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A Generalized Family of Ratio-Product Exponential Type Estimator Using Power Transformation

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Abstract

Usage of Auxiliary information has been widely addressed in literature with a view of enhancing the precision of the estimators of population parameters. this paper reviewed research works of some exponential and ratio type estimators for estimating the population parameters using the population mean of an auxiliary variable. In this research, a generalized family of ratio-product type estimator of population mean as well as the coefficient of kurtosis of two auxiliary variables x and z under stratified random sampling has been proposed using power transformation and exponential techniques. The Bias and MSE of proposed estimator have been derived up to second order of approximation. The conditions in which the proposed estimator is better than existing estimators were obtained. The empirical study shows that the proposed estimator is more efficient than existing estimators in reference to the validation statistics employed.

Keywords: Ratio, Exponential, Estimator, Auxiliary variable and Power Transformation.

INTRODUCTION

The problem of estimating population parameters such as the mean, variance, and ratio of two population means which is common in agriculture, economics, medicine, and population studies. The use of auxiliary information has been applied to improve the efficiency of the estimators of the population parameters, regardless of the sampling design. Various modifications and estimation of step ratio, product and regression type estimators have been suggested by various researchers in literature, utilizing one or more auxiliary variables to estimate the finite population mean under different sampling techniques. Cochran (1940) proposed the ratio estimator assuming that the study variable (y) and auxiliary variable (x) are positively correlated, and the population mean of the auxiliary variable is known. However, when the study variable (y) and the auxiliary variable (x) are negatively correlated then the ratio estimator does not perform well. In that situation, the product estimator envisaged by Robson (1957) is appropriate. Robson (1957) defined a product estimator which was examined by Murthy (1964). The product estimator is used when the correlation between the study and the auxiliary variables is negative.

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Sisodia and Dwivedi (1981), Upadhyaya and singh (1999). Singh *et al.* (2004) and Singh and Tailor (2005) later used known values of various parameters of an auxiliary variable in simple random sampling. When two auxiliary variables are present Perri (2007), Abu-Dayeh *et al.* (2003), Kadilar and Cingi (2005), Tailor *et al.* (2012), Singh and Kumar (2012), Sharma and Singh (2014), and Lu and Yan (2014) suggested some estimators for estimating the population mean. For recent development, exponential estimators have been widely studied by several authors including Lu *et al*. (2014) and Koyuncu and Ozel (2013) to minimize the mean square errors of estimators under different situations. A combined ratio and product are basic estimators of the population mean that use information about the auxiliary variable in stratified random sampling. Singh and Espejo (2003) suggested a ratio product estimator using a scalar, it is quite efficient in a negative correlation. Singh and Tailor (2005) and Tailor and Sharma (2009) proposed an estimator with respect to the ratio-cum-product of a finite population mean in simple random sampling. Tailor (2009) defined a ratio-product estimator in stratified random sampling. Koyuncu (2013), Yadav *et al.* (2014) among others suggested estimators using auxiliary information. In their paper they have tried to generalize Singh *et al*. (2008) and Yadav *et al.* (2014) estimators and suggested two exponential type estimators in stratified random sampling. Hansen et al. (1946) defined combined ratio estimator for estimating the population mean. Kadilar and Cingi (2003) defined a ratio-type estimator for population mean using coefficient of variation and coefficient of kurtosis of auxiliary variate. Singh et al. (2008) suggested a class of estimators of population mean using power transformation in stratified random sampling. Singh (1967) utilized information on population mean of two auxiliary variates and envisaged ratio-cum-product estimator for population mean. Tailor *et al*. (2012) studied Singh (1967) estimator in stratified random sampling. Tailor *et al*. (2013) discussed dual to ratio and product type exponential estimators of population mean in stratified random sampling. Parmar (2013) studied a ratio-cum-product estimator of population mean in stratified random sampling using coefficient of variation of auxiliary variates. Tailor *et al*. (2015) defined ratio estimators and ratio-cum-product estimators under stratified random sampling, which perform better than usual ratio and product estimators in simple random sampling under certain limitations.

This research employed the strategy of power transformation and incorporated the constant weight to Tailor *et al*. (2015) estimator and developed a new efficient estimator that is effective in enriching the accuracy of estimating population mean in two auxiliary variables under stratified random sampling

MATERIALS AND METHODS.

Procedure, Notations, and Definitions

Let's first consider a finite population $U = U_1, U_2, U_3, ...U_N$ of size N, which is divided into L strata of size $N_h \left(h = 1, 2..., L\right)$. Let y be the study variates and x and z be two auxiliary variates taking values y_{hi}, x_{hi}, and $z_{hi} \left(h = i, 2..., L; i = 1, 2...N_i\right)$ respectively, on the ith unit of the hth stratum. Here x is positively correlated with study variates y and z is negatively correlated with study variates y. A sample of size n_h is drawn from each stratum which constitutes a sample of size $n = \sum_{h=1}^{L} n_h$

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Let y be the study variates and x and z be two auxiliary variates taking values y_{hi} , x_{hi} and z_{hi} (h = 1, 2, …, L; i = 1, 2, …, N_h), respectively, on the i^{th} unit of the h^{th} stratum.
Here x is positively correlated with study variates y and z is negatively correlated with study variates y. A sample of size n_h is drawn from each stratum which constitutes a sample of size $n = \sum_{h=1}^{L} n_h$.

Now we define;

$\overline{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: h^{th} Stratum mean for the study variable Y.

$\overline{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} Stratum mean for the study variable X.

$\overline{Z} = \frac{1}{N_h}\sum_{i=1}^{N_h} z_{hi}$: h^{th} Stratum mean for the study variable Z.

$\overline{Y} = \sum_{h=1}^{L} W_h \overline{Y_h}$: Population mean of the study variable Y.

$\overline{X} = \frac{1}{N}\sum_{h=1}^{L} W_h \overline{X}$: Population mean of the study variable X.

$\overline{Z} = \sum_{h=1}^{L} W_h \overline{Z_h}$: Population mean of the study variable Z.

$\overline{y}_h = \frac{1}{n_h}\sum_{i}^{n_h} y_{hi}$: Sample mean for the study variable Y for h^{th} stratum.

$\overline{x}_h = \frac{1}{n_h}\sum_{i}^{n_h} x_{hi}$: Sample mean for the study variable X for h^{th} stratum.

$\overline{z}_h = \frac{1}{n_h}\sum_{i}^{n_h} z_{hi}$: Sample mean for the study variable Z for h^{th} stratum.

$W_h = \frac{N_h}{N}$ is the weight of h^{th} stratum.

Where;

$\overline{y}_{st}$, $\overline{x}_{st}$ and $\overline{z}_{st}$ are means of sample taken from h^{th} stratum for variates y, x and z respectively.

$\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y}_h$: Unbiased estimator of the population mean $\overline{Y}$

$\overline{x}_{st} = \sum_{h=1}^{L} W_h \overline{x}_h$: Unbiased estimator of the population mean $\overline{X}$

$\overline{z}_{st} = \sum_{h=1}^{L} W_h \overline{z}_h$: Unbiased estimator of the population mean $\overline{Z}$

The variance of unbiased estimator $\overline{y}_{st}$, $\overline{x}_{st}$ and $\overline{z}_{st}$ are;

$$Var(\overline{y}_{st}) = \sum_{h=1}^{L} W_h^2 \psi_h S_{yh}^2 \qquad (1)$$

$$Var(\overline{x}_{st}) = \sum_{h=1}^{L} W_h^2 \psi_h S_{xh}^2 \qquad (2)$$

$$Var(\bar{z}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{zh}^2 \tag{3}$$

Kadilar and Cingi (2003) defined y_{sidw}, y_{sh}, y_{upsi1} and y_{upsi2} in stratified random sampling respectively as

$$y_{sidw}^{st} = \bar{y}_{st}\left[\frac{W_h\left(\bar{X}_h + C_{xh}\right)}{\sum_{h=1}^{k} W_h\left(\bar{x}_h + C_{xh}\right)}\right] \tag{4}$$

$$y_{sh}^{st} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{k} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{k} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right] \tag{5}$$

$$y_{upsi_1}^{st} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{k} W_h\left(\bar{X}_h\beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{k} W_h\left(\bar{x}_h\beta_{2h}(x) + C_{xh}\right)}\right] \tag{6}$$

$$y_{upsi_2}^{st} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{k} W_h\left(\bar{X}_h C_{xh} + \beta_{2h}(x)\right)}{\sum_{h=1}^{k} W_h\left(\bar{x}_h C_{xh} + \beta_{2h}(x)\right)}\right] \tag{7}$$

To the first degree of approximation mean square errors of y_{sidw}, y_{sh}, y_{upsi1} and y_{upsi2} respectively are;

$$MSE(y_{sidw}^{st}) = \sum_{h=1}^{L} W^2{}_h \psi_h\left(S^2{}_{yh} + R_{sidw}^2 S_{xh}^2 - 2R_{sidw} S_{yxh}\right) \tag{8}$$

$$MSE(y_{sh}^{st}) = \sum_{h=1}^{L} W^2{}_h \psi_h\left(S_{yh}^2 + R_{sial}^2 S_{xh}^2 - 2R_{sial} S_{yxh}\right) \tag{9}$$

$$MSE(y_{upsi_1}^{st}) = \sum_{h=1}^{L} W^2{}_h \psi_h\left(S_{yh}^2 + R_{upsi_1}^2 \beta_{2h}^2(x) S_{xh}^2 - 2R_{upsi_1}\beta_{2h}(x) S_{yxh}\right) \tag{10}$$

$$MSE(y_{upsi_2}^{st}) = \sum_{h=1}^{L} W^2{}_h \psi_h\left(S_{yh}^2 + R_{upsi_2}^2 C_{xh}^2 S_{xh}^2 - 2R_{upsi_2} C_{xh} S_{yxh}\right)$$

(11)

Where, $R_{sidw} = \dfrac{\bar{Y}}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right)}$, $R_{sh} = \dfrac{\bar{Y}}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}$, $R_{upsi_1} = \dfrac{\bar{Y}}{\sum_{h=1}^{L} W_h\left(\bar{X}_h\beta_{2h}(x) + C_{xh}\right)}$

$$R_{upsi_2} = \frac{\bar{Y}}{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{xh} + \beta_{2h}(x)\right)} \tag{11}$$

Tailor *et al.* (2011) proposed a generalized ratio-cum-product estimator of population mean in stratified random sampling as;

$$\bar{y}_5 = \bar{y}_{st}\left[\alpha\left\{\frac{\sum_{h=1}^{L} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x))}{\sum_{h=1}^{L} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x))}\right\} + (1-\alpha)\left\{\frac{\sum_{h=1}^{L} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x))}{\sum_{h=1}^{L} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x))}\right\}\right] \tag{12}$$

$$Bias\left(y_{tailor}\right) = \frac{1}{X_{upsi_2}} \sum_{h=1}^{k} W_h^2 \psi_h C_{xh}\left[\alpha R_{upsi_2} C_{xh} S_{xh}^2 + (1-2\alpha) S_{yxh}\right] \tag{13}$$

$$MSE\left(y_{tailor}^{opt}\right)=\sum_{1}^{k}W_h^2\psi_h S_{yh}^2-\frac{\left(\sum_{h=1}^{k}W_h^2\gamma hC_{xh}S_{yxh}\right)^2}{\sum_{h=1}^{k}W_h^2\gamma hC_{xh}^2S_{xh}^2} \tag{14}$$

Combined ratio and product estimators for estimating the population mean $\overline{Y}$ in stratified random sampling are expressed as

$$\overline{y}_{RC}=\overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right) \tag{15}$$

$$\overline{y}_{PC}=\overline{y}_{st}\left(\frac{\overline{z}_{st}}{\overline{Z}}\right) \tag{16}$$

Bias and mean squared error of $\overline{y}_{RC}$ and $\overline{y}_{PC}$ are

$$B(\overline{y}_{PC})=\frac{1}{\overline{X}}\sum_{h=1}^{L}W_h^2\psi_h(R_1S_{xh}^2-S_{yxh}) \tag{17}$$

$$B(\overline{y}_{PC})=\frac{1}{\overline{Z}}\sum_{h=1}^{L}W_h^2\psi_h S_{yzh} \tag{18}$$

$$MSE(\overline{y}_{RC})=\sum_{h=1}^{L}W_h^2\psi_h(S_{yh}^2+R_1^2S_{xh}^2-2R_1S_{yxh}) \tag{19}$$

$$MSE(\overline{y}_{PC})=\sum_{h=1}^{L}W_h^2\psi_h(S_{yh}^2+R_2^2S_{zh}^2+2R_2S_{yzh}) \tag{20}$$

Where $R_1=\frac{\overline{Y}}{\overline{X}}$ and $R_2=\frac{\overline{Y}}{\overline{Z}}$

Singh et al. (2008) defined t_{Re} and t_{Pe} in stratified random sampling as;

$$t_{\mathrm{Re}}^{st}=\overline{y}_{st}\exp\left[\frac{\sum_{h=1}^{L}W_h(\overline{X}_h-\overline{x}_h)}{\sum_{h=1}^{L}W_h(\overline{X}_h+\overline{x}_h)}\right] \tag{21}$$

$$t_{Pe}^{st}=\overline{y}_{st}\exp\left[\frac{\sum_{h=1}^{L}W_h(\overline{z}_h-\overline{Z}_h)}{\sum_{h=1}^{L}W_h(\overline{z}_h+\overline{Z}_h)}\right] \tag{22}$$

To the first degree of approximation biases and mean squared errors of t^{st}_{Re} and t^{st}_{Pe} are;

$$B(t_{\mathrm{Re}}^{st})=\frac{1}{\overline{X}}\sum_{h=1}^{L}W_h^2\psi_h\left(\frac{3}{8}R_1S_{xh}^2-\frac{1}{2}S_{yxh}\right) \tag{23}$$

$$B(t_{Pe}^{st})=\frac{1}{\overline{Z}}\sum_{h=1}^{L}W_h^2\psi_h\left(-\frac{1}{8}R_2S_{zh}^2+\frac{1}{2}S_{yzh}\right) \tag{24}$$

$$MSE(t_{\mathrm{Re}}^{st})=\sum_{h=1}^{L}W_h^2\psi_h\left(S_{yh}^2+\frac{1}{4}R_1S_{xh}^2-R_1S_{yxh}\right) \tag{25}$$

$$MSE(t_{Pe}^{st})=\sum_{h=1}^{L}W_h^2\psi_h\left(S_{yh}^2+\frac{1}{4}R_2S_{zh}^2-R_2S_{yzh}\right) \tag{26}$$

Tailor et al. (2012) proposed a ratio-cum-product estimator of the population mean Y in stratified random sampling as:

$$\hat{\bar{Y}}_{RP}^{ST} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) = \sum_h^L W_h\bar{Y}_h\left(\frac{\sum_h^L W_h\bar{X}_h}{\sum_h^L W_h\bar{x}_h}\right)\left(\frac{\sum_h^L W_h\bar{z}_h}{\sum_h^L W_h\bar{Z}_h}\right) \quad (27)$$

$$MSE\left(\bar{y}_{RP}^{st}\right) = \sum_{h=1}^{K} W_h^2\psi_h\left(S_{yh}^2 + R_1^2S_{xh}^2 + R_2^2S_{zh}^2 - 2R_1S_{yxh} + 2R_2S_{yzh} - 2R_1R_2S_{xzh}\right) \quad (28)$$

Tailor et al. (2015) utilized the information of the coefficient of kurtosis of the auxiliary variables x and z, and proposed a ratio-cum-product estimator of the population mean Y under stratified random sampling as;

$$\hat{\bar{Y}}_{RP1}^{ST} = \sum_h^L W_h\bar{Y}_h\left(\frac{\sum_h^L W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_h^L W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right)\left(\frac{\sum_h^L W_h\left(\bar{z}_h + \beta_{2h}(x)\right)}{\sum_h^L W_h\left(\bar{Z}_h + \beta_{2h}(x)\right)}\right) \quad (29)$$

$$MSE\left(\bar{y}_{RP1}^{st}\right) = \sum_{h=1}^{K} W_h^2\psi_h\left(S_{yh}^2 + R_{12}^2S_{xh}^2 + R_{13}^2S_{zh}^2 - 2R_{12}S_{yxh} + 2R_{13}S_{yzh} - 2R_{12}R_{13}S_{xzh}\right) \quad (30)$$

Proposed Estimator

A New ratio-product exponential type estimator of population mean using the approach of power transformation to improve the inference of Tailor *et al.*, (2015) ratio type estimator. The proposed estimator is given as

$$\bar{y}_{ARP} = \bar{y}_{st}\left[\exp\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h(x)}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h(x)}\right)}\right)\right]^{\theta_1}\left[\exp\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \beta_{2h(z)}\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \beta_{2h(z)}\right)}\right)\right]^{\theta_2} \quad (31)$$

where θ_1 and θ_2 are constants that can takes values (0,1,-1) for designing different estimators; which are either real numbers or the function of the known parameters, $\beta_{2h}(x)$ and $\beta_{2h}(z)$ are coefficients of kurtosis of auxiliary variates x and z in h^{th} stratum respectively.

2.1.1 Properties (Biases and MSEs) for Proposed Estimator

In this section, the biases and MSEs for estimator is derived and discussed. Suitable specified unknown constant θ_1 and θ_2 are decided in such a manner that the mean square error of the proposed estimator is minimum. To achieve the asymptotic properties of the estimator, the following error terms are defined in line with the work of Singh *et al.,*2004;

$\bar{y}_h = \bar{Y}\left(1+\varepsilon_{yh}\right)$, $\bar{x}_h = \bar{X}\left(1+\varepsilon_{xh}\right)$ and $\bar{z}_h = \bar{Z}\left(1+\varepsilon_{zh}\right)$

$E\left(\varepsilon_{xh}\right) = E\left(\varepsilon_{yh}\right) = E\left(\varepsilon_{zh}\right) = 0$, $E\left(\varepsilon^2{}_{xh}\right) = \frac{1}{\bar{X}^2}\sum W_h^2\Psi_h S_{xh}^2$, $E\left(\varepsilon^2{}_{yh}\right) = \frac{1}{\bar{Y}^2}\sum W_h^2\Psi_h S_{yh}^2$ and $E\left(\varepsilon^2{}_{zh}\right) = \frac{1}{\bar{Z}^2}\sum W_h^2\Psi_h S_{zh}^2$

$$E\left(\varepsilon_{yh}\varepsilon_{xh}\right) = \Psi_h\rho_{yxh}C_{yh}C_{xh}\frac{\Psi_h S_{yxh}}{\bar{Y}_h\bar{X}_h}, E\left(\varepsilon_{xh}\varepsilon_{zh}\right) = \Psi_h\rho_{xzh}C_{xh}C_{zh}\frac{\Psi_h S_{xzh}}{\bar{X}_h\bar{Z}_h}, E\left(\varepsilon_{yh}\varepsilon_{zh}\right) = \Psi_h\rho_{yzh}C_{yh}C_{zh}\frac{\Psi_h S_{yzh}}{\bar{Y}_h\bar{Z}_h}$$

Express (31) in terms of error terms, we have;

$$\bar{y}_{ARP} = \sum W_h\bar{Y}_h(1+\varepsilon_{yh})\left[\exp\left(\frac{\sum W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum W_h\left(\bar{X}_h\left(1+\varepsilon_{xh}\right) + \beta_{2h}(x)\right)}\right)\right]^{\theta_1}\left[\exp\left(\frac{\sum W_h\left(\bar{Z}_h\left(1+\varepsilon_{zh}\right) + \beta_{2h}(z)\right)}{\sum W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right)\right]^{\theta_2} \quad (32)$$

$$\bar{y}_{ARP} = \bar{Y} + \bar{Y}\varepsilon_{yh}\left[\exp\left(\frac{\sum W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum W_h\left(\bar{X}_h + \beta_{2h}(x)\right) + \bar{X}\varepsilon_{xh}}\right)\right]^{\theta_1}\left[\exp\left(\frac{\sum W_h\left(\bar{Z}_h + \beta_{2h}(z)\right) + \bar{Z}\varepsilon_{zh}}{\sum W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right)\right]^{\theta_2} \quad (33)$$

$$\bar{y}_{ARP} = \bar{Y} + \bar{Y}\varepsilon_{yh}\left[\exp\left(\frac{1}{1 + \frac{\bar{X}\varepsilon_{xh}}{\sum W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}}\right)\right]^{\theta_1}\left[\exp\left(1 + \frac{\bar{Z}_h\varepsilon_{zh}}{\sum W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right)\right]^{\theta_2} \quad (34)$$

Let

$$\alpha_1 = \frac{\sum W_h \bar{X}_h}{\sum W_h\left(\bar{X}_h + \beta_{2h}(x)\right)} \quad \text{and} \quad \alpha_2 = \frac{\sum W_h \bar{Z}_h}{\sum W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}$$

(34) becomes;

$$\bar{y}_{ARP} = \bar{Y} + \bar{Y}\varepsilon_{yh}\left[\exp\left(1 - \alpha_1\varepsilon_{xh} + \alpha_1^2\varepsilon_{xh}^2\right)\right]^{\theta_1}\left[\exp\left(1 + \alpha_2\varepsilon_{zh}\right)\right]^{\theta_2} \quad (35)$$

$$\bar{y}_{ARP} = \left(\bar{Y} + \bar{Y}\varepsilon_{yh}\right)\left[1 - \frac{4}{5}\alpha_1\varepsilon_{xh} + \alpha_1^2\varepsilon_{xh}^2\right]^{\theta_1}\left[1 + \frac{4}{5}\alpha_2\varepsilon_{zh} + \frac{1}{5}\alpha_2^2\varepsilon_{zh}^2\right]^{\theta_2} \quad (36)$$

$$\bar{y}_{ARP} = \left(\bar{Y} + \bar{Y}\varepsilon_{yh}\right)\left[1 - \frac{4}{5}\alpha_1\theta_1\varepsilon_{xh} + \left(\frac{17 + 8\theta_1}{25}\right)\alpha_1^2\theta_1\varepsilon_{xh}^2 + \frac{4}{5}\alpha_2\theta_2\varepsilon_{zh} - \frac{16}{25}\alpha_1\alpha_2\theta_1\theta_2\varepsilon_{xh}\varepsilon_{zh} + \left(\frac{8\theta_2 - 3}{25}\right)\alpha_2^2\theta_2\varepsilon_{zh}^2\right] \quad (37)$$

$$\bar{y}_{ARP} = \bar{Y} + \bar{Y}\left[\begin{array}{l}\varepsilon_{xh} - \frac{4}{5}\alpha_1\theta_1\varepsilon_{xh} + \frac{4}{5}\alpha_2\theta_2\varepsilon_{zh} + \left(\frac{17 + 8\theta_1}{25}\right)\alpha_1^2\theta_1\varepsilon_{xh}^2 + \left(\frac{8\theta_2 - 3}{25}\right)\alpha_2^2\theta_2\varepsilon_{zh}^2 \\ -\frac{16}{25}\alpha_1\alpha_2\theta_1\theta_2\varepsilon_{xh}\varepsilon_{zh} - \frac{4}{5}\alpha_1\theta_1\varepsilon_{yh}\varepsilon_{xh} + \frac{4}{5}\alpha_2\theta_2\varepsilon_{yh}\varepsilon_{zh}\end{array}\right] \quad (38)$$

Subtract $\bar{Y}$ from both sides of (38) and take expectation to obtain the bias of $\bar{y}_{ARP}$ as:

$$Bias\left(\bar{y}_{ARP}\right) = \bar{Y}\left[\begin{array}{l}\left(\frac{17+8\theta_1}{25}\right)\alpha_1^2\theta_1\frac{1}{\bar{X}}\sum_{h=1}^{L}W_h^2\psi_h S_{xh}^2 + \left(\frac{8\theta_2 - 3}{25}\right)\alpha_2^2\theta_2\frac{1}{\bar{Z}}\sum_{h=1}^{L}W_h^2\psi_h S_{zh}^2 - \frac{16}{25}\alpha_1\alpha_2\theta_1\theta_2\frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L}W_h^2\psi_h\rho_{xzh}S_{xh}S_{zh} \\ -\frac{4}{5}\alpha_1\theta_1\frac{1}{\bar{X}\bar{Y}}\sum_{h=1}^{L}W_h^2\psi_h\rho_{xyh}S_{xh}S_{yh} + \frac{4}{5}\alpha_2\theta_2\frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{L}W_h^2\psi_h\rho_{yzh}S_{yh}S_{zh}\end{array}\right] \quad (39)$$

Subtract $\bar{Y}$ from both sides of (38), squaring, then take expectation to obtain the MSE of $\bar{y}_{ARP}$ as:

$$MSE\left(\bar{y}_{ARP}\right) = \mathrm{E}\left[\bar{Y}\left(\varepsilon_{yh} - \frac{4}{5}\alpha_1\theta_1\varepsilon_{xh} + \frac{4}{5}\alpha_2\theta_2\varepsilon_{zh}\right)\right]^2 \quad (40)$$

$$\begin{array}{l}MSE\left(\bar{y}_{ARP}\right) = \sum_{h=1}^{L}W_h^2\psi_h S_{yh}^2 + \frac{16}{25}\frac{\bar{Y}^2}{\bar{X}^2}\alpha_1^2\theta_1^2\sum_{h=1}^{L}W_h^2\psi_h S_{xh}^2 + \frac{16}{25}\frac{\bar{Y}^2}{\bar{Z}^2}\alpha_2^2\theta_2^2\sum_{h=1}^{L}W_h^2\psi_h S_{zh}^2 \\ -\frac{8}{5}\frac{\bar{Y}}{\bar{X}}\alpha_1\theta_1\sum_{h=1}^{L}W_h^2\psi_h\rho_{xyh}S_{xh}S_{yh} + \frac{8}{5}\frac{\bar{Y}}{\bar{Z}}\alpha_2\theta_2\sum_{h=1}^{L}W_h^2\psi_h\rho_{yzh}S_{yh}S_{zh} - \frac{32}{25}\frac{\bar{Y}\bar{Y}}{\bar{X}\bar{Z}}\alpha_1\alpha_2\theta_1\theta_2\sum_{h=1}^{L}W_h^2\psi_h\rho_{xzh}S_{xh}S_{zh}\end{array} \quad (41)$$

Let $R_1 = \frac{\bar{Y}}{\bar{X}}$ and $R_2 = \frac{\bar{Y}}{\bar{Z}}$,

$$A=\sum_{h=1}^{L}W_h^2\psi_h S_{yh}^2=V\left(\bar{y}_{st}\right),B=\sum_{h=1}^{L}W_h^2\psi_h S_{xh}^2,$$

$$C=\sum_{h=1}^{L}W_h^2\psi_h S_{zh}^2,\ D=\sum_{h=1}^{L}W_h^2\psi_h\rho_{xyh}S_{yh}S_{xh},$$

$$E=\sum_{h=1}^{L}W_h^2\psi_h\rho_{yzh}S_{yh}S_{zh}\text{ and }F=\sum_{h=1}^{L}W_h^2\psi_h\rho_{xzh}S_{xh}S_{zh}$$

$$\rho_{xyh}=\frac{S_{xyh}}{S_{xh}S_{yh}},\ \rho_{yzh}=\frac{S_{yzh}}{S_{yh}S_{zh}}\text{ and }\rho_{xzh}=\frac{S_{xzh}}{S_{xh}S_{zh}}$$

$$MSE\left(\bar{y}_{ARP}\right)=A+\frac{16}{25}R_1^2\alpha_1^2\theta_1^2B+\frac{16}{25}R_2^2\alpha_2^2\theta_2^2C-\frac{8}{5}R_1\alpha_1\theta_1D+\frac{8}{5}R_2\alpha_2\theta_2E-\frac{32}{25}R_1R_2\alpha_1\alpha_2\theta_1\theta_2F \quad (42)$$

Minimum MSEs of $\bar{y}_{ARP}$

The minimum MSE of at optimum values of $\bar{y}_{ARP}$ and is given by θ_1 and θ_2

To obtain the minimum MSE, $\bar{y}_{ARP}$ is partially differentiated with respect to θ_1 and θ_2 respectively and equate to zero as:

$$\frac{\partial MSE\left(\bar{y}_{ARP}\right)}{\partial\theta_1}=\frac{\partial MSE\left(\bar{y}_{ARP}\right)}{\partial\theta_2}=0 \quad (43)$$

$$\theta_1=\frac{\frac{5}{4}D+R_2\alpha_2\theta_2F}{R_1\alpha_1B} \quad (44)$$

$$\theta_2=\frac{R_1\alpha_1\theta_1F-\frac{5}{4}E}{R_2\alpha_2C} \quad (45)$$

Substitute (45) in ((44), we have

$$\theta_{1opt}=\frac{5\left(DC-EF\right)}{4R_1\alpha_1\left(BC-F^2\right)} \quad (46)$$

Put (44) into (45), we have

$$\theta_{2opt}=\frac{5\left(\left(DF-BE\right)\right)}{4R_2\alpha_2\left(BC-F^2\right)} \quad (47)$$

Substitute (46) and (47) in (42), the minimum MSE of $\bar{y}_{ARP}$ is obtained as

$$MSE\left(\bar{y}_{ARP}\right)_{\min}=A+\frac{4}{5}R_1\alpha_1\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}B+\frac{4}{5}R_2\alpha_2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}C-2\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}D$$
$$+2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}E-2\frac{\left(DC-EF\right)\left(DF-BE\right)}{\left(BC-F^2\right)^2}F \quad (48)$$

Theoretical Efficiency Comparison

Relevant conditions under which the proposed estimator $\left(\bar{y}_{ARP}\right)$ is better than the modified estimators of Tailor *et al.*, (2015) and the conventional estimators are presented below;

i. **Usual unbiased estimator $\bar{y}_{ST}$**

Recall

$$V\left(\bar{y}_{st}\right)=\sum_{h=1}^{L}W_{h}^{2}\psi_{h}S_{yh}^{2}=A$$

$MSE\left(\bar{y}_{ARP}\right)_{\min} < MSE\left(\bar{y}_{st}\right)$ If and only if:

$$\begin{aligned}&\frac{4}{5}R_1\alpha_1\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}B+\frac{4}{5}R_2\alpha_2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}C-2\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}D\\&\quad+2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}E-2\frac{\left(DC-EF\right)\left(DF-BE\right)}{\left(BC-F^2\right)^2}F<0\end{aligned} \tag{49}$$

ii. Combined ratio estimator $\widehat{Y}_{RC}^{ST}$

$MSE\left(\bar{y}_{ARP}\right)-MSE\left(\bar{y}_{RC}\right)<0$ If,

$$\begin{aligned}&A+\frac{4}{5}R_1\alpha_1\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}B+\frac{4}{5}R_2\alpha_2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}C-2\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}D\\&+2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}E-2\frac{\left(DC-EF\right)\left(DF-BE\right)}{\left(BC-F^2\right)^2}F-\left(A-R_1^2B+2R_1D\right)<0\end{aligned} \tag{50}$$

iii. Combined product estimator $\widehat{Y}_{PC}^{ST}$

$MSE\left(\bar{y}_{ARP}\right)-MSE\left(\bar{y}_{PC}\right)<0$ If,

$$\begin{aligned}&\frac{4}{5}R_1\alpha_1\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}B+\left(\frac{4}{5}\alpha_2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}-R_2\right)R_2C-2\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}D\\&+\left(\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}-R_2\right)2E-2\frac{\left(DC-EF\right)\left(DF-BE\right)}{\left(BC-F^2\right)^2}F<0\end{aligned} \tag{51}$$

iv. Kadilar and Cingi (2003) ratio type estimator $\widehat{Y}_{SER}^{ST}$ If,

$$\begin{aligned}&\left(\frac{4}{5}R_1\alpha_1\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}-R_{1SE}^2\right)B+\frac{4}{5}R_2\alpha_2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}C-\left(\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}-R_{1SE}\right)2D\\&+2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}E-2\frac{\left(DC-EF\right)\left(DF-BE\right)}{\left(BC-F^2\right)^2}F<0\end{aligned} \tag{52}$$

v. Kadilar and Cingi (2003) product type estimator of $\widehat{Y}_{SEP}^{ST}$

$MSE\left(\bar{y}_{ARP}\right)-MSE\left(\bar{y}_{SEP}\right)<0$ If,

$$\begin{aligned}&\frac{4}{5}R_1\alpha_1\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}B+\left(\frac{4}{5}R_2\alpha_2\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}-R_{2SE}^2\right)C-2\frac{\left(DC-EF\right)}{\left(BC-F^2\right)}D\\&+\left(\frac{\left(DF-BE\right)}{\left(BC-F^2\right)}-R_{2SE}\right)2E-2\frac{\left(DC-EF\right)\left(DF-BE\right)}{\left(BC-F^2\right)^2}F<0\end{aligned} \tag{53}$$

vi. Tailor et al. (2012) estimator $\widehat{Y}_{RP}^{ST}$

$MSE\left(\bar{y}_{ARP}\right)-MSE\left(\bar{y}_{RP}\right)<0$ If,

$$\left(\frac{4}{5}\alpha_1\frac{(DC-EF)}{(BC-F^2)}-R_1\right)R_1B+\left(\frac{4}{5}\alpha_2\frac{(DF-BE)}{(BC-F^2)}-R_2\right)R_2C-\left(\frac{(DC-EF)}{(BC-F^2)}-R_1\right)2D$$
$$+\left(\frac{(DF-BE)}{(BC-F^2)}-R_2\right)2E-\left(\frac{(DC-EF)(DF-BE)}{(BC-F^2)^2}-R_1R_2\right)2F<0 \quad (54)$$

Tailor et al. (2015) estimator $\hat{\bar{Y}}_{RP1}^{ST}$

$MSE(\bar{y}_{ARP})-MSE(\bar{y}_{RP1})<0$

$$\left(\frac{4}{5}\alpha_1\frac{(DC-EF)}{(BC-F^2)}-R_{1SE}R_1\right)B+\left(\frac{4}{5}\alpha_2\frac{(DF-BE)}{(BC-F^2)}-R_{2SE}R_2\right)C-\left(\frac{(DC-EF)}{(BC-F^2)}-R_{1SE}\right)2D$$
$$+\left(\frac{(DF-BE)}{(BC-F^2)}-R_{2SE}\right)2E-\left(\frac{(DC-EF)(DF-BE)}{(BC-F^2)^2}-R_{1SE}R_{2SE}\right)2F<0 \quad (56)$$

From equations (49)-(56), we obtained the conditions under which the proposed estimator performed better than the usual unbiased estimator, combined ratio estimator $\hat{\bar{Y}}_{RC}^{ST}$, Tailor et al. (2012) estimator $\hat{\bar{Y}}_{RP}^{ST}$ and Tailor et al. (2015) estimator $\hat{\bar{Y}}_{RP1}^{ST}$.

EMPIRICAL STUDY

Population I [Source: Murthy (1967)]
Y: Output, X: Fixed capital, Z: Number of workers
Population 2 [Source: National Horticulture Board (2010)]
N=10, n=5

Population 2 [Source: National Horticulture Board (2010), page 208] on All India area,

$n_1=2, n_2=2$	$N_1=5, N_2=5$	$S_{yz1}=-411.16$	$S_{yz2}=-1536.24$
$\bar{X}_1=214.40$	$\bar{Y}_1=1925.80$	$\bar{Z}_1=51.80$	$S_{yx1}=39360.68$
$\bar{X}_2=333.80$	$\bar{Y}_2=3115.60$	$\bar{Z}_2=60.60$	$S_{yx2}=22356.50$
$S_{x1}=74.87$	$S_{y1}=615.92$	$S_{z1}=0.75$	$S_{zx1}=-38.08$
$S_{x2}=66.35$	$S_{y2}=340.38$	$S_{z2}=4.84$	$S_{zx2}=-287.92$

Production and productivity of Cashew-nut.

Table 1: MSE and PRE using Population 1

Estimators	MSE	PRE
$\bar{y}_{st}$	37141.2	100
$\hat{\bar{Y}}_{RC}^{ST}$	119383.7	31.11077
$\hat{\bar{Y}}_{PC}^{ST}$	25382.91	146.3236
$\hat{\bar{Y}}_{SER}^{ST}$	118232.4	31.41373
$\hat{\bar{Y}}_{SEP}^{ST}$	25347.69	146.527
$\hat{\bar{Y}}_{RP}^{ST}$	419159.3	8.86088
$\hat{\bar{Y}}_{RP1}^{ST}$	184201.6	20.16334
$\bar{y}_{ARP}$	8573.731	433.1977

From Table 1, the proposed estimator $\bar{y}_{ARP}$ has less mean squared error and high relative efficiency than that of usual unbiased estimator $\bar{y}_{st}$, combined ratio $\hat{\bar{Y}}_{RC}^{ST}$ and product estimators $\hat{\bar{Y}}_{PC}^{ST}$, Kadilar and Cingi (2003) $\hat{\bar{Y}}_{SER}^{ST}$ and $\hat{\bar{Y}}_{SEP}^{ST}$, Tailor *et.al* (2012) $\hat{\bar{Y}}_{RP}^{ST}$ and Tailor *et.al* (2015) $\hat{\bar{Y}}_{RP1}^{ST}$ estimator.

Table 2: MSE and PRE using Population 2

Estimators	MSE	PRE
$\bar{y}_{ST}$	0.114516	100
$\hat{\bar{Y}}_{RC}^{ST}$	0.1156092	99.05444
$\hat{\bar{Y}}_{PC}^{ST}$	0.216607	52.8681
$\hat{\bar{Y}}_{SER}^{ST}$	0.1156821	98.99195
$\hat{\bar{Y}}_{SEP}^{ST}$	0.2186503	52.37403
$\hat{\bar{Y}}_{RP}^{ST}$	0.2249535	50.90653
$\hat{\bar{Y}}_{RP1}^{ST}$	0.2197911	52.10221
$\bar{y}_{ARP}$	0.1140415	100.4161

From Table 2, we came up with a conclusion that MSE and PRE of the proposed estimator has less and high relative efficient than all the other considered estimators respectively. So, we can say that proposed estimator is better than the usual unbiased estimator $\bar{y}_{st}$, combined ratio $\hat{\bar{Y}}_{RC}^{ST}$ and product estimators $\hat{\bar{Y}}_{PC}^{ST}$, Kadilar and Cingi (2003) $\hat{\bar{Y}}_{SER}^{ST}$ and $\hat{\bar{Y}}_{SEP}^{ST}$, Tailor *et.al* (2012) $\hat{\bar{Y}}_{RP}^{ST}$ and Tailor *et.al* (2015) $\hat{\bar{Y}}_{RP1}^{ST}$ estimator.

CONCLUSION

This paper has proposed a generalized family of ratio-product exponential type estimator using power transformation to estimate the population mean of the study variable using the knowledge of the population mean as well as the coefficient of kurtosis of two auxiliary variables x and z under stratified random sampling. Its properties were studied and other related estimators were revealed. This means that the proposed estimator is more efficient than other considered estimators as shown in Tables. Therefore, it can be concluded that if information on the coefficient of kurtosis of the auxiliary variables is available for each stratum, then the proposed estimator performs well and more efficiently than other considered estimators.

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