https://dx.doi.org/10.4314/dujopas.v9i4a.31
ISSN (Print): 2476-8316
ISSN (Online): 2635-3490
Dutse Journal of Pure and Applied Sciences (DUJOPAS), *Vol. 9 No. 4a December 2023*

Enhanced Variance Estimation Techniques for Addressing Non-Response and Measurement Error Situations

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Abstract
The use of relevant information from auxiliary variables at the estimation stage and design stage to obtain reliable and efficient estimates is a common practice in a sample survey. Several Estimators of population variance have been suggested. However, these estimators do not consider the situation of non-response due to non-availability of respondents, refusal to respond, presence of hard-core respondents or due to non-understanding of the question thereby, reducing the efficiency of the estimators and the parameters of the auxiliary variable $\bar{X}, S_X^2$ *used are sensitive to outliers or extreme values which can either lead to underestimation or overestimation. To address the aforementioned observations, classes of variance estimators under the simultaneous influence of non-response and measurement errors using outlier-free parameters as well as calibration approaches were proposed. The properties (Bias and MSE) of the modified estimators were derived up to the first order of approximation using the Taylor series approach. The efficiency conditions of the proposed estimators over the existing estimators considered in the study were established. The empirical studies were conducted using simulation and the results revealed that the proposed class of estimators have minimum MSEs and higher PREs among all the competing estimators. These imply that the proposed estimators are more efficient and can produce a better estimate of the population mean compared to other existing estimators considered in the study.*

Keywords; Non-response, Estimators, Measurement error, Calibration, Efficiency

INTRODUCTION
The population parameters of interest that are often estimated in sampling survey includes population mean, population variance, population proportion, population total etc. Various estimators can be used to estimate these parameters and they include; Ratio, Product, Regression, Exponential ratio, Exponential product, Dual to ratio, Dual to product and many more. The Ratio estimators can be used when there is a strong positive correlation between the study and the auxiliary variables and it was initiated by Cochran (1940). The product

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estimator can also be used when there is a negative correlation between auxiliary and study variables, Murthy (1964). Regression Estimator can be applied for the estimation of parameters for both situations, Singh and Singh (2001b), Khan and Shabbir (2017), Southwick and Robinson (1957), Subramani and Kumarapandiyan (2012, 2015), Shabbir and Gupta (2010), Olufadi and Kadilar (2014), Prasad and Singh (1992), Muili et al (2018), Kadilar and Cingi (2006), Koyuncu (2013), Grover (2010), Audu et al. (2016a, b, c), Upadhyaya and Singh (1999). Other authors that have worked in this direction are Reed and Das (1988), Audu and Singh (2015), Deville and Särndal (1992), Haq *et al.*(2017) and many more for the estimation of population mean. In many studies, variance estimation is a crucial concern and the variance estimator uses auxiliary information to improve the efficiency of estimators. One of its applications is that it is used to study variations of produce or yields in the manufacturing and pharmaceutical industries, (Audu *et al.*, 2022). Ratio, product and regression estimators are formulated using auxiliary variables. It is widely accepted that the information on auxiliary variables such as the population mean, variance, kurtosis, skewness etc. enhance the efficiency of estimators. Authors like Das *et al.* (1978), Srivastava and Jhajj (1981), Isaki(1983), Upadhyaya and Singh (1983), Singh *et al.* (1988), Biradar and Singh (1994), Singh and Singh (2001a), Lin *et al.* (2003), Singh and Vishwakarma (2008), Singh *et al.* (2011), Audu and Singh (2015), Ishaq *et al.* (2020), Yunusa *et al.* (2022), Olayiwola *et al.* (2021), Adejumobi *et al.* (2022) have developed variance estimators using auxiliary information. It is a well-known fact in sample surveys that there are two major non-sampling error problems encountered by survey statisticians during a survey. The problems are non-response and measurement errors. These errors are so serious that their occurrence may lead any estimation strategy to either overestimation or underestimation. For example, in a mail questionnaire survey where respondents are expected to give their actual age, instead, they give an approximate age. Hence, there is an occurrence of measurement error in this situation. Also, when we use an instrument that is faulty during a survey, the observations to be obtained will not be accurate, hence measurement error is present. Apart from measurement error, there is another error in the survey which is non-response. This error arises as a result of respondents not responding to item(s) of a questionnaire either due to a lack of understanding of the question or, not willing to disclose vital information, and as a result of this, the questions are unanswered. Thus, non-response arises. Hansen and Hurwitz (1946) were the first to consider the problem of non-response in estimation theory by using the technique of sub-sampling from non-respondents. The estimation of unknown population parameters with the presence of non-response and measurement errors influences the properties of the estimators to a greater extent. In statistical analysis, it is often assumed that all the observations are measured correctly, but in reality, it is not true. Several authors like, Cochran (1968), Shalabh (1997), Srivastava (2002), Allen *et al.* (2003), Singh and Karpe (2010), Singh *et al.* (2011), and Singh *et al.* (2022) have considered the influence of measurement error on the estimators of mean and variance under simple and stratified random sampling. Authors such as Kumar and Krishna (2011), Singh *et al.* (2018), Audu *et al.* (2021), Audu *et al.* (2022), defined estimators under the simultaneous influence of non-response and measurement errors respectively.

Various techniques have been adopted to improve the efficiency of variance estimators and other parameters of the study variable in sampling literature, they include the use of unknown weight, power transformation, linear combination of estimators, exponential transformation, logarithmic transformation, use of non-conventional robust measures, use of conventional and non-conventional measures, etc. Another technique that can provide an improved estimate of population variance is Calibration.

Calibration estimation is a technique that makes use of auxiliary information to modify the original design weights in order to improve the accuracy of the estimators. It is also a method of adjusting the original strata weights by reduction of a given distance measure based on a set of calibration constraints. Works in this field on mean estimation were done by Tracy *et al.* (2003), Singh *et al.* (2018), Koyuncu (2018), Zaman and Bulut (2019), Garg and Pachori (2020), Shahzad *et al.* (2021). Singh (2001) were the first to extend the Calibration approach to a stratified sampling design. Others include: Arnab and Singh (2005), Särndal (2007), Kim and Park (2010), Clement *et al.* (2014), Koyuncu and Kadilar (2016). Outliers or extreme values are the observations in a dataset that appear to be inconsistent with the rest of that dataset, (Lemonaki, 2021). Zaman and Bulut (2020) used robust methods for the estimation of mean and variance in the presence of outliers or extreme values. Abid *et al.* (2018) Variance estimators use traditional central moments yet, their estimators are sensitive to outliers. To resolve this problem, L-moments were introduced. L-moments are based on linear combinations of order statistics. Singh *et al.* (2018) adopted L-moments for estimation of population mean considering calibrations approach, under stratified random sampling, other authors include Zaman and Bulut (2020), Zaman and Bulut (2021), Shahzad *et al.* (2021) adopted the L-moment and calibration approach for variance estimation under stratified random sampling.

Recently, Singh *et al*. (2021) suggested variance estimators which were studied under measurement error without considering the influence of non-response error in their study. In this present study, classes of variance estimators under the simultaneous influence of measurement errors and non-response using a calibration approach are proposed.
The usual variance estimator in the presence of measurement error is given by (1.1),

$$t_{st1(e)} = \sum_{h=1}^{L} W_h^2 \frac{\hat{s}_{y(e)h}^2}{n_h} \tag{1.1}$$

The bias and mean square errors are given by (1.2) and (1.3) respectively

$$Bias\ (t_{st1(e)}) = 0 \tag{1.2}$$

$$MSE\left(t_{st1(e)}\right) = \sum_{h=1}^{L} \frac{\left(W_h S_{y(e)h}\right)^4}{n_h^3} \left|A\gamma_h\right| \tag{1.3}$$

Singh *et al.* (2021) proposed variance estimators in the presence of measurement errors under stratified random sampling. Three classes of estimators for the estimation of population variance under stratified random sampling when both the study and auxiliary variable are characterized with measurement errors are given by (1.4), (1.5) and (1.6) respectively as

$$\hat{s}_a^2 = \sum_{h=1}^{L} \left(\frac{W_h^2}{n_h}\right) \hat{s}_{y(e)h}^2 a_h\left(l_h\right) \tag{1.4}$$

$$\hat{s}_b^2 = \sum_{h=1}^{L} \left(\frac{W_h^2}{n_h}\right) \hat{s}_{y(e)h}^2 b_h\left(m_h\right) \tag{1.5}$$

$$\hat{s}_c^2 = \sum_{h=1}^{L} \left(\frac{W_h^2}{n_h}\right) \hat{s}_{y(e)h}^2 c_h\left(l_h m_h\right) \tag{1.6}$$

The biases and mean square errors of the estimators are given by (1.7), (1.8), (1.9), (1.10), (1.11) and (1.12) respectively as

$$Bias(\hat{s}_a^2)=\sum_{h=1}^{L}\left(\frac{W_h^2}{n_h^2}\right)S_{yh}^2\left[\lambda_{21h}C_{xh}a_{1h}(1)+\frac{1}{2}\frac{C_{Xh}^2}{\theta_{Xh}}a_{2h}(1)\right] \tag{1.7}$$

$$Bias(\hat{s}_b^2)=\sum_{h=1}^{L}\left(\frac{W_h^2}{n_h^2}\right)S_{yh}^2\left[(\lambda_{22h}-1)b_{1h}(1)+\frac{A_{Xh}b_{2h}(1)}{2}\right] \tag{1.8}$$

$$Bias(\hat{s}_c^2)=\sum_{h=1}^{L}\left(\frac{W_h^2}{n_h^2}\right)S_{yh}^2\left[\begin{array}{l}\lambda_{21h}C_{Xh}c_{1h}(1,1)+(\lambda_{22h}-1)c_{2h}(1,1)\\ +\frac{1}{2}\left(\frac{C_{Xh}^2c_{11h}(1,1)}{\theta_{Xh}}+2\lambda_{03h}C_{Xh}c_{12h)}(1,1)+A_{Xh}c_{22h}(1,1)\right)\end{array}\right] \tag{1.9}$$

$$MSE(\hat{s}_a^2)=\sum_{(h=1)}^{L}\frac{(W_hS_{yh})^4}{n_h^3}\left(A_{Yh}-\lambda_{21h}^2\theta_{Xh}\right) \tag{1.10}$$

$$MSE(\hat{s}_b^2)=\sum_{(h=1)}^{L}\frac{(W_hS_{yh})^4}{n_h^3}\left[A_{Yh}-\frac{(\lambda_{22h}-1)^2}{A_{Xh}}\right] \tag{1.11}$$

$$MSE(\hat{s}_c^2)=\sum_{(h=1)}^{L}\frac{(W_hS_{yh})^4}{n_h^3}\left[A_{Yh}-\lambda_{21h}^2\theta_{Xh}-\frac{[\lambda_{21h}\theta_{Xh}\lambda_{03h}-(\lambda_{22h}-1)]^2}{\left(A_{Xh}-\lambda_{03h}^2\theta_{Xh}\right)}\right] \tag{1.12}$$

Where, $\mathrm{A}_{Yh}=\gamma_{2yh}+\gamma_{2uh}\frac{s_{uh}^4}{s_{Yh}^4}+\frac{2}{\theta_{Yh}^2}$, $\mathrm{A}_{Xh}=\gamma_{2xh}+\gamma_{2vh}\frac{s_{vh}^4}{s_{Xh}^4}+\frac{2}{\theta_{Xh}^2}$

$$\beta_{2h}(Y)=\lambda_{40h}=\frac{\mu_{40h}}{\mu_{20h}^2},C_{Xh}=\frac{S_{Xh}}{\bar{X}_h},\beta_{2h}(X)=\lambda_{04h}=\frac{\mu_{04h}}{\mu_{02h}^2},\gamma_{2Yh}=\lambda_{40h}(Y)-3,\gamma_{2uh}=\beta_{2h}(u)-3,$$

$$\gamma_{2Xh}=\lambda_{04h}(X)-3,\gamma_{2vh}=\beta_{2h}(v)-3,\theta_{Yh}=\left(\frac{S_{yh}^2}{S_{yh}^2+s_{uh}^2}\right),\theta_{Xh}=\frac{S_{xh}^2}{S_{xh}^2+s_{vh}^2},\delta_{rsh}=\frac{\mu_{rsh}}{\left(\mu_{20h}^r\mu_{02h}^r\right)^{\frac{1}{2}}}$$

$$\mu_{rsh}=\frac{1}{N_h}\sum_{j=1}^{Nh}\left(y_{hj}-\bar{Y}_h\right)^r\left(x_{hj}-\bar{X}_h\right)^s,\ a_h(l_h)=a_h(1)+(l_h-1)a_{1h}(1)+\frac{1}{2}(l_h-1)^2a_{2h}(l),$$

$$l_h=\frac{\bar{x}_h}{\bar{X}_h},b_h(m_h)=b_h(1)+(m_h-1)b_{1h}(h)+\frac{1}{2}(m_h-1)^2b_{2h}(1),m_h=\frac{s_x^2}{s_h^2}$$

$$c_h(l_h,m_h)=\left[\begin{array}{l}c_h(1,1)+(l_h-1)c_{1h}(1,1)+(m_h-1)C_{2h}(1,1)\\ +\frac{1}{2}\left\{(l_h-1)^2c_{11h}(1,1)+2(l_h-1)(m_h-1)c_{12h}(1,1)+(m_h-1)^2c_{22h}(1,1)\right\}\\ +\frac{1}{6}\left\{(l_h-1)^3c_{111h}(l_h^*,m_h^*)+3(l_h-1)^2)(m_h-1)c_{112h}(l_h^*,m_h^*)\right)\\ +3(l_h-1)(m_h-1)^2c_{122h}(l_h^*,m_h^*)+(m_h-1)^3c_{222h}(l_h^*,m_h^*)\}]\end{array}\right]$$

$$,a_{1h}=\frac{-\lambda_{21h}\theta_{Xh}}{C_{Xh}},b_{1h}=\frac{-(\lambda_{21h}-1)}{A_{Xh}},c_{1h}=\frac{\lambda_{03h}(\lambda_{22h}-1)-\lambda_{21h}A_{Xh}}{\left[C_{Xh}\left(\frac{A_{Xh}}{\theta_{Xh}}\right)-\lambda_{03h}^2\right]},c_{2h}=\frac{\left[\frac{\lambda_{03h}(\lambda_{21h}-1)(\lambda_{22h}-1)}{\theta_{Xh}}\right]}{\left(\frac{A_{Xh}}{\theta_{Xh}}\right)-\lambda_{03h}^2}$$

$$\mathrm{E}\left(\varepsilon_{oh}^{2}\right)=\frac{A_{Yh}}{n_h},\ \mathrm{E}\left(\varepsilon_{1h}^{2}\right)=\frac{A_{Xh}}{n_h},\ \mathrm{E}\left(\varepsilon_{2h}^{2}\right)=\frac{C_{Xh}^{2}}{n_h\theta_{Xh}},\ \mathrm{E}\left(\varepsilon_{0h}\varepsilon_{1h}\right)=\frac{1}{n_h}\left(\lambda_{22h}-1\right),$$

$$\mathrm{E}\left(\varepsilon_{1h}\varepsilon_{2h}\right)=\frac{1}{n_h}\left(\lambda_{03h}C_{Xh}\right),\ \mathrm{E}\left(\varepsilon_{0h}\varepsilon_{2h}\right)=\frac{1}{n_h}\left(\lambda_{21h}C_{Xh}\right).$$

Their study confirmed that the proposed estimators are better than the usual unbiased estimators theoretically. Singh *et al.* (2021) also concluded that the presence of measurement errors incorporates larger mean square error than the absence of measurement error, through exhibiting the impact of measurement error on the MSE of the estimators. However, the study does not consider the simultaneous influence of measurement error and non-response. Additionally, the parameters of the auxiliary $\bar{X}_h, S_{x_h}^2$ used in the estimators can easily be influenced by the presence of outliers or extreme values in the data thereby leading to underestimation or underestimation.

MATERIALS AND METHODS

Proposed Variance Calibration Estimator

Having studied the work of Singh *et al.* (2021) variance estimators, the following class of variance estimators in the presence of measurement errors and non-response is proposed.

$$T_{pi}=\sum_{h=1}^{L}\frac{\alpha_h^{*2}}{n_h}s_{y(e)h,}^{*2},\ i=1,2,3,\ldots,10 \tag{2.1}$$

$$\left.\begin{aligned} &min\ z=\sum_{h=1}^{L}\frac{\left(\alpha_h^{*2}-W_h^2\right)^2}{\theta_{hi}W_h^2}\\ &s.t\quad \sum_{h=1}^{L}\alpha_h^{*2}=\sum_{h=1}^{L}W_h^2\\ &\sum_{h=1}^{L}\alpha_h^{*2}\hat{\Delta}_{x(e)h}^{*}=\sum_{h=1}^{L}W_h^2\Delta_{xh}^{*}\end{aligned}\right\} \tag{2.2}$$

$$Where\ s_{y(e)h}^{*2}=\frac{n_{1h}s_{y(e)h}^{2}+n_{2h}s_{yh2(e)h}^{2}}{n_h}\ and,\ s_{x(e)h}^{*2}=\frac{n_{1h}s_{x(e)h}^{2}+n_{2h}s_{xh2(e)h}^{2}}{n_h}$$

$$\bar{y}_{(e)h}^{*}=\frac{n_{1h}\bar{y}_{1(e)h}+n_{2h}\bar{y}_{h_2(e)h)}}{n_h},\ \bar{x}_{(e)h}^{*}=\frac{n_{1h}\bar{x}_{(e)h}^{*}+n_{2h}\bar{x}_{h_2(e)h}}{n_h}.$$

$\hat{\Delta}_{x(e)h}^{*}$ *and* Δ_{xh}^{*} are the sample and population characteristics of the auxiliary variable in the h[th] stratum based on population L-moments $\hat{\lambda}_{1xh},\hat{\lambda}_{2xh},\hat{\lambda}_{3xh},\hat{\lambda}_{4xh}$ and $\lambda_{1xh},\lambda_{2xh},\lambda_{3xh},\lambda_{4xh}$ respectively in the presence of non-response and measurement error.

To obtain bias and MSE of the estimators, T_{pi} = 1, 2, …5, function in (2.1) was used.

$$Bias(T)=2^{-1}\left(\sum_{i=1}^{q}\sum_{j=1}^{q}\Delta_{ij(h)}E(\hat{\theta}_{i(h)}-\theta_{i(h)})(\hat{\theta}_{j(h)}-\theta_{j(h)})\right) \tag{2.3}$$

Where, q is the number of sample variances in the estimators, q = 2

$$\hat{\theta}_{1(h)} = s^{*2}_{y(e)h},\ \hat{\theta}_{2(h)} = s^{*2}_{x(e)h},\ \theta_{1(h)} = S^2_{y(h)},\ \theta_{1(h)} = S^2_{x(h)}\ .\ \Delta_{ij(h)} = \left[\frac{\partial^2 T}{\partial\hat{\theta}_{i(h)}\partial\hat{\theta}_{j(h)}}\right]\Bigg|_{\hat{\theta}_{i(h)}=S^2_{y(h)},\ \hat{\theta}_{j(h)}=S^2_{x(h)}}$$

The MSEs of the estimators is obtained using function in (14)

$$MSE(T) = \Delta_{jh}\sum\nolimits_{jh}\Delta^T_{jh}, \tag{2.4}$$

where $\Delta_{ah} = \left[\frac{\partial T}{\partial\hat{\lambda}^2_{2y(e)h}}\ \frac{\partial T}{\partial\hat{\lambda}_{1x(e)h}}\right]\Bigg|_{\lambda^2_{2yh}=\hat{\lambda}^2_{2y(e)h},\ \lambda_{1xh}=\hat{\lambda}_{1x(e)h}}$

$$\sum\nolimits_{ah} = \begin{bmatrix} Var\left(\hat{\lambda}^{*2}_{2y(e)h}\right) & Cov\left(\hat{\lambda}^{*2}_{2y(e)h}, \hat{\lambda}^{*}_{1x(e)h}\right) \\ Cov\left(\hat{\lambda}^{*2}_{2y(e)h}, \hat{\lambda}^{*}_{1x(e)h}\right) & Var(\hat{\lambda}^{*}_{1x(e)h}) \end{bmatrix}$$

$$Var(s^{*2}_{y(e)h}) = \sum_{h=1}^{L}\frac{[W_h S_{y(h)}]^4}{n_h^3}[K_{1(h)}H_{y(h)}) + K_{2(h)}H_{y(h2)h}]$$

$$H_{y(h)} = \lambda_{40(h)} + \gamma_{40(h)}S^4_{u(h)}S^{-4}_{y(h)} + 2(1 + S^2_{u(h)}S^{-2}_{y(h)}\ )^2$$

$$H_{y(2)h} = \lambda_{40(2)h} + \gamma_{40(2)h}S^4_{u(2)h}S^{-4)}_{y(2)h} + 2(1 + S^2_{u(2)h}S^{-2}_{y(2)h})^2$$

$$Var(s^{*2}_{x(e)h}) = \sum_{h=1}^{L}\frac{[W_h S_{y(h)}]^4}{n_h^3}[K_{1(h)}H_{x(h)}) + K_{2(h)}H_{x(h2)h}]$$

$$H_{x(h)} = \lambda_{04(h)} + \gamma_{40(h)}S^4_{v(h)}S^{-4}_{x(h)} + 2(1 + S^2_{v(h)}S^{(-2)})^2$$

$$H_{x(2)h} = \lambda_{04(2)h} + \gamma_{40(2)h}S^4_{v(2)h}S^{-4}_{x(2)h} + 2\left(1 + S^2_{v(2)h}S^{(-2)}_{x(2)h}\right)^2$$

$$Cov\left(s^{*2}_{y(e)h}, s^{*2}_{x(e)h}\right) = \sum_{h=1}^{L}\left[K_{1(h)}\lambda_{22(h)} + K_{2(h)}\lambda_{22(2)h}\right]$$

$$K_{(h)} = n_h^{-1} - N_h^{-1}, K_{2(h)} = \frac{W_{2h}(f_h - 1)}{n_h}, W_{2h} = \left(\frac{N_2}{N}\right)_h$$

To obtain the calibration weight estimators and properties of the estimator T_{pi}, we define the Lagrange function as

$$L_{pi} = \sum_{h=1}^{L}\frac{\left(\alpha_h^{*2} - W_h^2\right)^2}{\theta_{ih}W_h^2} - 2a_1\left(\sum_{h=1}^{L}\alpha_h^{*2} - \sum_{h=1}^{L}W_h^2\right) - 2a_2\left(\sum_{h=1}^{L}\alpha_h^{*2}\hat{\Delta}^*_{x(e)h} - \sum_{h=1}^{L}W_h^2\Delta^*_{x(h)}\right) \tag{2.5}$$

where a_1 and a_2 are Lagrange's multipliers.

Differentiate (2.5) partially with respect to α_h^{*2}, a_1 and a_2 respectively and equate to zero to obtain (2.6), (2.7) and (2.8) after simplification.

$$\alpha_h^{*2} = W_h^2 + a_1\theta_{ih}W_h^2 + a_2\theta_{ih}W_h^2\hat{\Delta}^*_{x(e)h}, \tag{2.6}$$

$$\sum_{h=1}^{L}\alpha_h^{*2} = \sum_{h=1}^{L}W_h^2, \tag{2.7}$$

$$\sum_{h=1}^{L}\alpha_h^{*2}\hat{\Delta}^*_{x(e)h} = \sum_{h=1}^{L}W_h^2\Delta^*_{x(h)}, \tag{2.8}$$

Substitute (2.6) into (2.7) and (2.8) and simplify to generate two simultaneous equations in (2.9) and (2.10) respectively as

$$a_1\sum_{h=1}^{L}\theta_h W_h^2 + a_2\sum_{h=1}^{L}\theta_h W_h^2\hat{\Delta}^*_{x(e)h} = 0, \tag{2.9}$$

$$a_1\sum_{h=1}^{L}\theta_{ih} W_h^2\hat{\Delta}^*_{x(e)h} + a_2\sum_{h=1}^{L}\theta_{ih} W_h^2\hat{\Delta}^*_{x(e)h} = \sum_{h=1}^{L} W_h^2\left(\Delta^*_{x(h)} - \hat{\Delta}^*_{x(e)h}\right), \tag{2.10}$$

Solving equations (2.9) and (2.10) simultaneously, the results obtained are,

$$a_1 = \frac{-\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}_{x(e)h}\left(\sum_{h=1}^{L} W_h^2\Delta^*_{xh} - \sum_{h=1}^{L} W_h^2\ \hat{\Delta}^*_{x(e)h}\right)}{\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\right)\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^{*2}_{x(e)h}\right) - \left(\sum_{h=1}^{L}\theta_{ih} W_h^2\hat{\Delta}^*_{x(e)h}\right)^2}, \tag{2.11}$$

$$a_2 = \frac{\left(\sum_{h=1}^{L}\theta_{ih} W_h^2\right)\left(\sum_{h=1}^{L} W_h^2\Delta^*{}_{x(h)} - \sum_{h=1}^{L} W_h^2\hat{\Delta}^*_{x(e)h}\right)}{\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\right)\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^{*2}_{x(e)h}\right) - \left(\sum_{h=1}^{L}\theta_h W_h^2\hat{\Delta}^*_{x(e)h}\right)^2}, \tag{2.12}$$

Substituting (2.11) and (2.12) into (2.6) and simplify, we obtained the calibration weights as,

$$\alpha_h^{*2} = W_h^2 + \theta_{ih} W_h^2 \frac{\left[\hat{\Delta}^*_{x(e)h}\sum_{h=1}^{L} W_h^2\theta_h - \sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^*_{x(e)h}\right]\left[\sum_{h=1}^{L} W_h^2\left(\Delta^*_{x(h)} - \hat{\Delta}^*_{x(e)h}\right)\right]}{\left[\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^{*2}_{x(e)h}\right)\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\right) - \left(\sum W_h^2\theta_{ih}\hat{\Delta}^*_{x(e)h}\right)^2\right]}, \tag{2.13}$$

By substituting (2.13) into calibration schemes defined in (2.1), we obtained the proposed estimators $T_{pi,}$ i= 1, 2, 3, ..5 as

$$T_{pi} = \sum_{h=1}^{L}\frac{W_h^2}{n_h}s^{*2}_{y(e)h} + \beta_i\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\Delta^*_{x(h)} - \hat{\Delta}^*_{x(e)h}\right), \tag{2.14}$$

where $\beta_i = \dfrac{\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^*_{x(e)h}s^{*2}_{y(e)h}\right)\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\right) - \left(\sum_{h=1}^{L} W_h^2\theta_{ih}s^{*2}_{y(e)h}\right)\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^*_{x(e)h}\right)}{\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^{*2}_{x(e)h}\right)\left(\sum_{h=1}^{L} W_h^2\theta_{ih}\right) - \left(\sum_{h=1}^{L} W_h^2\theta_{ih}\hat{\Delta}^*_{(e)h}\right)^2}$,

Table 4.1: Members of the proposed estimators T_{pi}

i	β_i	θ_{ih}	$\Delta^*_{x(h)}$	$\hat{\Delta}^*_{x(e)h}$	Estimators
1	β_1	1	λ_{1xh}	$\hat{\lambda}_{1x(e)h}$	$T_{p1} = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s^{*2}_{y(e)h} + \beta_1\left(\lambda_{1xh} - \hat{\lambda}_{1x(e)h}\right)\right)$
2	β_2	$\left(\hat{\lambda}_{1x(e)h}\right)^{-1}$	λ_{1xh}	$\hat{\lambda}_{1x(e)h}$	$T_{p2} = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s^{*2}_{y(e)h} + \beta_2\left(\lambda_{1xh} - \hat{\lambda}_{1x(e)h}\right)\right)$
3	β_3	$\left(\hat{\lambda}_{2x(e)h}\right)^{-1}$	λ_{1xh}	$\hat{\lambda}_{1x(e)h}$	$T_{p3} = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s^{*2}_{y(e)h} + \beta_3\left(\lambda_{1xh} - \hat{\lambda}_{1x(e)h}\right)\right)$
4	β_4	$\left(\hat{\lambda}^2_{2x(e)h}\right)^{-1}$	λ_{1xh}	$\hat{\lambda}_{1x(e)h}$	$T_{p4} = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s^{*2}_{y(e)h} + \beta_4\left(\lambda_{1xh} - \hat{\lambda}_{1x(e)h}\right)\right)$
5	β_5	$\left(\frac{\hat{\lambda}_{2x(e)h}}{\hat{\lambda}_{1x(e)h}}\right)^{-1}$	λ_{1xh}	$\hat{\lambda}_{1x(e)h}$	$T_{p5} = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s^{*2}_{y(e)h} + \beta_5\left(\lambda_{1xh} - \hat{\lambda}_{1x(e)h}\right)\right)$

To obtain the bias of the estimators T_{pi}, take expectation of (2.16), under the assumption that

$$E\left(\beta_i\right)=\beta_i^*, \text{ where } \beta_1^*=\frac{\left(\sum_{h=1}^{L}W_h^2\Delta_{xh}S_{yh}^2\right)\left(\sum_{h=1}^{L}W_h^2\right)-\left(\sum_{h=1}^{L}W_h^2S_{yh}^2\right)\left(\sum_{h=1}^{L}W_h^2\Delta_{xh}\right)}{\left(\sum_{h=1}^{L}W_h^2\Delta_{xh}^2\right)\left(\sum_{h=1}^{L}W_h^2\right)-\left(\sum_{h=1}^{L}W_h^2\Delta_h\right)^2} \quad (2.15)$$

$$E\left(T_{pi}\right)=\sum W_h^2E\left(s_{y(e)h}^{*2}\right)+\beta_i\sum W_h^2\left(\Delta_{x(h)}^*-E\left(\hat{\Delta}_{x(e)h}^*\right)\right) \quad (2.16)$$

$$Since, E\left(\hat{\Delta}_{x(e)h}^*\right)=\Delta_{x(h)}^*$$

$$E\left(T_{pi}\right)=\sum_{h=1}^{L}W_h^2S_{yh}^2+\beta_i\sum_{h=1}^{L}W_h^2\left(\Delta_{x(h)}^*-\Delta_{x(h)}^*\right)=\sum_{h=1}^{L}W_h^2S_{yh}^2 \quad (2.17)$$

This implies that the proposed estimator is unbiased estimators

Differentiating T_{pi} i = 1, 2, 3, 4, 5 partially with respect to $s_{y(e)h}^{*2}$ and $\bar{x}_{(e)h}^*$, we obtained

$$\frac{\partial T_{pi}}{\partial s_{y(e)h}^{*2}}=\sum_{h=1}^{L}W_h^2=1 \quad (2.18)$$

$$\frac{\partial T_{pi}}{\partial \bar{x}_{(e)h}^*}=-\beta_i\sum_{h=1}^{L}W_h^2=-\beta_i \quad (2.19)$$

The mean square errors of the estimators T_{pi}, i = 1, 2, 3, 4, 5. are obtained as

$$MSE\left(T_{pi}\right)=\Delta_{jh}\sum_{h=1}^{L}{}_{jh}\Delta_{jh}^T \quad (2.20)$$

$$MSE\left(T_{pi}\right)=\left[1 \quad -\beta_i\right]\begin{bmatrix}Var\left(s_{y(e)h}^{*2}\right) & Cov\left(s_{y(e)h}^{*2} \quad \bar{x}_{(e)h}^*\right)\\ Cov\left(\bar{x}_{(e)h}^* \quad s_{y(e)h}^{*2}\right) & Var\left(\bar{x}_{(e)h}^*\right)\end{bmatrix}\begin{bmatrix}1\\ -\beta_i\end{bmatrix} \quad (2.21)$$

$$MSE\left(T_{pi}\right)=Var\left(s_{y(e)h}^{*2}\right)-2\beta_iCov\left(s_{y(e)h}^{*2} \quad \bar{x}_{(e)h}^*\right)+\beta_i^2Var\left(\bar{x}_{(e)h}^*\right) \quad (2.22)$$

2.2 Efficiency Comparison

In this section, conditions for the efficiency of the new estimators over Singh *et al,* (2021) are established.

The proposed estimator T_{pi} are more efficient than Singh *et al,* (2021) estimators if

$$MSE\left(T_{pi}\right)<MSE\left(\hat{S}_a^2\right) \quad i=1,2,3,4,5 \quad (3.1)$$

$$Var\left(s_{y(e)h}^{*2}\right)-2\beta_iCov\left(s_{y(e)h}^{*2}, \quad \bar{x}_{(e)h}^*\right)+\beta_i^2Var\left(\bar{x}_{(e)h}^*\right)<\sum_{h=1}^{L}\frac{\left(W_h^2S_{yh}\right)^4}{n_h^3}\left(A_{yh}-\lambda_{21h}^2\theta_{x(h)}\right) \quad (3.2)$$

$$Var\left(s_{y(e)h}^{*2}\right)-2\beta_iCov\left(s_{y(e)h}^{*2}, \quad s_{x(e)h}^{*2}\right)+\beta_i^2Var\left(s_{x(e)h}^{*2}\right)<\sum_{h=1}^{L}\frac{\left(W_h^2S_{yh}\right)^4}{n_h^3}\left(A_{yh}-\lambda_{21h}^2\theta_{x(h)}\right) \quad (3.3)$$

$$Var\left(s_{y(e)h}^{*2}\right)-2\beta_iCov\left(s_{y(e)h}^{*2}, \quad \bar{x}_{(e)h}^*\right)+\beta_i^2Var\left(\bar{x}_{(e)h}^*\right)<\sum_{h=1}^{L}\frac{\left(W_h^2S_{yh}\right)^4}{n_h^3}\left(A_{yh}-\frac{\left(\lambda_{21h}^2-1\right)^2}{A_{x(h)}}\right) \quad (3.4)$$

$$Var\left(s_{y(e)h}^{*2}\right)-2\beta_i Cov\left(s_{y(e)h}^{*2}, s_{y(e)h}^{*2}\right)+\beta_i^2 Var\left(s_{x(e)h}^{*2}\right)<\sum_{h=1}^{L}\frac{\left(W_h^2 S_{yh}\right)^4}{n_h^3}\left(A_{yh}-\frac{\left(\lambda_{21h}^2-1\right)^2}{A_{x(h)}}\right) \quad (3.5)$$

$$Var\left(s_{y(e)h}^{*2}\right)-2\beta_i Cov\left(s_{y(e)h}^{*2}, \ \bar{x}_{(e)h}^{*}\right)+\beta_i^2 Var\left(\bar{x}_{(e)h}^{*}\right)<MSE\left(\hat{S}_a^2\right)-\frac{\left[\lambda_{21}\theta_{x(h)}\lambda_{03h}-\left(\lambda_{22h}-1\right)\right]}{A_{x(h)}} \quad (3.6)$$

$$Var\left(s_{y(e)h}^{*2}\right)-2\beta_i Cov\left(s_{y(e)h}^{*2}, \ s_{x(e)h}^{*2}\right)+\beta_i^2 Var\left(s_{x(e)h}^{*2}\right)<MSE\left(\hat{S}_a^2\right)-\frac{\left[\lambda_{21}\theta_{x(h)}\lambda_{03h}-\left(\lambda_{22h}-1\right)\right]}{A_{x(h)}} \quad (3.7)$$

Empirical Study

In this section, simulation studies to assess the performance of the proposed estimators T_{pi} with respect to Singh *et al.* (2021) estimators under the effect of measurement error only were conducted. Data of size 1000 units were generated for the study population using functions defined in Table 3. A sample of size 100 was selected by the method of Simple Random Sampling without replacement (SRSWOR) 1000 times. The Biases, MSEs and PREs of the considered estimators were computed using (4.1) (4.2) (4.3).

$$Bias(T)=\frac{1}{1000}\sum_{j=1}^{1000}\left(T-\bar{Y}\right) \quad (4.1)$$

$$MSE(T)=\frac{1}{1000}\sum_{j=1}^{1000}\left(T-\bar{Y}\right)^2 \quad (4.2)$$

$$PREs(T)=\left(\frac{MSE(t_O)}{MSE(T)}\right)\times 100 \quad (4.3)$$

where T are any of the existing or proposed estimators.

Table 1: Population Used for Simulation Study

Population	Auxiliary Variable (X)	Study Variable (Y)
1	$X_h \sim N(N,\mu_h,\sigma_h)\mu_1=10,\sigma_1=40,$ $\mu_2=30,\sigma_2=70,\mu_3=20,\sigma_3=50,$ $X_h \sim gamma(\mu_h,\lambda_h)\mu_1=\frac{1}{3},\lambda_1=\frac{5}{7},$ $\mu_2=\frac{1}{2},\lambda_2=\frac{3}{4},\mu_3=\frac{1}{5},\lambda_3=\frac{5}{6},$	$Y_h=0.5X_h+0.5X_h^2+e_h$ Where, $e_h \sim N(0,1)$
2	$X_h \sim N(N,\mu_h,\sigma_h),\mu_1=10,\sigma_1=40,$ $\mu_2=30,\sigma_2=70,\mu_3=20,\sigma_3=50,$ $X_h \sim LN(\mu_h,\sigma_h),\ \mu_1=1,\sigma_1=2,\mu_2=2,$ $\sigma_2=3,\mu_3=1,\sigma_3=5$	

RESULTS AND DISCUSSION

Table 2: Biases, MSEs and PREs of the proposed and existing estimators using data from pop 1

Estimators	Biases	MSEs	PREs
Sample mean t_0	1.516123e+44	1.094586e+89	100
Singh et al. (2021)			
S^2_{a1}	2.726755e+44	3.540569e+89	30.91553
S^2_{a2}	1.360998e+45	8.820555e+90	1.240949
S^2_{a3}	-9.041514e+43	3.892808e+88	281.1815
S^2_{a4}	-9.041514e+43	3.892808e+88	281.1815
S^2_{b1}	1.524502e+44	1.106718e+89	98.90377
S^2_{b2}	1.507744e+44	1.082521e+89	101.1146
S^2_{b3}	7.458111e+44	2.648734e+90	4.132486
S^2_{b4}	7.559851e+44	2.721492e+90	4.022006
S^2_{b5}	7.559851e+44	2.721492e+90	4.022006
S^2_{c1}	1.556495e+44	1.153656e+89	94.87974
S^2_{c2}	-4.593109e+44	1.004763e+90	10.89397
S^2_{c3}	8.919776e+43	3.789692e+88	288.8324
S^2_{c4}	3.077535e+44	5.426608e+89	20.17072
S^2_{c5}	1.216811e+44	3.440941e+89	31.81066
S^2_{c6}	1.439426e+46	2.748697e+93	0.003982199
Proposed estimator T_{Pi}			
T_{p1}	3.034099e+43	4.383693e+87	2496.949
T_{p2}	3.034433e+43	4.38466e+87	2496.398
T_{p3}	3.034434e+43	4.384661e+87	2496.398
T_{p4}	1.51616e+44	1.094639e+89	99.99512
T_{p5}	3.034098e+43	4.383691e+87	2496.951

Table 2 displays the outcomes of biases, mean squared errors (MSEs), and percentage relative efficiency (PREs) for several existing and suggested estimators, using population 1 as defined in Table 2 as the basis. The findings indicate that, with the exception of T_{P4}, all of the proposed estimators exhibit lower MSEs and higher PREs in comparison to the existing estimators considered in this investigation. Moreover, T_{P4} falls short in this regard. Consequently, the proposed estimators T_{p1}, T_{p2}, T_{p3} and T_{p5} under both calibration and L-Moment techniques prove to be more efficient than the competing estimators in the study. They are better suited for producing more accurate estimates of the population parameters, particularly in scenarios involving measurement errors

Table 3: Biases, MSEs and PREs of the proposed and existing estimators using data from pop 2

Estimators	Biases	MSEs	PREs
Sample mean t_0	2.363867e+17	2.771558e+35	100
Singh et al. (2021)			
S^2_{a1}	NaN	NaN	NaN
S^2_{a2}	1.676693e+18	1.415123e+37	1.958527
S^2_{a3}	-1.869346e+17	1.76151e+35	157.3399
S^2_{a4}	-1.869346e+17	1.76151e+35	157.3399
S^2_{b1}	2.378271e+17	2.805544e+35	98.78861
S^2_{b2}	2.349463e+17	2.737779e+35	101.2338
S^2_{b3}	1.097652e+18	5.994453e+36	4.623537
S^2_{b4}	1.113641e+18	6.170415e+36	4.491687
S^2_{b5}	1.113641e+18	6.170415e+36	4.491687
S^2_{c1}	NaN	NaN	NaN
S^2_{c2}	-6.378649e+17	2.089354e+36	13.26514
S^2_{c3}	NaN	NaN	NaN
S^2_{c4}	3.670738e+17	9.33445e+35	29.69171
S^2_{c5}	1.751355e+17	2.475958e+35	111.9388
S^2_{c6}	6.500598e+17	3.983928e+37	0.6956847
Proposed estimator T_{Pi}			
T_{p1}	7.060793e+16	2.465067e+34	1124.333
T_{p2}	6.980662e+16	2.466054e+34	1123.884
T_{p3}	7.064596e+16	2.467694e+34	1123.136
T_{p4}	2.41163e+17	2.885744e+35	96.04309
T_{p5}	7.05868e+16	2.46368e+34	1124.967

Table 3 shows the outcomes regarding biases, mean squared errors (MSEs), and percentage relative efficiencies (PREs) for various existing and newly proposed estimators derived from population 2, as defined in Table 3. The results indicate that, in general, the proposed estimators exhibit the smallest MSEs and the highest PREs compared to the existing estimators in this research, with the exception of T_{P4} that is inferior to certain existing estimators (S^2_{a3}, S^2_{a4}, S^2_{b1}, S^2_{b2} and S^2_{c5}). Specifically, the proposed estimators (T_{p1}, T_{p2}, T_{p3}, T_{p5}) outperform all considered existing estimators except for one instance where they perform worse than a few of them. Consequently, the proposed estimators in this context are more effective than their competitors in the study and are likely to yield superior estimates of population parameters, especially in scenarios involving measurement errors.

CONCLUSION

From the results of the empirical study, it was obtained that the proposed estimators are more efficient than other estimators considered in the study and therefore it is recommended for use for estimating population variance in the presence of non-response and measurement error in stratified random sampling.

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