the exit of employees from an organization which may be voluntary or involuntary. The concept of wastage in manpower planning models in literature is being expanded in this study by restricting the concept of wastage to only employees who got to the highest rank in their careers before their exit from service. A new approach which involves backward recursive procedure in dynamic programming (DP) is formulated and applied to evaluate the wages of retirees who exit from service. The mathematical analysis of the proposed model indicates that the wages of employees who got to the highest rank in their profession earn higher wages compare to other employees recruited to replace retirees in all the periods of manpower cost being considered.*

**Keywords:** Manpower, retirees, employment, dynamic programming.

## Introduction

Manpower can be defined as human beings involved in the production of goods and services in an industry, Urhoma (2009). Manpower planning is an attempt to match the supply of people with the jobs available for them. Rao (1990) remarks that activities of manpower planning include: retention of employees, optimum utilization of employees, training, transfer, promotion, dismissal, retirement/wastage etc. According to Yan and Chen (2008), manpower planning objective is to ensure that the demand and supply of manpower in both short and long terms in an industry coincide optimally. Consequently, Gregoriades (2000), remarks that human resources are the most valuable, crucial and volatile resources which organizations utilize to achieve their goals. Failure to put sufficient and qualified people in right positions at the right time and in right quantity can lead to business failure.

The essential factors such as recruitment, wastage and promotion must be looked into when planning the workforce. Since manpower planning depends on the non-predictive behavior of human beings and also uncertain environment, stochastic models which incorporate several factors such as recruitment, wastage, promotion, demotion etc. are often being used in formulating manpower planning models, Kurakhashvili and Pavliashvili (2023). According to Jacquier (2010), manpower planning problems are usually formulated with a single objective function: the total cost of the process over a finite time horizon with the objective of minimize the cost such that the constraints of the problem are met.

As stated in Cole (2005), manpower planners must be able to assess the present and future manpower requirement and come out with recruitment and promotion schedules to fill

vacancies created due to exit of employees. Some factors which could lead to manpower loss in a business organization include resignation, retrenchment, accident, retirement death and dismissal, Ogumeyo (2014). Manpower planning consists of a number of discrete manpower ranks which follow a fixed probability with staff moving from one lower rank to a higher rank, Robin and Harrison (2007). Raghavendra (1991) and Ekoko (2006), developed manpower models which use transition matrix with fixed probability in discrete time to determine staff promotions. Anderson (2001) and Ogumeyo, (2014) remark that recruitment and wastage schedule should be planned such that as employees exit from the organization through retirement or resignation, other employees are being promoted or recruited to fill the vacancies that exist.

The principal objective of manpower planning is to model the migration of staff from one grade to another in discrete time which could be as a result of recruitment, promotion or retirement, (Robbin and Harrison, 2007). Gregoriades (2000) reported that the three factors responsible for staff transition or migration are recruitment, promotion and wastage. According to Vijaya and Jaikar (2019), wastages are considered as “exits of manpower from an organization which cannot be controlled”. Wastages can be voluntary withdrawal of manpower from the existing grade (due to resignation, retrenchment) and involuntary (due to ill health, death, retirement, etc, Vijaya and Jaikar, 2017). Wastage is a factor that decides the stability of a manpower system of any organization, Ogumeyo and Okogun, (2023).

Manpower planning models in literature that have studied wastage are found in Price (1980), Raghavendra (1991), Yadavlli (2002), Olanrewaju *et al.* (2019), Song *et al.* (2023), Ogumeyo and Okogun (2023). A manpower model for retirement (or wastage) of academic staff is developed in Olanrewaju *et al.* (2019). A stochastic retirement model which depends on length of service and tenure of employees is discussed in Agrafiotis (1983).

The shortcoming of the Olanrewaju *et al.* (2019) is that, it considers two phases of retirement: First, ten years to the retirement date and secondly, after the retirement age of sixty-five years. Thus, evaluation of wastage/retirement is being done in piece mill and takes a long time to ascertain the accumulated amount. On the other hand, Ogumeyo and Okogun (2023) model considers wastage as staff that exit the organization per period under consideration either voluntarily or involuntarily with recruitment. In this research, wastage which is also known as exit staff in manpower planning literature is being considered only for staff who got to the highest rank in their careers from a general point of view. Hence, this paper is an extension of the manpower planning models in Olanrewaju *et al.* (2019) and Ogumeyo and Okogun (2023).

## **Materials and Methods**

### **Model Assumptions**

The proposed model has the following assumptions:

- (a) Wastage is limited to only employees at the highest rank of their career.
- (b) Recruitment ( $s'_j$ ) and wastage ( $s_j$ ) costs for each period is fixed.
- (c) Minimum and maximum employees in terms numbers at initial and final period are fixed.

### **Notations**

- $u_j$  = number of staff that retired at the peach of their career in  $j$  period.
- $v_j$  = number of employees recruited to replace them in  $j$
- $s_j$  = savings from each retiree in period  $j$  as a result of their exit.

- $s'_j =$  average cost wages per recruited staff in period  $j$ .
- $b =$  initial number of employees in the firm at the initial period
- $B =$  total number of employees at the final period

**Model Description**

It is assumed in many literatures including Raghavendra (1991), that promotion of employees from a lower rank to a higher rank follow s a fixed matrix probability in a discrete time. This implies the existence of a transition probability matrix in which an employee in a lower grade move to a higher grade he or she gets promoted on the job. In this model,  $(i, j = 1, 2, \dots, k)$  is used to represent various grades in period  $t$ . Using the parameters  $b, B, s_j(t)$  and  $s'_j(t)$  as being defined above in this section, the objective of the model is to find the total sum of the differences between savings from each retired staff in period  $j$  as a result of their exit from the organization and the average cost per recruited staff who fill their vacant positions in period  $j$ .

**Model Formulation**

The objective of the proposed model is to determine the optimal values of  $u_j$  and  $v_j$  so that the revenue earned by the organization is maximized. The parameters  $u_j(t), v_j(t), s_j(t)$  and  $s'_j(t)$  are assumed to be constant with a given time intervals. The process involves finding the sum of the difference between  $su_j$  and  $sv_j$  i.e

$$\sum_{j=1}^n (s_j u_j - s'_j v_j)$$

We can state the objective function as:

$$Max z = \sum_{j=1}^n (s_j u_j - s'_j v_j) \tag{1}$$

The objective function of the manpower planning problem has two sets of employees' constraints which are:

(a) The excess recruitment constraints: This constraint implies that the total number of recruited employees in the first  $i$  periods should not be more than the number employees who exit the organization  $(B - b)$ . That is

$$\sum_{j=1}^i (v_j - u_j) = - \sum_{j=1}^i u_j + \sum_{j=1}^i v_j \leq B - b, \quad i = 1(1)n \tag{2}$$

Where  $(v_j - u_j) > 0$  is the number of employees by which the recruited staff exceeds the number of exit employees in the firm in period  $j$ . Another constraint is understaffing constraints. This states that the number of employees which exit the firm for the first  $(i - 1)$  periods and period  $i$  should not be more than the initial number of employees in the firm. This is essential because every organization need human resources in order to function. This can be expressed mathematically as:

$$\sum_{j=1}^{i-1} (u_j - v_j) + u_i = \sum_{j=1}^i u_j - \sum_{j=1}^{i-1} v_j \leq b, \quad i = 1(1)n \tag{3}$$

Where  $(u_j - v_j) > 0$  denotes the number of excess employees in period  $j$ .

The left hand side of equation (3) is the total number of staff in the manpower system minus exit staff in the first  $(i - 1)$  periods plus the number employees who exited the manpower system in period  $i$ . The second summation in equation (3) for  $i = 1$  disappears.



$$\sum_{i=k}^n x_i - \sum_{i=k+1}^n y_i \geq -s'_k, \quad k = 1(1)n \tag{9}$$

$$x_i, y_i \geq 0, \quad i = 1(1)n \tag{10}$$

If  $k = n$ , the second summation in equation (9) disappears. Hence, we introduce new variables  $X_k$  and  $Y_k$  as follows:

$$X_k = \sum_{i=k}^n x_i, \quad k = 1(1)n \tag{11}$$

$$Y_k = \sum_{i=k}^n y_i, \quad k = 1(1)n \tag{12}$$

We observe that by the dual dynamic programming problem, if  $x_i$  and  $y_i$  are non-negative,  $X_k$  and  $Y_k$  will become non-negative. Although, non-negativity of  $X_k$  and  $Y_k$  will not guaranty the non-negativity of  $x_i$  and  $y_i$  for every  $i$  (i.e.  $x_i \geq 0$  and  $y_i \geq 0, \forall i$ ). Definition of  $X_k$  and  $Y_k$ , shows that non-negativity of  $x_i$  and  $y_i$  will be possible if we augment the dual linear programming problem, stated in terms of  $X_k$  and  $Y_k$  by the constraints:

$$X_k \geq X_{k+1}, \quad k = 1(1) n-1 \tag{13}$$

$$Y_k \geq Y_{k+1}, \quad k = 1(1) n-1 \tag{14}$$

The constraints in equations (13) and (14) disappear whenever  $k = n$  since  $X_{n+1} = Y_{n+1} = 0$ . This is the reason why we have other constraints  $2(n-1)$  in equations (13) and (14).

If we substitute for  $\sum_{i=1}^n x_i$  and  $\sum_{i=1}^n y_i$  in the dual dynamic programming problem in equations (7)-(10) and putting constraints (13) and (14), we will get the dual system of equations in (15).

$$\left. \begin{aligned} \text{Min } W &= (B-b)X_1 + bY_1 \\ \text{s.t.} & \\ X_k &\geq Y_{k+1} - s'_k, \quad k = 1(1)n \\ X_k &\geq X_{k+1}, \quad k = 1(1)n-1 \\ X_k &\geq 0, \quad k = 1(1)n \\ Y_k &\geq X_k + c_k, \quad k = 1(1)n \\ Y_k &\geq Y_{k+1}, \quad k = 1(1)n-1 \\ Y_k &\geq 0, \quad k = 1(1)n \end{aligned} \right\} \tag{15}$$

This is the dual dynamic programming problem which begins with the first period while  $X_1$  and  $Y_1$  denote the least values in the solution region.

The dual dynamic programming problem in system (15) contains different smaller problems which involve the pairs  $(X_k, Y_k), k = 1(1)n$ . For example the smaller problem concerning  $X_1$  and  $Y_1$  is:

$$\left. \begin{array}{l}
 \text{Min } W = (B - b)X_1 + bY_1 \\
 \text{s.t.} \\
 X_1 \geq X_2 - s'_1 \\
 X_1 \geq X_2 \\
 X_1 \geq 0 \\
 Y_1 \geq X_1 + s_1 \\
 Y_1 \geq Y_2 \\
 Y_1 \geq 0
 \end{array} \right\} \quad (16)$$

The  $X_2$  and  $Y_2$  values are determined if we use backward recursive approach of dynamic programming technique. Similarly,  $n$  solutions for the pairs  $(X_k, Y_k)$ ,  $k = 1(1)n$  of the other sub-problems can be obtained by beginning from the last ( $n$ th) pair  $(X_n, Y_n)$  with sub-problem stated as follows:

$$\left. \begin{array}{l}
 \text{Min } W = (B - b)X_n + bY_n \\
 \text{s.t.} \\
 X_n \geq -c'_n \\
 X_n \geq 0 \\
 Y_n \geq X_n + s_n \\
 Y_n \geq 0
 \end{array} \right\} \quad (17)$$

**Backward Recursive Algorithm**

The backward recursive algorithm is as follow:

**Step 1**

Compute for the values of  $(X_k, Y_k)$ ,  $k = 1(1)n$  nth sub-problem in (17)

This implies consideration of equations (18)

$$\left. \begin{array}{l}
 X_n \geq -s'_n \\
 X_n \geq 0
 \end{array} \right\} \Rightarrow \quad (18)$$

The solution region in system (18) is  $X_n \geq 0$ . In order to minimize the value of  $W$  in (17), it implies that  $X_n$  must take the least value in the solution region.

$$\begin{array}{l}
 \text{i.e. } X_n = \max(-s'_n, 0) \\
 X_n = \max(-s'_n, 0) = 0
 \end{array} \quad (19)$$

$$\left. \begin{array}{l}
 Y_n \geq X_n + s_n \\
 Y_n \geq 0
 \end{array} \right\} \text{ solution region is } Y_n \geq (X_n + s_n) \text{ and } Y_n = \max(X_n + s_n, 0) \quad (20)$$

**Step 2**

Compute  $k^{\text{th}}$  problems ( $k = (n - 1), (n - 2), \dots, 3, 2, 1$ ) for their optimal values as follows:

$$\left. \begin{aligned}
 &Min \ W = (B - b)X_k + bY_k \\
 &s.t. \\
 &X_k \geq Y_{k+1} - s'_k \\
 &X_k \geq X_{k+1} \\
 &X_k \geq 0 \\
 &Y_k \geq X_k + s_k \\
 &Y_k \geq Y_{k+1} \\
 &Y_k \geq 0
 \end{aligned} \right\} \tag{21}$$

$X_k$  and  $Y_k$  optimal values are determined as follows:

$$\left. \begin{aligned}
 &X_k \geq Y_{k+1} - s'_k \\
 &X_k \geq X_{k+1} \\
 &X_k \geq 0
 \end{aligned} \right\} \tag{22}$$

The solution region in system (22) is  $X_k \geq \max(Y_{k+1} - s'_k, X_{k+1}, 0)$

That is  $X_k = \max(Y_{k+1} - s'_k, X_{k+1}, 0)$ ,  $k = (n - 1), (n - 2), \dots, 3, 2, 1$

$$\left. \begin{aligned}
 &Y_k \geq X_k + s_k \\
 &Y_k \geq Y_{k+1} \\
 &Y_k \geq 0
 \end{aligned} \right\} \tag{23}$$

The solution region in system (23) is  $Y_k \geq \max(X_k + s_k, Y_{k+1}, 0)$

That is  $Y_k = \max(X_k + s_k, Y_{k+1}, 0)$ ,  $k = (n - 1), (n - 2), \dots, 3, 2, 1$

$$\left. \begin{aligned}
 &X_k = \max(Y_{k+1} - s'_k, X_{k+1}, 0) \\
 &Y_k = \max(X_k + s_k, Y_{k+1}, 0)
 \end{aligned} \right\} k = (n - 1), (n - 2), \dots, 3, 2, 1 \tag{24}$$

From the above analysis, we observe that the values of  $X_k$  in each sub-problem must be evaluated before we can assess the optimal values of  $Y_k$ . We also recall that  $X_k$  and  $Y_k$  form the partial sums of  $s_k$  and  $s'_k$  in system (24).

**Step 3**

To obtain the optimal value of the objective function in equation (7), we have to substitute the values of  $X_1$  and  $Y_1$  into it.

**2.2 Derivation of Salaries of Staff at the Peak of their Career**

The effect of restricting wastage to only staff who reach the peak of their career will produce equations (25) and (26) by induction from the step 2 of the algorithm.

$$X_1 = s_n - s'_{n-1} + s_{n-1} - s'_{n-2} + s_{n-2} - s'_{n-3} + \dots + s_2 - s'_1 = \sum_{j=2}^n s_j - \sum_{j=1}^{n-1} s'_j \tag{25}$$

$$Y_1 = s_n - s'_{n-1} + s_{n-1} - s'_{n-2} + s_{n-2} - s'_{n-3} + \dots - s'_1 + s_1 = \sum_{j=1}^n s_j - \sum_{j=1}^{n-1} s'_j \tag{26}$$

In order to get the solution of the primal dynamic programming problem we have to put  $X_1$  and  $Y_1$  values into the objective function  $[(B - b)X_1 + bY_1]$  of the dual dynamic programming problem.

**Numerical Example**

The mean yearly salary ( $s_j$ ) of senior staff that got to the peak of their career for the year 2012 to 2024 and the mean yearly salary  $s'$  of recruited staff to fill vacancies created due to wastage for the year 2012 to 2024.

**Table 1:** Average monthly salary for senior and newly recruited staff.

Year	2001 (1)	2002 (2)	2003 (3)	2004 (4)	2005 (5)	2006 (6)	2007 (7)	2008 (8)	2009 (9)	2010 (10)	2011 (11)	2012 (12)
$s_j$	127104	131223	135211	140421	142995	159213	162372	179084	180512	182750	184152	187289
$s'_j$	74372	76911	80625	83179	88370	91372	94246	96960	99124	102629	113893	118413

In 2012, XYZ higher institution had 230 senior staff. Determine the optimal annual number of staff on wastage and recruitment that will maximize total accruable revenue to the institution in the next 12 years (i.e. by the year 2036) when the number staff at the peak of their career is expected to reach 600 based on the current salary trend stated in Table 1.

**Solution**

A close look at the wastage ( $s_j$ ) and recruitment ( $s'_j$ ) costs in Table 1 reveals that the senior staff problem satisfies the condition of the manpower planning problem of dynamic programming model discussed in section 3 where  $s'_j > 0$ ,  $s_j > s'_j$  and  $s_{j+1} > s'_j \forall j$ . That is the wastage costs of staff at the peak of their career are consistently higher than the corresponding recruitment costs. By applying equation (25) and (26) to the given data in Table 1, we have

$$X_1 = \sum_{j=2}^{12} s_j - \sum_{j=1}^{11} s'_j = 866604 \text{ and } Y_1 = \sum_{j=1}^{12} s_j - \sum_{j=1}^{11} s'_j = 792232$$

The objective function value is given as:

$$z = (B - b)X_1 + bY_1 = 370(866604) + 230(792232)$$

~~= ₦502856840~~ which is the total amount accruable to the firm based on the recruitment/wastage policy **cost** of staff that reached the peak of their career of the manpower planning problem before retirement.

$$\text{But } (B - b)X_1 + bY_1 = \sum_{j=1}^{12} (s_j x_j - s'_j y_j) \tag{27}$$

$$\begin{aligned} \text{LHS: } & 370(s_2 + s_3 + \dots + s_{12}) - 370(s'_1 + s'_2 + \dots + s'_{11}) \\ & + 230(s_1 + s_2 + \dots + s_{12}) - 230(s'_1 + s'_2 + \dots + s'_{11}) \\ & = 230s_1 + 600s_2 + \dots + 600s_{12} - 600s'_1 - 600s'_2 - \dots - 600s'_{11} \end{aligned}$$

$$\text{RHS: } x_1s_1 + x_2s_2 + \dots + x_{12}s_{12} - y_1s'_1 - y_2s'_2 - \dots - y_{11}s'_{11}$$

$$\begin{aligned} \Rightarrow \quad & x_1 = 230, \quad x_2 = x_3 = x_4 = \dots = x_{12} = 600 \\ & y_1 = y_2 = \dots = y_{12} = 600 \text{ and } z = \text{₦}502856840 \\ & \text{as optimal solution.} \end{aligned}$$

**Results and Discussion**

The primal objective function is  $(B - b)X_1 + bY_1$  and the dual objective function is  $\sum_{j=1}^{12} (s_j x_j - s'_j y_j)$ . From the result analysis of the numerical illustration in Section 4, the objective functions of the dual and the primal are equal. That is



$$(B-b)X_1 + bY_1 = \sum_{j=1}^{12} (s_j x_j - s'_j y_j) = 502856840.$$

Which conform to the duality theorem in linear programming problems.

### Conclusion

The shortcoming of the Olanrewaju *et al.* (2019) is that, it considers two phases of retirement: First, ten years to the retirement date and secondly, after the retirement age of sixty-five years. Thus, evaluation of wastage/retirement is being done in piece mill and takes a long time to ascertain the accumulated wastage cost. On the other hand, Ogumeyo and Okogun (2023) model considers wastage as staff that exit the organization per period under consideration either voluntarily or involuntarily accompany by recruitment. In this paper, we have examined the financial implication for having senior staff who attain the highest rank in their career before exiting from their job. This is very much like the existing models developed in Olanrewaju *et al.* (2019) and Ogumeyo and Okogun (2023). A backward recursive dynamic programming model to determine wastage cost accruable to the organization as result of their exit from service has been proposed and illustrated with a numerical example. It has been observed that such staff earns higher salaries than other staff.

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