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Developing Exp-FIGARCH Hybrid Models for Time Series Modelling

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Abstract
In this paper, we introduced a new hybrid model namely Exponential Autoregressive-Fractional Integrated Generalized Autoregressive Conditional Heteroscedasticity (ExpAR-FIGARCH) model and study financial data. The Daily Nigeria All Share Stock Index that exhibit nonlinear, volatility and long memory effect were analyzed in the study. The existing ExpAR-Generalized Autoregressive Conditional Heteroscedasticity (ExpAR-GARCH) model were estimated and compared with the proposed ExpAR-FIGARCHmodel. Results showed that the new hybrid model is better based on efficient parameters, serial correlation analysis and forecast measures of accuracy. Therefore, as a conclusion, the current study indicates that the ExpAR-FIGARCHmodel performed better compared to the ExpAR-GARCHhybrid model. Therefore, the ExpAR-FIGARCHmodel is a better option for modeling nonlinear, volatility and long memory characteristics of time series. Future study should focus on the application of the developed hybrid ExpAR-FIGARCHmodel using health, meteorological and economic data.

Keywords: Long Memory, Nonlinear, ExpAR-FIGARCH model, Serial Correlation, Volatility.

Introduction
A time series is said to be an exponential pattern when each successive value increases (or decreases) by the similar value. Ozaki (1980) introduced a mean model known as the Exponential Autoregressive (ExpAR) models to study nonlinear and exponential patterns. As some times series are known to be nonlinear and volatile, estimating the mean model alone for example ExAR, would lead to poor modeling. This is because the volatility and other important components of time series that regularly dwelled in many time series are not captured and eliminated. In view of this, Katsiampa. (2014) introduced the ExpAR-Autoregressive Conditional Heteroscedasticity (ExpAR-ARCH) and ExpAR-Generalized ARCH (ExpAR-GARCH) models to study these two characteristics of time series concurrently.

Numerous hybrid time series models were proposed and developed. For example, Weiss (1984) was the first to study on hybrid modeling and Autoregressive Moving Average-Autoregressive Conditional Heteroscedasticity(ARMA-ARCH) hybrid model Similarly, Ballie

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et al.,(1996) introduced the hybrid model called Autoregressive Fractional Integral Moving Average-Generalized Autoregressive Conditional Heteroscedasticity (ARFIMA-GARCH) model to study the LM and variance in the inflation of USA concurrently. The square and absolute returns are found to exhibit slowly decaying autocorrelations, therefore, ARFIMA-FIGARCH and ARFIMA-Fractional Integrated Asymmetric Power Autoregressive Conditional Heteroscedasticity (ARFIMA-FIAPARCH) models were suggested by Baillie *et al.,*(1996). Also, Belkhouja *et al.* (2008) proposed the extension of the ARFIMA into the ARFIMA-GARCH models with a time varying GARCH specification. Cheung and Chung (2009) developed the ARFIMA model with Normal Mixture GARCH (NM-GARCH) process, called the ARFIMA-NM-GARCH model to study the series that exhibited LM and volatility.

Similarly, Fofana *et al.*(2014) formed a hybrid specification called the Regime Switching ARFIMA-GARCH (RS-ARFIMA-GARCH) models to account for structural change in a series that exhibited both LM and volatility. Ambach and Ambach (2018) have introduced periodic known as ARFIMA-P-GARCH process. It is important to note that the hybrid models of ARFIMA discussed in the literature were designed to study time series with fractional differencing value less than unity ($0 < d < 1$). Jibrin (2019) developed the Autoregressive Fractional Unit Root Integral Moving Average-GARCH (ARFURIMA-GARCH) model and Boubaker *et al.* (2020) developed and introduced the ARFIMA-Wavelet local Linear Wavelet neural Network (ARFIMA-WLLNN) and ARFIMA-Hyperbolic Asymmetric Power Autoregressive Conditional Heteroscedasticity (ARFIMA-HYAPARCH) models. Again, Hybrid ARFIMA–Artificial Neural Network(ARFIMA-ANN) model was developed by Al-Gounmeein and Ismail (2021).

There are some financial indices that are known to be nonlinear, volatile and long memory (Rahman and Jibrin (2018), Benrhmach *et al.* (2020), Ojeda *et al.,* (2021), de Oliveira *et al.,* (2022), Jibrin *et al.,* (2022), and Jiang *et al.,* (2023)). The ExpAR-ARCH, ExpAR-GARCH and other hybrid models introduced in the time series literature lack the strength to handle these three identified time series characteristics at the same time. Having said this, the aim of the current study is to develop the ExpAR-Fractional Integrated GARCH (ExpAR-FIGARCH) model for the study of nonlinear, volatility and long memory in time series. This aim would be achieved based on the following objectives: to develop the ExAR-FIGARCH hybrid model, estimate the parameters of the developed hybrid model and compare with the existing ExAR-GARCH hybrid model. Some of the proposed hybrid model properties include white noise residuals, efficient parameters and minimum forecast accuracy measures.

Methodology

The ExpAR model of order p denoted by ExpAR(p) can be defined as:

$$Y_t = \left(\phi_j + \eta_j e^{-\lambda Y_{t-1}^2}\right) Y_{t-j} + \varepsilon_t. \qquad (1)$$

Many time series exhibits trend, volatilityand long memory effect (Liang *et al.,* (2022) and Fameliti and Skintzi(2022)). Besides handling trend, it is clear that the Exp-AR model lack requirements of handling time series with volatilityand long memory characteristics.

The ProposedExpAR-FIGARCH Model

The current study assumed that the components of the model in (1)

a. Cannot capture the volatility and long memory in a financial or any time series$\{Y_t\}$, $t = 1, \dots, T$.
b. Have residuals $\{\varepsilon_t\}$, $t = 1, \dots, T$that are serially correlated and heteroscedastic.

c. Lack the requirements to account for the high degree of autocorrelation that might existsin volatility of a time series as observed by Gurgul *et al.,* (2021), Gonzaga (2022), Gil-Alana *et al.,* (2023) and Aliyu *et al.,* (2023) for similar mean models.

Having observed this, the current study wants to introduce the ExpAR-FIGARCH hybrid model. The ExpAR(p)-FIGARCH(u,d_v,z) is for the study of the nonlinear, volatilityand long memory in time series. The ε_t in equation (1) is a stochastic process that can be expressed as:

$$\varepsilon_t = m_t\sigma_t, \qquad (2)$$

where $E(m_t) = 0$, $Var(m_t) = 1$ and σ_t is positive and changes with respect to time, t. This implies that the process, $\{m_t\}$, is assumed to be serially uncorrelated expressed as:

$$m_t \sim iid(0,1) \qquad (3)$$

Thus, the conditional variance σ_t is non-stationary process that changes over time. In view of this, Baillie *et al.,* (1996) introduced the FIGARCH(u,d_v,z) model to study σ_t as:

$$\sigma_t^2 = \omega[1-\psi(B)]^{-1} + \{1-[1-\psi(B)]^{-1}\alpha(B)(1-B)^{d_v}\}\varepsilon_t^2, \qquad (4)$$

Now, to develop the hybrid ExpAR-FIGARCH model, from equation (4), consider the function

$$f\big(\psi(B)\big) = [1-\psi(B)]^{-1}, \qquad (5)$$

The Taylor series expansion for equation (5) is

$$f\big(\psi(B)\big) = [1-\psi(B)]^{-1} = 1 + \psi(B) + \big(\psi(B)\big)^2 + \cdots \qquad (6)$$

Let consider part of the expansion in (6) $[1-\psi(B)]^{-1} = 1$ and substituting in equation (4). Then, we can have

$$\sigma_t^2 = \omega + \{1-\alpha(B)(1-B)^{d_v}\}\varepsilon_t^2. \qquad (7)$$

However, $\alpha(B) = 1-\alpha(B)-\beta(B)$(Lopes, 2008). Therefore, the FIGARCH(u,d_v,z) can be expressed as:

$$\sigma_t^2 = \omega + \big\{\big(\alpha(B)+\beta(B)\big)(1-B)^{d_v}\big\}\varepsilon_t^2. \qquad (8)$$

Similarly, equation (8) can be expressed as:

$$\sigma_t = \big[\omega + \big\{\big(\alpha(B)+\beta(B)\big)(1-B)^{d_v}\big\}\varepsilon_t^2\big]^{\frac{1}{2}}. \qquad (9)$$

Again, substituting equation (9) in (2), the following is obtained

$$\varepsilon_t = m_t\big[\omega + \big\{\big(\alpha(B)+\beta(B)\big)(1-B)^{d_v}\big\}\varepsilon_t^2\big]^{\frac{1}{2}}. \qquad (10)$$

Finally, let $\varepsilon_t = m_t\big[\omega + \big\{\big(\alpha(B)+\beta(B)\big)(1-B)^{d_v}\big\}\varepsilon_t^2\big]^{\frac{1}{2}}$in equation (1) so that the ExpAR(p)-FIGARCH(u,d,z) can be represented as:

$$Y_t = \big(\phi_j + \eta_j e^{-\lambda Y_{t-1}^2}\big)Y_{t-j} + m_t\big[\omega + \big\{\big(\alpha(B)+\beta(B)\big)(1-B)^{d_v}\big\}\varepsilon_t^2\big]^{\frac{1}{2}}. \qquad (11)$$

where$Y_t$is dependent variable,ϕ_j, η_j *(for* $j = 1, \dots, p$*)*,$\lambda$and $\omega$are unknown parameters to be estimated from Y_t. $m_t$is the error termsthat are independent and identically distributed random variables, $p$is the order of the model and$\lambda$is defined as the scaling parameter.The$\alpha(B) = \alpha_1 B^1 + \cdots, \alpha_r B^r$ and$\beta(B) = \beta_1 B^1 + \cdots, \beta_s B^s$are called characteristics polynomial and all their roots are expected to lie in the unit root circle while Lis the lag

operator. The $c > 0, \phi_j \geq 0$ for $j = 1, \dots, p$, $\alpha_k \geq 0$ for $k = 1, \dots, r$, $\beta_l \geq 0$ for $l = 1, \dots, z$, and $d_v$are parameters of the model to be estimated.
In addition, when $p = u = v = 1$, the ExpAR(*1*)-FIGARCH(*1*,d_v,*1*) can be shown to be

$$Y_t = \left(\phi_1 + \eta_1 e^{-\lambda Y_{t-1}^2}\right)Y_{t-1} + m_t[\omega + \{(\alpha_1 + \beta_1)(1-B)^{d_v}\}\varepsilon_t^2]^{\frac{1}{2}}. \quad (12)$$

Diagnostic Tests

After estimation of the proposed ExAR-FIGARCH or any time series models, it will be paramount to test for the adequacy of the model before considering further analysis such as forecasting. It is at this stagethat the serial correlation analysis of the residuals would be carried out. The Autoregressive Conditional Heteroscedastic-Lagrange Multiplier (ARCH-LM), Portmanteau and normality are some of the diagnostic tests employed in the literature of time series.

ARCH-LM Test

The ARCH-LM test of Ljung and Box (1978) for testing heterogeneity of residuals is defined as:

$$Q = M(M+2)\sum_{i=1}^{M} \frac{\rho_i}{(M-i)}, \quad (13)$$

where Q is the estimated test statistic value, M sample size, and ρ_i is the sample degree of relationship between residuals.

Jarque-Bera Test

Jarque and Bera (1987) proposed a test for non-normality of observations and the test statistic is given as:

$$JB = \frac{T}{6}[T^{-1}\sum_{t=1}^{T}(\hat{\varepsilon}_t^2)^3]^T + \frac{T}{24}[T^{-1}\sum_{t=1}^{T}(\hat{\varepsilon}_t^2)^4 - 3]^T. \quad (14)$$

The test statistic has an asymptotic χ^2 distribution and the null hypotheses is rejected if the p-value is less than significant level α.

Measures of Forecast Accuracy

In this study, Mean Square Error (MSE) and Root Mean Square Error (RMSE) are the forecasting performance methods considered.

Mean Square Error

The MSE is obtained by calculatingthe difference among actual and fitted observations that are both squared and averaged over the sample. It is defined as:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(Y_t - \hat{Y}_t\right)^2. \quad (15)$$

Root Mean Square Error

The RMSE is similar to the MSE but the square root of the MSE is considered as expressed below in (16).

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Y_t - \hat{Y}_t\right)^2}. \quad (16)$$

Where Y_t is the actual value and$\hat{Y}_t$ is the forecasts output and n is the sample size.

Data and Analysis

Daily index for Nigeria All Share Stock Index (DNGNASSI) is used to determine the best of the class of models considered. The considered models are ExAR, ExAR-GARCH and the proposed hybrid ExAR-FIGARCH model.

Table 1: Descriptive Statistics for Daily MNWNWS and DNGNASSI

Statistics	DNGNASSI
Minimum	8111.00
Maximum	66371.19
Mean	28292.31
Std. Dev.	11541.23
Skewness	0.72
Kurtosis	0.60
Jarque-Bera Test	476.21(0.0000)

Table 1 presents the descriptive statistics and normality test for the DNGNASSI index. The mean and standard deviation for DNGNASSI is 28292.31 and 11541.23respectively. The kurtosis, which measures the risk of investment in DNGNASSIis 0.6 indicating high positive risk investor should take in the Nigeria stock exchange market. The skewness and Jarque-Bera statistic for DNGNASSI indicate non-normality for the series.

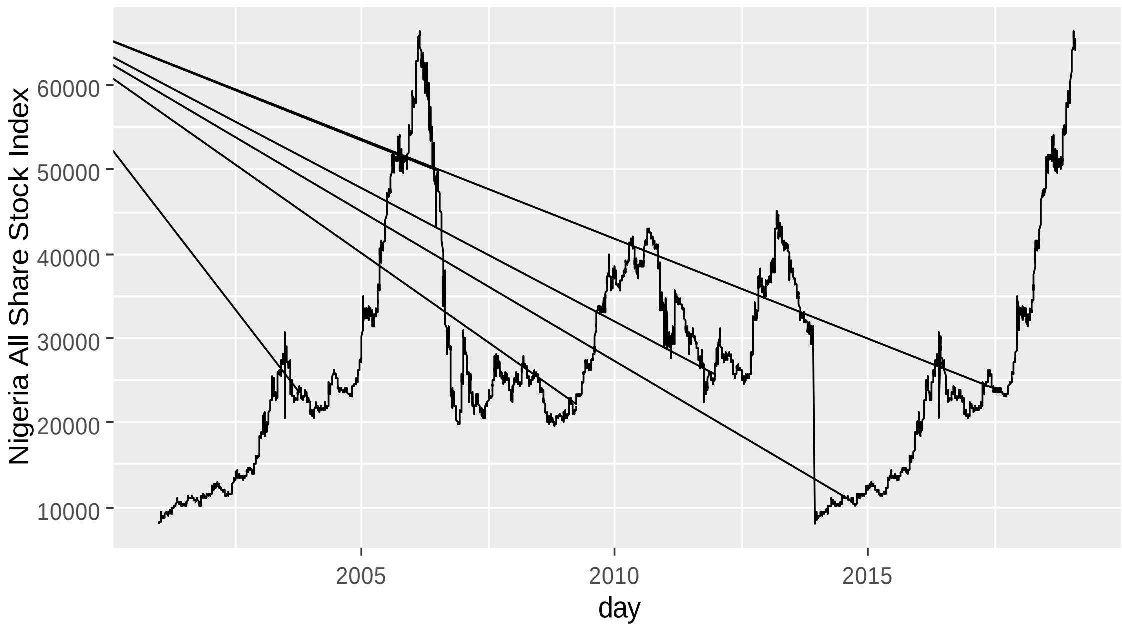

Figure 1: Plot of Daily Time Series of Daily Nigeria AllShare Stock Index.

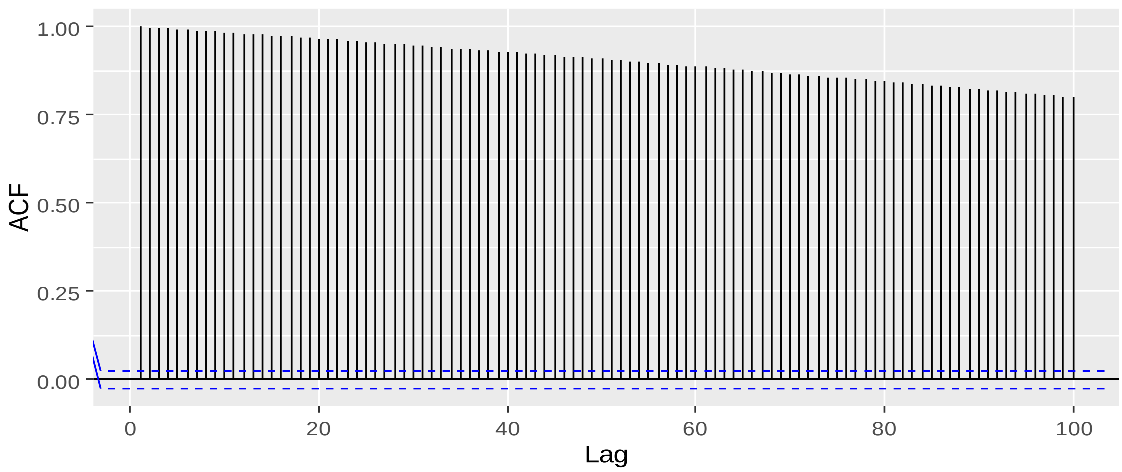

Figure 2: Plot of ACF of Daily Nigeria All Share Stock Index.

The time series plot of the Daily Nigeria All Share Stock Index is shown in figure 1. The time series graphs exhibit unstable trend behavior.The Autocorrelation Function (ACF) of the DNGNASSI is shown in figure 2. The autocorrelations indicate a slow decay which is evidence of long memory process. Therefore, on the average, the DNGNASSI are not stationary.Having said this, the trend behavior observed in DNGNASSI series indicates the ExAR is a candidate model to study time series with this type of attributes.

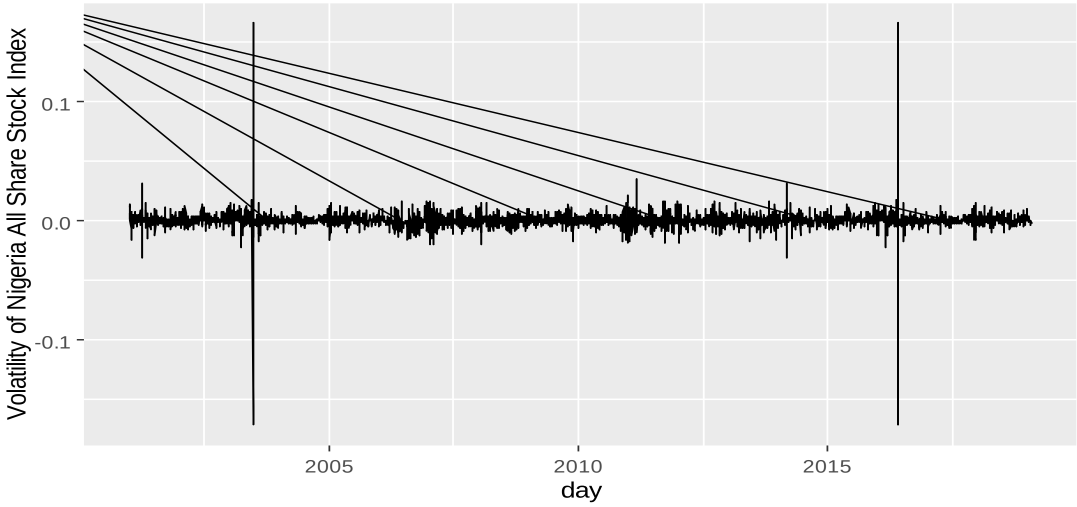

Figure 3: Plot of Daily Volatility of the Daily Nigeria All Share Stock Index.

The volatility plot of the DNGNASSI is shown in figure 3. The plot indicatesevidence of volatility clustering indicating that the series is volatile. The observed volatility shows that volatility models such as GARCH and FIGARCH are other candidate models to study the DNGNASSI series.

ExAR(p)Model Estimation and Diagnostic tests

The ExAR modeling based on model estimation and diagnostic analysis are carried out in this section. Theestimation of ExAR(1) and ExAR(2) modelfor the DNGNASSI and serial correlation analysis results are displayed in Table 2 and 3.All the parameters in the ExAR models estimated using the DNGNASSI are insignificant due to large standard errors of the parameters. The serial correlation analysis results show that the residuals of the estimated mean models, ExAR(1) and ExAR(2) are heteroscedastic and non-normal. This is because the p-values are less than the 0.01 significance level.This suggests the two models are inadequate to be considered for studying the DNGNASSIseries as they failed to eliminate or reduce the noise signals.

Table 2: ExAR(1) Models Estimation and Diagnostic Analysis

ExAR(1) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
C	32560.477	193.113	168.61	0.0000
λ_1	0.375	0.015	25.00	0.0000
ϕ_1	-32010.263	725.428	-44.13	0.0000
ARCH-LM Test=4710.2(0.0000) and Jarque-Bera Test = 2694.9(0.0000)				

Table3: ExAR(2) Models Estimation and Diagnostic Analysis

ExAR(2) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
C	32540	2.649e+10	0.000	0.9999
λ_1	0.7441	13.28	0.056	0.9550
λ_2	0.7500	13.47	0.056	0.9560
ϕ_1	-5812000	2.649e+10	0.000	0.9999
ϕ_2	5794000	189.1	30635.664	0.0000
ARCH-LM Test=4709.6(0.0000) and Jarque-Bera Test = 2523.5(0.0000)				

Statistical facts that explainedthe heterogeneity and non-normality in model residuals are high noise signals,size of volatility, outliersand volatility clustering as seen in Figure 3. The GARCH and FIGARCH known as volatility and long memory volatility models respectively could be joined with the ExAR models to form hybrid model.This could helpin eliminating the observed unwanted signals. It could also assists in improving the fitting of the ExAR model to the DNGNASSI data. Having said this, an analyses by considering hybrid model of ExAR-GARCHand ExAR-FIGARCH models would be carried out and discussed in the next section.

Hybrid ExAR-GARCH and ExAR-FIGARCH Modeling

This section discusses the estimation and diagnostic tests of hybrid model; ExAR-GARCH and ExAR-FIGARCH using the DNGNASI. Before estimating the hybrid models that involved the long memory volatility model,FIGARCH, it is important to investigate the presence of long memory in the volatility of the DNGNASSI.

Table 4: Long Memory Parameter Estimation

Data	Volatility
DNGNASSI	0.2507

The long memory in the volatility of DNGNASSI was further estimated and is displayed in Table 4. The Geweke and Porter-Hudak (GPH)long memory estimation method produced the fractional differencing value to be 0.2507 for the volatility of DNGNASSI. Thisvalue confirmed the long memory attributes in the original series and the volatility of DNGNASSI and indicating the suitability of considering the FIGARCH model. The results of the parameters estimation of ExAR-GARCH models are shown in Table 5 and 6 and ExAR-FIGARCH in Table 7 and 8.The hybrid models estimated are assumed to be normal and Student-t-distribution because of the heteroscedasticity of the residuals of the ExAR models.

Table5: ExAR(1)-GARCH(1,1) Models Estimation

ExAR(1) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
c	32560.477	193.113	168.61	0.0000
λ_1	0.375	0.015	25.00	0.0000
ϕ_1	-32010.263	725.428	-44.13	0.0000
GARCH(1,1) Components with m_t assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	768.7024	0.0278	27634.7115	0.0000
α_1	0.0524	0.0002	228.6802	0.0000
β_1	0.8817	0.0009	937.7579	0.0000
GARCH(1,1) Components with m_t assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	786.4462	0.0280	28065.2225	0.0000
α_1	0.2338	0.0005	483.6179	0.0000
β_1	0.7652	0.0009	870.3150	0.0000
v	4.1752	0.0020	2044.9993	0.0000

Table6: ExAR(2)-GARCH(1,1) Models Estimation

ExAR(2) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
c	32540	2.649e+10	0.000	0.9999
λ_1	0.7441	13.28	0.056	0.9550
λ_2	0.7500	13.47	0.056	0.9560
ϕ_1	-5812000	2.649e+10	0.000	0.9999
ϕ_2	5794000	189.1	30635.664	0.0000
GARCH(1,1) Components with m_t assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	768.7025	0.0278	27664.7823	0.0000
α_1	0.0524	0.0002	228.6083	0.0000
β_1	0.8817	0.0009	938.3341	0.0000
GARCH(1,1) Components with m_t assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	786.4462	0.0280	28066.0349	0.0000
α_1	0.2338	0.0005	481.9565	0.0000
β_1	0.7652	0.0009	877.2057	0.0000
v	4.1752	0.0020	2052.8112	0.0000

Table 7:ExAR(1)-FIGARCH(1,1) Models Estimation

ExAR(1) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
c	32560.477	193.113	168.61	0.0000
λ_1	0.375	0.015	25.00	0.0000
ϕ_1	-32010.263	725.428	-44.13	0.0000
FIGARCH(1,1) Components with m_t assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	664.60	0.0259	25698.195	0.0000
α_1	0.0001	0.0000	11.430	0.0000
β_1	0.8071	0.0009	898.247	0.0000
d_v	0.9592	0.0010	979.477	0.0000
FIGARCH(1,1) Components with m_t assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	476.6062	0.0219	21791.0217	0.0000
α_1	0.1618	0.0004	400.3620	0.0000
β_1	0.8234	0.0009	909.1440	0.0000
d_v	0.9782	0.0010	988.8625	0.0000
v	4.1839	0.0021	2040.1837	0.0000

Table8: ExAR(2)-FIGARCH(1,1) Models Estimation

ExAR(2) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
c	32540	2.649e+10	0.000	0.9999
λ_1	0.7441	13.28	0.056	0.9550
λ_2	0.7500	13.47	0.056	0.9560
ϕ_1	-5812000	2.649e+10	0.000	0.9999
ϕ_2	5794000	189.1	30635.664	0.0000
FIGARCH(1,1) Components with m_t assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	664.600	0.0257	25820.6895	0.0000
α_1	0.0001	0.0000	11.435	0.0000
β_1	0.8071	0.0009	896.701	0.0000
d_v	0.9592	0.0010	980.005	0.0000
FIGARCH(1,1) Components with m_t assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(>|t|)
ω	476.6062	0.0218	21866.4207	0.0000
α_1	0.1618	0.0004	401.7389	0.0000
β_1	0.8234	0.0009	906.3555	0.0000
d_v	0.9782	0.0010	986.4570	0.0000
v	4.1839	0.0020	2048.7065	0.0000

All parameters of the hybrid models specifically the ExAR-FIGARCH that were assumed to be Student-t-distribution comes with smaller standard errors than parameters of the ExAR models. Thisindicates the goodness-of-fit and adequacy of the proposed hybrid ExAR-FIGARCH model.

The Diagnostic and Foreacst Accuracy Measures of Hybrid Models

Table 9: Diagnostic Analysis of the Hybrid Models

Candidate Models	Residuals as Normal Distribution		Residuals as Student-t- Distribution	
	ARCH-LM Test	Jarque-Bera–Test	ARCH-LM Test	Jarque-Bera-Test
ExAR(1)-GARCH(1,1)	1.4629(0.4029)	4281921(0.0000)	3.5124(0.1029)	4337831(0.0000)
ExAR(2)-GARCH(1,1)	1.7731(0.2514)	4852370(0.0000)	1.8179(0.2271)	3389123(0.0000)
ExAR(1)-FIGARCH(1,1)	1.9413(0.2001)	5952411(0.0000)	**0.1985(0.9055)**	1071651(0.0000)
ExAR(2)-FIGARCH(1,1)	1.5179(0.3251)	6037621(0.0000)	1.1321(0.2142)	3528192(0.0000)

The ARCH-LM test which is a serial correlation analysis investigates the homoscedasticity of residuals of a time series models. The p-values of the ARCH-LM test of the hybrid models in Table 9 are larger than the p-values of the ExAR mean models as shown in Table 2 and 3.This results show evidence of improvement in model fitting as a results of introducing the FIGARCH model to the ExAR models.Also, the residuals of the ExAR(1)-FIGARCH(1,1) is homocedastic and therefore the mlodel could be considered in producing relibale forecasts. However, Results of the Jarque-Bera test show evidence of non-normality in all the mean and hybrid model residuals due to zero p-values of Jarque-Bera test statistics.

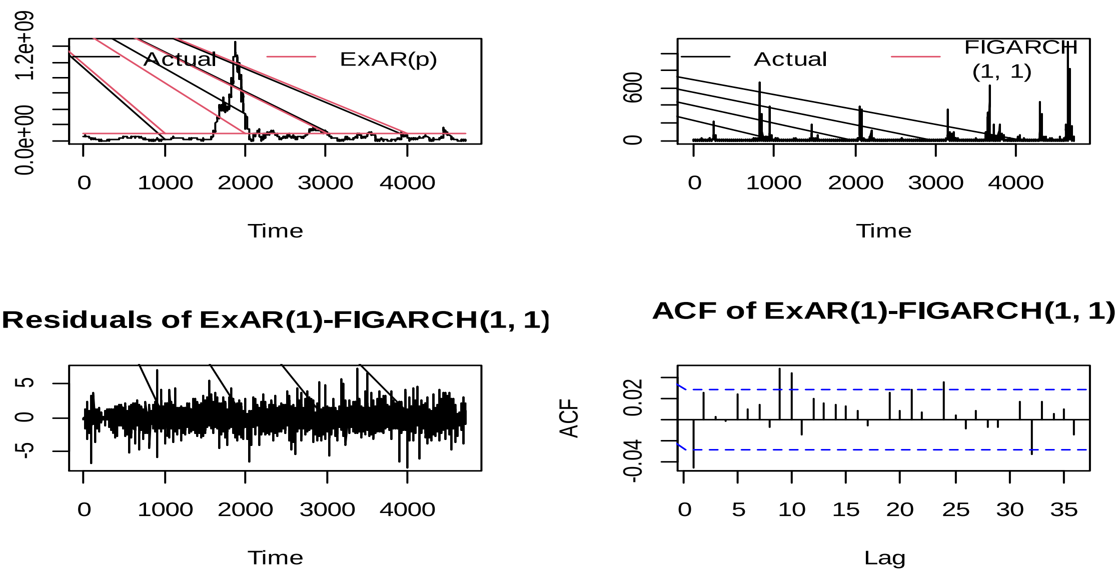

Figure 4: Diagnostic Plots of ExAR(1)-FIGARCH(1,1) Fitted to DNGNASSI.

The diagnostic plots and in-sample forecasts for the ExAR(1)-FIGARCH(1,1) are shown in Figure 4. The in sample forecast plot (right top panel) show that the actual volatility coincide with the fitted or observed volatilities for the ExAR(1)-FIGARCH(1,1) model.

Table 10: Forecast Accuracy Measures for Hybrid Models

Candidate Models	Residuals as Normal Distribution			Residuals as Student-t-Distribution		
	MSE	RMSE	MAE	MSE	RMSE	MAE
ExAR(1)-GARCH(1,1)	11.8613	3.4440	1.8827	5.9879	2.4470	0.9887
ExAR(2)-GARCH(1,1)	13.4134	3.6624	2.7162	7.7525	2.7843	1.2426
ExAR(1)-FIGARCH(1,1)	6.9602	2.6382	1.6901	**1.3418**	**1.1583**	**0.1461**
ExAR(2)-FIGARCH(1,1)	8.6651	2.9437	1.9042	5.7241	2.3925	0.7479

The MSE, RMSE and Mean Absolute Error (MAE) are used in the literature to evaluate performance (Hyndman and Athanasopolous(2013) and Papailias and Dias (2015)). The forecast accuracy measures results of DNGNASSI using the hybrid ExAR-GARCH and ExAR-FIGARCH models as shown in Table 10. Compared to the hybrid ExAR-GARCH models, the ExAR-FIGARCH model produces the minimum and better forecast performancesbased on the assumptions that their residuals are Student-t-distributed. Again, the ExAR(1)-FIGARCH(1,1) produces the same accuracy measures; MSE, RMSEand MAE as 1.3418, 1.1583 and 0.1461 respectively.In view of this, the ExAR(1)-FIGARCH(1,1) is chosen as the best model. Consequently, the chosen model which is equivalent to the hybrid ExpAR(1)-FIGARCH($1,d_v,1$) model in equation (12) has estimated parameters $\phi_1 = 32560.48$,$\eta_1 = -32010.26$,$\lambda = 0.38$,$\omega = 476.61$,$\alpha_1 = 0.16$,$\beta_1 = 0.82$and$d_v = 0.98$as shown in Table 7. Therefore, the fitted model is

$$Y_t = (32560.48 - 32010.26e^{-0.38})Y_{t-1} + m_t[476.61 + \{(0.16 + 0.82)(1 - B)^{0.98}\}\varepsilon_t^2]^{\frac{1}{2}}.$$

Conclusion

This paper presented a new hybrid ExAR-FIGARCH model for the study of nonlinearity and long memory in variance components. The models calibration was achieved using DNGNASSI. In addition, the proposed hybrid model is compared with existing ExAR-GARCH competing method. The results from the comparative analysis using parameters standard errors, serial correlation analysis, and forecast accuracy measures revealed that the hybrid ExAR-FIGARCH model is the best. The diagnostic confirms that the residuals from the ExAR-FIGARCH model can be regarded as serially uncorrelated and homoscedastic.

This study contributes to the literature on nonlinear, volatility and long memory time series modeling. Since many financial and economic data are non-stationary, if the series show long memory characteristics, they are predictable. In this study, it can be stated that a novel and efficient hybrid model was developed for eliminating nonlinear, volatility and long memory in time series. Specifically, the proposed hybrid ExAR-FIGARCH model can be used to fit the data with nonlinear, volatile and long memory characteristics. In addition, this hybrid model contributes to the study of the nonlinear, volatilities and long memory in financial time series. This study can be a reference to other research on the nonlinear, volatilities and long memory modeling. Future works may focus on the application of the developed hybrid ExAR-FIGARCH model using major health, meteorological and economic data. The non-normality was observed in both the mean and hybrid model residuals and this could be due to the high impact of the noise signals, volatility clustering and asymmetric effects that dwelled in many time series data. Future research should consider developing hybrid ExAR-Asymmetric Power Autoregressive Conditional Heteroscedastic (ExAR-APARCH) or ExAR-Fractional Integrated Asymmetric Power Autoregressive Conditional Heteroscedastic (ExAR-FIAPARCH) models to account for these major unwanted volatility facts.

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