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Fuzzy Linear Regression Modelling of Federal Government Budget Performance

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Abstract

Every government spends huge resources to see that its budget achieves the desired and stated objectives. This can never be seen and achieved till the budget performance over the period in question is analyzed and evaluated. In this paper, a fuzzy linear regression model capable of predicting the budget performance of Nigeria, assuming that residuals are due to system fuzziness is fitted with a threshold value of zero to a data on budget variables published by the Central Bank of Nigeria. The aim is to appropriately determine and assess the worst and best budget performance that may be achieved based on the socioeconomic variables. The empirical results reveal that on average, the budget performance lies between ₦160.645BN (worst) and ₦412.568BN (best) respectively for the considered period. In addition, a unit change in the transfers, social and community services, and administration will lead to a corresponding increase in the budget performance based on the following ranges: $(\mathbf{5.0792, 6.0224})$, $(\mathbf{0.0966, 0.2616})$, $(\mathbf{0.2366, 0.2366})$,*and* $(\mathbf{0.6386, 0.6386})$ *respectively.*

Keywords – Fuzzy linear regression, Budget performance, Microeconomic indicator, Socio-economic variables.

INTRODUCTION

Nigeria with its deep economic, cultural and religious diversity is extremely relevant for the government at the centre to allocate its scarce resources efficiently and effectively. Various responsibilities such as upgrading defence capabilities, providing proper educational facilities, mitigating regional disparities and clashes, reduction and eradication of poverty, eradication of insurgency, kidnapping etc. are expected by the government. To shoulder these responsibilities to its fullest, government must prepare a budget as well as evaluate the performance of the budget at the end of the period in question. In nutshell, government budget is a deliberate action plan by the government ahead of time in establishing its anticipated expenses and revenues over a period of one year.

According to Iheanacho (2016), budget is an estimate of expenditures and revenues of the government over a specific period usually a year. It is also viewed as an annual financial statement which outlines the estimated and expected government revenues for the

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forthcoming fiscal year. The structure of Nigerian Public expenditure can broadly be categorized into capital and recurrent expenditure. The recurrent expenditure are government expenses on administration such as wages, salaries, interest on loans, maintenance etc. whereas expenses on capital projects like roads, airports, education, telecommunication, electricity generation etc. are referred to as capital expenditure (Muritala & Taiwo, 2011). It is a general view that public expenditure either recurrent or capital expenditure can be growth enhancing. Therefore, government expenditure should have multiplier effects on the economy which should also be felt by society or by all and sundry.

Revenue and expenditure budget is an instrument of public financial management institutions, planning the necessary funds (Carmen-mihaela, *et al*., 2010) . It is also viewed as an annual financial statement which outlines the estimated government and expected revenues for the forthcoming fiscal year. Saidu & Utiya (2016) looked at government budget from economic perspective as fiscal policy which contains package of several blueprints of the government aimed to achieve specific goals. Every budget performance can be analyzed when the achievement of the objectives set aside in the budget blueprints are accomplished which squarely depend on many factors. Some of the factors as outlined by Iheanacho (2016), and Olatunji *et al.* (2017) are: the political will for effective implementation, qualities of national institutions, awarding contracts to appropriate institutions or personnel, effective supervision and evaluation of results, completion of projects, social stability and absence of corruption etc. The impact of the budget should be measured in terms of Gross Domestic Product (GDP) growth. The Bureau of Economic Analysis (BEA) gives a clear definition of GDP: Gross domestic product is the value of the goods and services produced by the nation's economy less the value of the goods and services used up in production. The budget performance of a government will stimulate corresponding economic performance. Hence, governments spend huge resources to see that its budget achieves the desired and stated objectives. However, this can never be seen and achieved till the budget performance over the period in question is evaluated and analysed. Thus, the impact should be seen and deduced from the final values of goods and services produced in the economy **(**Edame & Fonta, 2014; Nworji *et al*., 2012; Ogbonna & Azubuike, 2018). (Dynan & Sheiner, 2018).

A number of studies have modelled the relationship between government spending and economic growth in recent years. For instance, Gemmell *et al.* (2015) examined the impacts of GDP on changes in total government expenditure, alongside changes in the shares of spending devoted to various categories. While Nusron *et al.* (2022) examined the impact of three predictor variables, namely: accountability, transparency, and oversight on-budget performance. Similarly, Pramudiati *et al.* (2023) concluded that the better the accountability, transparency and supervision, the better of budget performance based on the findings that accountability, transparency, and supervision positively influence the performance of the value for money concept budget. Additionally, Bandiyono & Utami (2019) conducted a research on budget performance in relation to budget quality, knowledge of systems and procedures, as well as human resource competence. The authors discovered that human resource competence, knowledge systems, and procedures influence budget performance positively. However, budget quality does not influence the budget performance significantly. More so, Nurmala *et al.* (2023) conducted a research with the purpose to determine the effect of the size of the budget on budget performance with changes to the budget as a moderator. The results of the study showed that there was a positive effect of the amount of budget variable on budget absorption before and during Covid-19. Additionally, Adafin *et al.* (2021) investigated risk factors impacting project budget performance in New Zealand.

Furthermore, Fölster & Henrekson (1999), Chinweoke *et al.* (2014), Abu-Eideh (2015) and Nwamuo (2019) have examined the impact of government spending on economic growth.
This paper utilises data on transfers, social and community services, economic services, administration, and Gross Domestic Product (GDP) aiming to appropriately determine and assess the worst and best Federal Government of Nigeria's budget performance estimates through Fuzzy linear regression analysis.

MATERIALS AND METHODS

Description of Data

A sample of forty-two (42) observations (1981 to 2022) used for this study were obtained from Central Bank of Nigeria (2022a; 2022b). The first thirty-nine (39) observations are used to formulate the model, while the last three (3) observations are used to validate the estimated model results. The data is composed of: transfers (x_1), social and community services (x_2), economic services (x_3), administration (x_4), and GDP as a proxy for budget performance (y) (measured in gross domestic product (GDP)) in billion naira units. We utilize production and operations management software (POMS) for Windows to analyse the data.

Fuzzy Linear Regression Model

Fuzzy linear regression is a fuzzy type of classical regression analysis in which some elements of the model are represented by fuzzy numbers (Alsoltany & Alnaqash, 2015). In other words, the fuzzy regression method estimates the effects of the explanatory variables on the response variable as a range. The functional relationship between the dependent and independent variables as reported in Tanaka *et al.* (1982) is presented as follows:

$$\widetilde{\boldsymbol{Y}} = \widetilde{\boldsymbol{A}}_{\boldsymbol{0}} + \widetilde{\boldsymbol{A}}_{\boldsymbol{1}}\boldsymbol{x}_{\boldsymbol{1}} + \widetilde{\boldsymbol{A}}_{\boldsymbol{2}}\boldsymbol{x}_{\boldsymbol{2}} + \ldots + \widetilde{\boldsymbol{A}}_{\boldsymbol{p}}\boldsymbol{x}_{\boldsymbol{p}} \quad (1)$$

In matrix form;

$$\hat{Y} = \tilde{A}_0 + \tilde{A}X \quad (2)$$

Where:

$\tilde{Y}$ is the fuzzy output,

$X = (x_1, x_2, \ldots, x_p)^T$ p-dimensional crisp input vector,

$\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_P)^T$ fuzzy vector of coefficients presented in the form of a symmetric triangular fuzzy number denoted by $\tilde{A}_j = [c_j, w_j]$, respectively c_j and w_j are its centre and width. The description of its membership function is as follows:

$$\mu_{\tilde{A}_j}(a_j) = \begin{cases} 1 - \frac{|c_j - a_j|}{w_j} & if\ c_j - a_j \le a_j \le c_j + a_j \\ 0 & otherwise \end{cases} \quad (3)$$

Similarly, the triangular membership function $\tilde{Y}$ is given by :

$$\mu_Y(y) = \begin{cases} Max(0, 1 - \frac{|y - C^T X|}{W^T X}) & if\ X \neq 0 \\ 1 & if\ X = 0, y \neq 0 \\ 0 & if\ X = 0, y = 0 \end{cases} \quad (4)$$

The problem of finding the fuzzy regression coefficient is formulated by Tanaka *et al*. (1982) as a linear programming such that the total vagueness is minimised. That is,

$$
\begin{aligned}
& Min\ S = \sum_{i=1}^{n}\sum_{j=0}^{p} w_j \left| x_{ij} \right| \\
st \quad & \sum_{j=0}^{p} c_j x_{ij} - (1+h)\sum_{j=0}^{p} w_j \left| x_{ij} \right| \le y_i \ \forall i = 1, \dots, n \\
& \sum_{j=0}^{p} c_j x_{ij} + (1-h)\sum_{j=0}^{p} w_j \left| x_{ij} \right| \ge y_i \ \forall i = 1, \dots, n \\
& w \ge 0, x_{i0} = 1; i = 1, \dots, n
\end{aligned} \tag{5}
$$

Where, $W^T = (w_0, w_1, w_2, \dots w_p)$ and $C^T = (c_0, c_1, c_2, \dots c_p)$ are unknown variables vectors. Equation (1) can be rewritten in possibilistic form as follows:

$$\tilde{Y} = (c_0, w_0) + (c_1, w_1)x_1 + (c_2, w_2)x_2 + \dots + (c_p, w_p)x_p \tag{6}$$

This expression makes it possible to forecast the best and worst possible values of $\tilde{Y}$ based on predetermined values of the related explanatory factors, $X = (x_1, x_2, \dots, x_p)$.

RESULTS AND DISCUSSION

In this section, we solved the linear programming (LP) problem, that is, equation (5) with threshold level of $h = 0$ to achieve best prediction values using POM software for Windows in order to determine the minimal fuzziness of the model. To ensure stability of forecasts, the natural logarithm of the data was taken after deleting the data observation of the year, 2002 since the natural logarithm of zero is undefined. The empirical results based on the 39 observations indicated that the actual data observations of 1981, 1983 and 2003 were located outside the predicted interval of possibility. Consequently, the linear constraints generated from these observations were deleted as suggested in Ishibuchi & Tanaka (1988). The renewed empirical results presented in three phases are as follows:

Phase I: Determining the fuzzy parameters: The central values and widths of each fuzzy parameter in equation (1) for $h = 0$ were obtained and presented in Table 1 along with the corresponding upper bound (UB) and lower bound (LB) respectively. Additionally, the fuzzy linear regression prediction equation for the budget performance ($\tilde{y}$) of Nigeria against the related budget variables is provided in equation (11).

It can be observed in Table 1 as well as equation (11), the spreads of the explanatory variables are all zero except that of social and community services. This implies that the budget performance interval of possibility can be suitably predicted when the social and community services coefficient is between 0.0966 and 0.2616 respectively. Whereas, the transfers, economic services, and administration maintain the exact centre values, because the width values are zero (see column 3). It is identified that the most effective variable that influences budget performance is administration. Hence, increasing spending on administration increases the budget performance.

Table 1. Central and widths values for the estimated fuzzy parameters

Fuzzy parameters	Centre	Width	UB	LB
A_o	5.5508	0.4716	6.0224	5.0792
A_1	0.1791	0.0825	0.2616	0.0966
A_2	0.2366	0.0000	0.2366	0.2366
A_3	0.0000	0.0000	0.0000	0.0000
A_4	0.6386	0.0000	0.6386	0.6386

$$GDP\ (\tilde{y}) = (5.5508, 0.4716) + (0.1791, 0.0825)x_{i1} + (0.2366, 0.0000)x_{i2} + (0.6386, 0.0000)x_{i4} \tag{11}$$

Here, equation (11) indicates that a unit change in the Transfers, Social and community services, and Administration will lead to a corresponding increase in the budget performance based on the following ranges: $(\mathbf{5.0792, 6.0224})$, $(\mathbf{0.0966, 0.2616})$, $(\mathbf{0.2366, 0.2366})$,and

$(\mathbf{0.6386}, \mathbf{0.6386})$ respectively. It is reasonable to note that economic services is eliminated in equation (11) because it does not contribute significantly to budget performance, while retaining the rest of the explanatory variables fixed.

Phase II: Prediction of bounds: Using equation (11), the predicted worst and best possible budget performance (GDP) values for the considered time range were obtained and presented in Table 2 as well as Figure 1 along with the actual observation values. The lower bound (LB) and the upper bound (UB) are the worst and best values respectively.

Table 2. Predicted Interval of Possibility Values

Year	Actual GDP	LB	UB	Year	Actual GDP	LB	UB
1982	5.004	4.552	5.648	2001	9.016	8.927	10.586
1984	5.111	3.990	5.111	2004	9.805	9.296	10.694
1985	5.236	4.721	5.844	2005	10.049	9.610	10.957
1986	5.289	4.312	5.564	2006	10.321	9.762	11.245
1987	5.500	5.402	6.441	2007	10.454	10.033	11.494
1988	5.755	5.710	6.810	2008	10.595	10.158	11.572
1989	6.028	6.022	7.277	2009	10.680	10.398	12.223
1990	6.204	6.204	7.600	2010	10.924	10.214	11.832
1991	6.380	6.236	7.676	2011	11.062	10.144	11.968
1992	6.809	6.630	8.136	2012	11.193	10.054	11.919
1993	7.137	7.024	8.495	2013	11.302	10.371	12.156
1994	7.478	7.176	8.681	2014	11.409	10.041	11.626
1995	8.039	7.647	9.253	2015	11.464	10.079	11.859
1996	8.315	7.726	9.374	2016	11.538	9.759	11.538
1997	8.394	8.393	9.959	2017	11.652	10.506	12.326
1998	8.477	8.477	10.064	2018	11.768	10.777	12.649
1999	8.609	8.609	10.334	2019	11.889	11.063	13.010
2000	8.863	8.777	10.355				

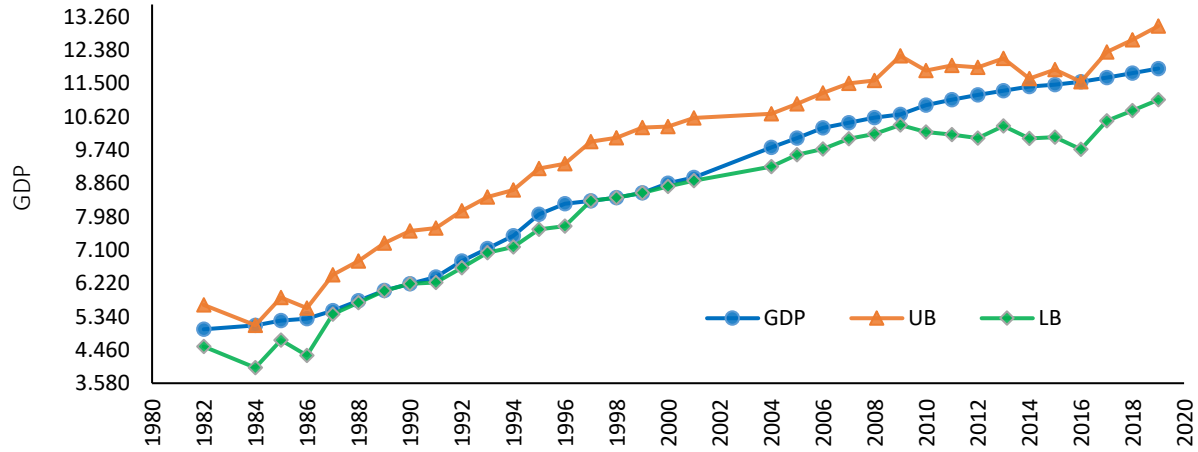

Figure 1: Time Plot of Actual GDP along with UB and LB

Furthermore, an out-of-sample prediction also indicated that the actual GDP values are located in the prediction bounds as it can be seen in Table 3. This implies the estimated model is suitable for future budget performance forecast based on a predetermined or anticipated values of the related explanatory variables.

Table 3. The result of the estimated out of sample values

Year	Actual GDP	LB	UB
2020	11.9463	10.7235	12.6131
2021	12.0787	11.1501	13.1122
2022	12.2178	11.3610	13.3589

Phase III: Bound assessment: The assessment is to check whether the estimated lower and upper prediction ranges contained the actual observation values. From Tables 2 and 3 as well as Figure 1, it can be observed that the actual GDP values are located within the predicted bounds. However, the lower bound values are closer to the actual observation values compared to the upper bound values.

CONCLUSION

This study analyzed the budget performance of Federal Government of Nigeria measured in terms of gross domestic product. A Fuzzy Linear Regression model is formulated on the basis of triangular membership function that resulted into a linear programming problem and solved using POMS for Windows. The results revealed that, in terms of the centre values (point estimates), administration could have more impact on the budget performance if the rest of the predictor variables are held constant. On the other hand, the most effective variable that influences budget performance is social and community services because it has widest possibility interval of the estimated fuzzy coefficients.

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