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Effect of Viscous Dissipative Fluid in A Slit Micro-Channel with Heated Super-Hydrophobic Surface

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Abstract

The analytical solution of heat transfer and hydro magnetic flow for an incompressible, electrically-conducting, and viscous dissipative fluid traveling vertically through an isothermally heated parallel plate in a slit micro-channel is performed. One channel surface was super-hydrophobic slip and temperature jump, whereas the other did not. This study also examined the effects of super-hydrophobic slip and a temperature jump on characteristic flow of magneto-hydrodynamics (MHD) free convection flow with viscous dissipative fluid. The perturbation technique was used to solve the coupled nonlinear and coupled equations analytically. With the aid of illustrative graphs, the results were presented, and the effects of the relevant controlling parameters namely Brinkman number, magnetic field, velocity slip condition and temperature jump effects were analysed for temperature, velocity, skin friction and Nusselt number distributions when either wall is heated by constant wall temperature. The result demonstrated that the activities of Brinkman number, Br, substantially increase fluid velocity, whereas growing levels of magnetic field effect was seen to slow down the flow movement. In addition, the fluid velocity is higher for greater values of temperature jump and velocity slip effects respectively. Furthermore, this research can prove very useful for designers in improving the performances of mechanical systems when viscous dissipation is involved, as well as heat transfer in micro-channels, as it is in combustion.

Keywords: Super-hydrophobic slip, Temperature jump, Viscous dissipation, Magneto-hydrodynamics (MHD), Slit micro-channel.

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INTRODUCTION

Magneto-hydrodynamics (MHD) is the study of electrically conducting liquids, such as salty water, electrolytes, plasma, and liquid metals. This type of fluid has a number of engineering and industrial applications such as in growth of crystals, reactor cooling, magnetic drug targeting, MHD sensors, and power generation. MHD is dependent on the intensity of magnetic induction (Jawad *et al*. 2021). The experimental investigation of modern MHD flow in a laboratory was first carried out by Hartmann and Lazarus (1937). This study provided the basic knowledge for the development of many MHD devices, such as MHD pumps, MHD generators, brakes, flow meters, plasma studies, and geothermal energy extraction. Since then, many works have been carried out to investigate the impacts of MHD on free convection flow through various channels. In view of this, Hamza *et al*. (2023) recently investigated the impact of MHD free convection of a chemically reacting fluid using homotopy perturbation technique. Jha *et al.* (2023) echoed the significance of magnetic field on a free convection of an incompressible viscous fluid in an upstanding vertical channel having point/line heat generation/absorption at different channel positions. Jha *et al.* (2012) discussed the unsteady as well as steady state free convection Couette flow of reactive viscous fluid in a vertical channel formed between two infinite vertical parallel porous plates. Saeed and Gul (2021) examined the MHD Casson Nanofluid flow for mass and heat transfer past a moving sheet. Reddy *et al*. (2018) analysed the effects of cross diffusion for the flow of non-Newtonian fluids over an extended surface using a non-uniform heat source and sink. In this analysis, the authors have studied the combined effects of fractional and irregular for heat on the flow system and used the Fehlberg method to determine the solution of the modelled problem. Sravanthi and Gorla (2018) presented the influence of the source/sink and chemical response on the Maxwell MHD nano-liquid flow. In this discussion, the authors considered the flow past an exponentially convective stretched surface and employed the semi-analytical technique homotopy analysis method (HAM) to determine the solution of the modeled problem. Gurivireddy *et al*. (2016) highlighted the effect of thermal diffusion on MHD heat and mass transfer flow through a semi-infinite moving vertical porous plate with heat generation and chemical reaction. With these concerns in mind, Ojemeri *et al*. (2023) recently investigated the hydromagnetic flow of an electrically conductive Casson fluid driven by radiation factor in an upstanding porous channel. Hamza *et al*. (2022) emphasized on the impact of MHD flow of a chemically reacting fluid that is convectively heated in a vertical channel imagined with a porous medium. While the unsteady state situation was investigated using a numerical scheme, the steady-state component was obtained using the homotopy perturbation approach. Obalulu *et al.* (2021) discussed the influence of Arrhenius energy and exothermic chemical reaction of a combustible fluid in the coexistence variable electric conductivity and magnetic field effect flowing between two upright plates. They discovered that the presence of variable electric conductivity and concentration buoyancy improve the fluid velocity whereas the activation energy and magnetic field has the reverse behaviour. Ojemeri and Hamza (2022) proposed a MHD-free convection flow of a chemically reactive fluid contemplated with heat generation/absorption effect in a microchannel using the homotopy perturbation procedure. Hamza *et al.* (2023) investigated the implication of Arrhenius-controlled heat transfer enhancement affected by an induced magnetic field in a microchannel.

Scientists, technologists, and engineers are paying much attention to the evaluation of the result of the new combination using hydro-magnetic natural convection flow in a super-hydrophobic (SHO) micro-channel. Oil and gas companies, semiconductor manufacturing facilities, and companies that assemble small equipment on SHO surfaces have the ability to reduce drag in a flow because of the enormous slip obtained from liquid/solid interfaces, making it a particularly relevant parameter to gauge the extent of drag reduction depending on the slip length (Jha and Gwandu 2020). In view of these considerations, Hamza *et al*. (2023) recently scrutinized the consequences of thermal radiation and super-hydrophobicity on a free convection of an electrically conducting fluid across an upstanding microchannel influenced by a transverse magnetic field. Ojemeri and Onwubuya (2023a) presented the exploration of viscous dissipative MHD fluid in the coexistence of suction/injection effect and mixed convection through a heated super-hydrophobic microchannel. Later, Ojemeri and Onwubuya (2023b) recently describe the analysis of steady mixed convection flow of Arrhenius-controlled, incompressible and electrically conducting fluid along an isothermally heated superhydrophobic microchannel due to heat source/sink. Jha and Gwandu (2017) conducted a theoretical investigation of MHD natural convection in a vertical slit micro-channel having a super-hydrophobic slip effect and temperature jump. Their research showed that the greatest upward velocity obtained by heating the super-hydrophobic wall is less than that attained by heating the no-slip surface whenever there is a temperature leap and no super-hydrophobic slip, or both. The maximum velocities are equivalent when neither is present. Later, Jha and Gwandu (2019) investigated the free convection flow of an electrically conducting fluid in a vertical slit microchannel affected by super-hydrophobic slip and temperature jump effects using the non-linear Boussinesq approximation methods. Raising the temperature jump coefficient, according to the computational results, contributes to a decrease in temperature when the super-hydrophobic surface is heated and increases the temperature when the no-slip surface is heated. Jha and Gwandu (2020) built on their previous work, Jha and Gwandu (2017) by proposing an analytical investigation of free convection airflow across porous plates heated alternately, one channel with no slip and the other super-hydrophobic. Ramanuja *et al.* (2020) explored free convection flow in an isothermally heated channel with super-hydrophobic slip on one surface and a temperature rise but no slip on the opposite side. Hatte and Pitchumani (2020) used a fractional rough surface characterization to thoroughly and explicitly describe the impact of heat transfer flow inside a cylinder with non-wetting surfaces. The approach examines the dynamic stability of the air-fluid interaction in the asperities of air-infused super-hydrophobic surfaces. Their findings show that, contrary to prevalent belief, super-hydrophobicity, defined by the largest contact angles, does not always result in peak convective heat transfer behaviour and that, under specific fluid flow conditions, hydrophobic surfaces can provide excellent thermal performance.

The focus of this paper is to therefore expand the work of Jha and Gwandu (2017) to theoretically investigate the impact of MHD-free convection flow of viscous dissipative fluid in a vertical parallel plate that is constantly heated in a slit microchannel having a super-hydrophobic surface. A semi-analytical approach (the perturbation method) was employed to solve the dimensionless nonlinear and coupled governing equations. The behaviors of controlling parameters on temperature, velocity, heat transfer rate, and sheer stress were computed and discussed with the help of illustrative graphs. Internal heating is a type of mechanical energy dissipation that is spurred on by viscous forces in molecular fluid-particle exchanges. This particular kind of mechanical energy dissipation has a major effect on the fluid's hydrodynamic and thermodynamic behavior, and its attributes have a wide range of applications in the lubrication, food processing, and food preservation industries, as well as in the cooling of electrical appliances, exploration for petroleum products, and other industries.

MATERIALS AND METHODS

Imagine an electrically conducting fluid traveling gradually upward within a vertical parallel plate microchannel heated alternatively by wall constant temperature. Due to a particular micro-engineering treatment, one of the surfaces is exceedingly difficult to wet (super-hydrophobic). The opposite wall (no-slip surface) was unaltered. As shown in figure 1, the super-hydrophobic wall is kept at $y_0 = 0$, while the no-slip surface is kept at $y_0 = L$. Since the ultimate focus is on the super-hydrophilicity of a surface rather than the flow behavior, various temperature jump and slip conditions were applied to the plates. Following Jha and Gwandu (2017, 2020), the leading equations for the current problem, employing the Boussinesq buoyancy approximation with boundary conditions, assuming that the fluid is affected by viscous dissipation effect, can be written in dimensional form as follows:

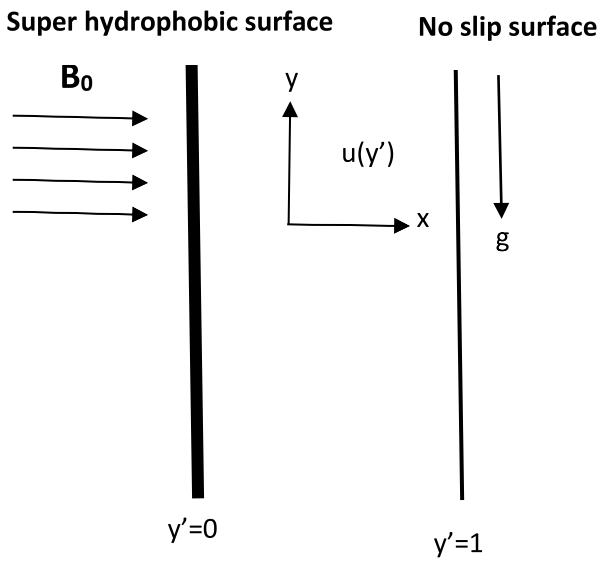

Figure 1: Schematic diagram of the flow configuration

$$V\frac{d^2u'}{dy'^2} + g\beta(T' - T_O) - \frac{\sigma B_o^2 u'}{\rho} = 0 \quad (1)$$

$$\frac{k}{\rho c_\rho}\frac{d^2T'}{dy'^2} + \frac{v}{c_\rho}\left(\frac{du'}{dy'}\right)^2 = 0 \quad (2)$$

Where y' and x' are the dimensional distances along and perpendicular to the plate. u' and T' are the dimensional velocity and temperature. V , k, ρ , c_p , β and g are the dimensional kinematic viscosity, thermal conductivity, density, specific heat at constant pressure, thermal expansion coefficient, and acceleration due to gravity of the fluid, respectively. We assume that the appropriate boundary conditions of the model are:

$$\left.\begin{array}{l} \left.\begin{array}{r} u(y') = \lambda'\dfrac{du'}{dy'} \\ T(y') = T_h + \gamma'\dfrac{dT'}{dy'} \end{array}\right\} at\ y' = 0 \\ \left.\begin{array}{r} u(y') = 0 \\ T(y') = T_o \end{array}\right\} \quad at\ y' = L \end{array}\right\} \quad (3)$$

To solve equations (1-3), we employ the dimensionless quantities and parameters:

$$u = \frac{u'}{U}, y = \frac{y'}{h}, \theta = \frac{T' - T_0}{T_w - T_o}, x = \frac{x'v}{Uh^2}, M^2 = \frac{\sigma\beta_o^2 h^2}{\rho v}, (Y, \gamma, \lambda) = (Y', \gamma', \lambda')/h$$

$$B_r = \frac{hH}{k} \tag{4}$$

Using the dimensionless quantities in equation (4), the basic equations (1) to (3) become:

$$\frac{d^2U}{dy^2} + \theta - M^2U = 0 \tag{5 }$$

$$\frac{d^2\theta}{dy^2} + Br\left(\frac{du}{dy}\right)^2 = 0 \tag{6}$$

The initial and boundary conditions in dimensionless forms are:

$$\theta(0) = 1 + \gamma\frac{d\theta}{dy}, \qquad u(0) = \lambda\frac{du}{dy} \tag{7}$$

$$\theta(1) = 1, \quad u(1) = 0$$

Where M is the magnetic field intensity, Br = EcPr is the Brinkman number, γ is the temperature jump coefficient and λ is the velocity slip condition.

Method of Solution

The momentum and energy equations can be reduced to the set of ordinary differential equations, which are solved analytically by perturbation method.

$$Let \left.\begin{aligned} \theta &= \theta_o + B_r\theta_1 \\ U &= U_o + B_rU_1 \end{aligned}\right\} \tag{8}$$

Substituting $U\ and\ \theta$ into Equations (5-7) and comparing the coefficients of $Br^0 and\ B_r$, the following set of nonlinear differential equations have been derived as:

$$B_r^o: \frac{d^2U_o}{dy^2} + \theta_o - M^2U_o = 0 \tag{9}$$

$$B_r: \frac{d^2U_1}{dy^2} + \theta_1 - M^2U_o = 0 \tag{10}$$

$$B_r^o: \frac{d^2\theta_o}{dy^2} = 0 \tag{11}$$

$$B_r: \frac{d^2\theta_1}{dy^2} + \left(\frac{dU_o}{dy}\right)^2 = 0 \tag{12}$$

The transformed boundary condition becomes:

$$\left.\begin{aligned} &\left.\begin{aligned} U_o &= \lambda\frac{dU_o}{dy} \\ U_1 &= \lambda\frac{dU_1}{dy} \end{aligned}\right\} at\ y = 0 \\ &\left.\begin{aligned} U_o &= 0 \\ U_1 &= 0 \end{aligned}\right\} at\ y = 1 \end{aligned}\right\} \tag{13}$$

$$\left.\begin{array}{l}\left.\begin{array}{l}\theta_o = 1 + \gamma \frac{d\theta_o}{dy} \\ \theta_1 = \gamma \frac{d\theta_1}{dy}\end{array}\right\} at\ y = 0 \\ \left.\begin{array}{l}\theta_o = 1 \\ \theta_1 = 0\end{array}\right\} at\ y = 1\end{array}\right\} \quad (14)$$

Solutions for the temperature and velocity gradients have been obtained as follows:

$$\theta_o = C_1 y + C_2 \quad (15)$$

$$\theta_1 = -\frac{b_1}{4m^2}e^{2my} - \frac{b_2}{m^2}e^{my} - \frac{b_3}{m^2}e^{-my} - \frac{b_4}{4m^2}e^{-2my} - b_5\frac{y^2}{2} + C_7 y + C_8 \quad (16)$$

$$U_0 = C_3 e^{my} + C_4 e^{-my} + C_5 y + C_6 \quad (17)$$

$$U_1 = D_1 e^{my} + D_2 e^{-my} + D_3 e^{2my} + D_4 y e^{my} + D_5 y e^{-my} + D_6 e^{-2my} + D_7 y^2 + D_8 y + D_9 \quad (18)$$

Since $\theta = \theta_0 + B_r\theta_1$ (19)

Then

$$\theta = C_1 y + C_2 + B_r\left[-\frac{b_1}{4m^2}e^{2my} - \frac{b_2}{m^2}e^{my} - \frac{b_3}{m^2}e^{-my} - \frac{b_4}{4m^2}e^{-2my} - b_5\frac{y^2}{2} + C_7 y + C_8\right] \quad (20)$$

also, since $U = U_0 + B_r U_1$

(21)

then

$$U = C_3 e^{my} + C_4 e^{-my} + C_5 y + C_6 + B_r[D_1 e^{my} + D_2 e^{-my} + D_3 e^{2my} + D_4 y e^{my} + D_5 y e^{-my} + D_6 e^{-2my} + D_7 y^2 + D_8 y + D_9] \quad (22)$$

The heat transfer rate and skin friction at both plates are derived as follows:

$$\frac{d\theta}{dy}\ |_{y=0} = C_1 + B_r\left[-\frac{b_1}{2m} - \frac{b_2}{m} + \frac{b_3}{m} + \frac{b_4}{2m} + C_7\right] \quad (23)$$

$$\frac{d\theta}{dy}\ |_{y=1} = C_1 + B_r\left[-\frac{b_1}{2m}e^{2m} - \frac{b_2}{m}e^{m} + \frac{b_3}{m}e^{-m} + \frac{b_4}{2m}e^{-2m} - b_5 + C_7\right] \quad (24)$$

$$\frac{dU}{dy}\ |_{y=0} = C_3 m - C_4 m + C_5 + B_r[D_1 m - D_2 m + 2D_3 m + D_4 + D_5 - 2D_6 m + D_8] \quad (25)$$

$$\frac{dU}{dy}\ |_{y=1} = C_3 m e^{m} - C_4 m e^{-m} + C_5 + B_r[D_1 m e^{m} - D_2 m e^{-m} + 2D_3 m e^{2m} + D_4 m e^{m} + D_4 e^{m} + D_5 m e^{-m} + D_5 e^{-m} - 2D_6 m e^{-2m} + 2D_7 + D_8] \quad (26)$$

RESULTS AND DISCUSSION

The steady state governing equations are solved analytically using regular perturbation series method. Various graphs were plotted with the aid of a user-friendly software program (MATLAB 2015a) to illustrate the effects of pertinent parameters on the fluid flow and thermal profile. The influence of the Brickman number (Br) on the dimensionless temperature and velocity gradients for fixed value of $\lambda = \gamma = 1$) is displayed in figures 2 and 3. It clearly reveals that, from these figures, the temperature and velocity of the fluid grow with an increase in the local Brickman number (Br) significantly. Higher Brickman number mean correspondingly higher degrees of convective heating at the lower channel wall and consequently lead to high temperature and velocity at the lower plate. Makinde and Aziz (2011) reported that, increasing the local Brickman numbers leads to the stronger convective heating at the lower channel wall which results in higher surface temperatures causing the thermal effect to penetrate deeper into the quiescent fluid, consequently making the fluid motion to escalate. The deviation pattern of the velocity with respect to the displacement y is shown in Figure 4, and it is similar to what was found in previous studies by other researchers (Jha and Gwandu (2017); Jha and Gwandu (2019)). This shows that as particles move away from the super-hydrophobic surface they tend to have higher velocities for some time, then, the velocity falls before reaching the center. But if both the surfaces are not super-hydrophobic, then, the velocity does not rise much and will not drop until around the center of the channel. The response of varying magnetic field number on the fluid motion is illustrated in figure 5. It was observed that a decaying effect on the fluid movement is visible when the level of magnetic number is raised. This is attributable to the Lorentz force, which appears when a magnetic field imposes itself on an electrically conducting fluid and a drag force is created. Due to this force, fluid movement dwindles near the plate; all other forces, including the Lorentz force, opposes the fluid velocity, as a result when the fluid comes to rest. The actions of Brickman number (Br) on the rate of heat transfer coefficient are depicted in Figure 6. The rate of heat transfer demonstrates a growing effect when Br is increased at the both surfaces. The actions of Brickman number (Br) on the drag force is showcased in figure 7. It is revealed from Figure 7a that the fictional factor is stronger at y=0 when the Brickman number is enhanced while an opposite phenomenon is witnessed at y=1 as portrayed in Figure 7b. The influence of magnetic field on the skin friction is illustrated in Figure 8. It can be clearly seen that the impact of MHD on the sheer stress demonstrate similar trends on the both plates but this effect is higher at the plate (y = 0).

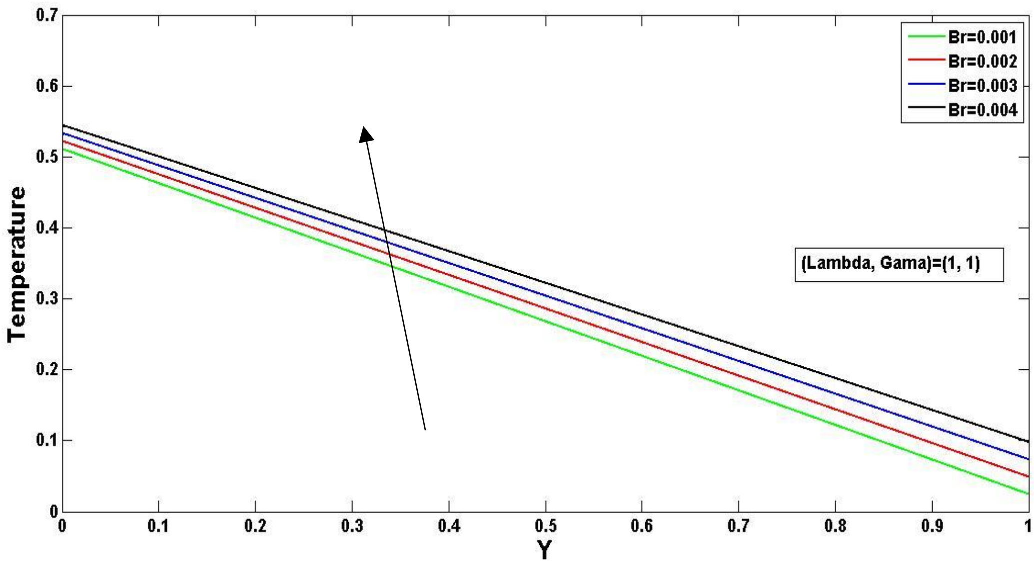

Figure 2: Deviation of temperature for Br

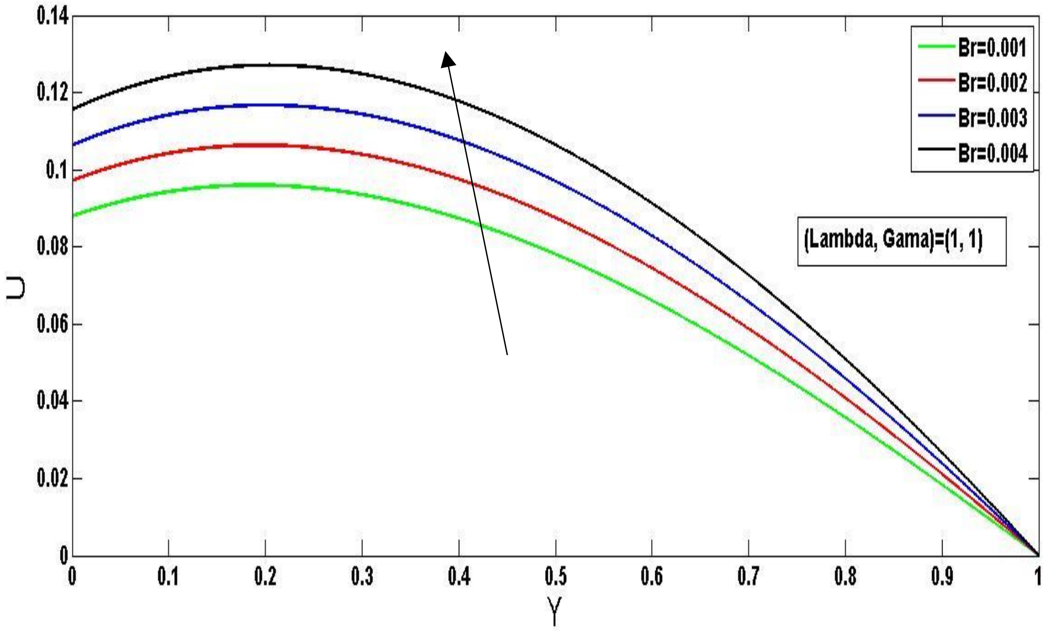

Figure 3: Deviation of velocity for Br

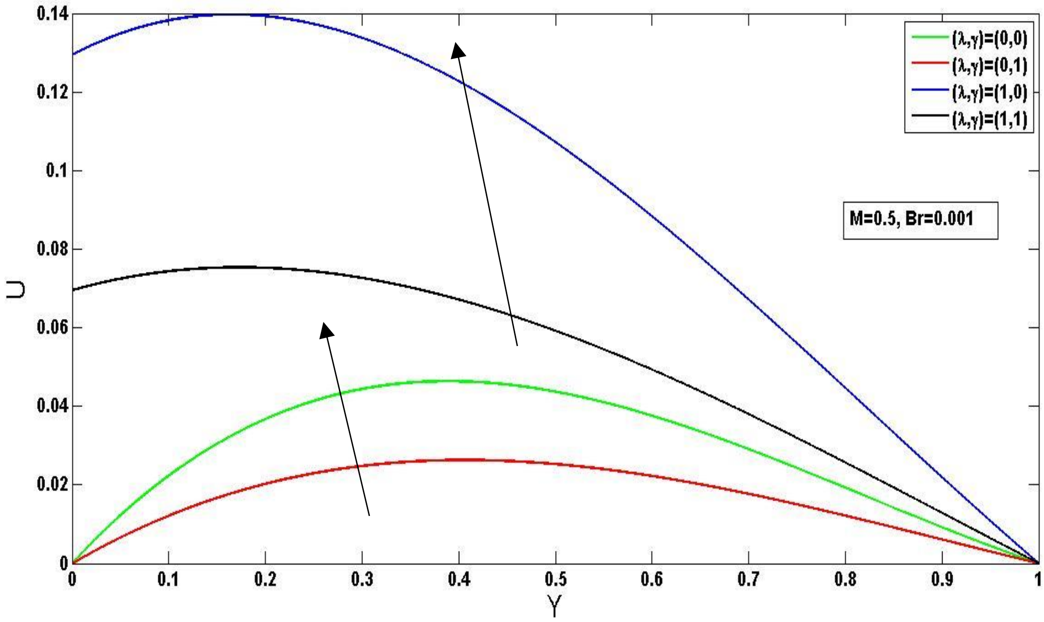

Figure 4: Deviation of velocity for the displacement y

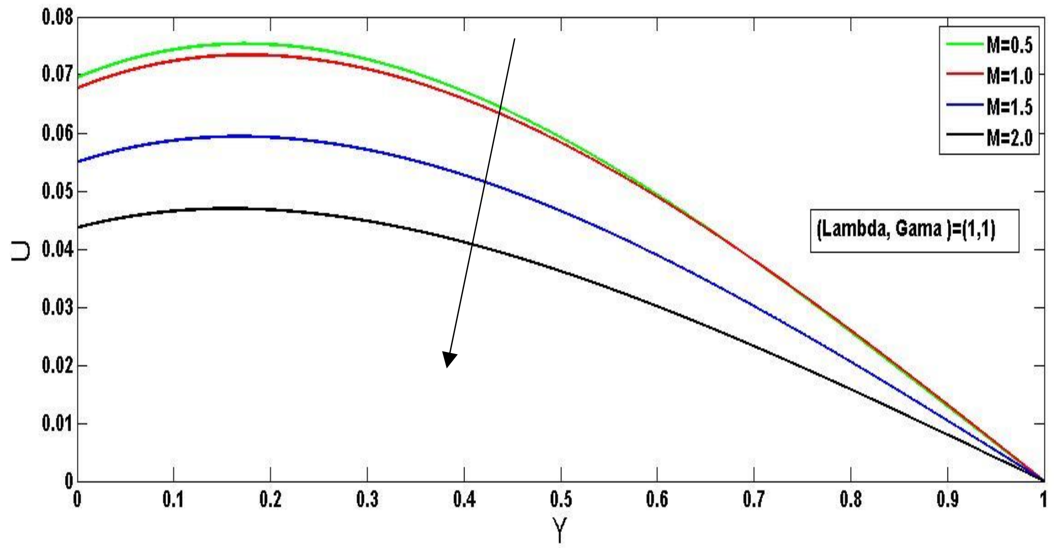

Figure 5: Deviation of velocity for MHD

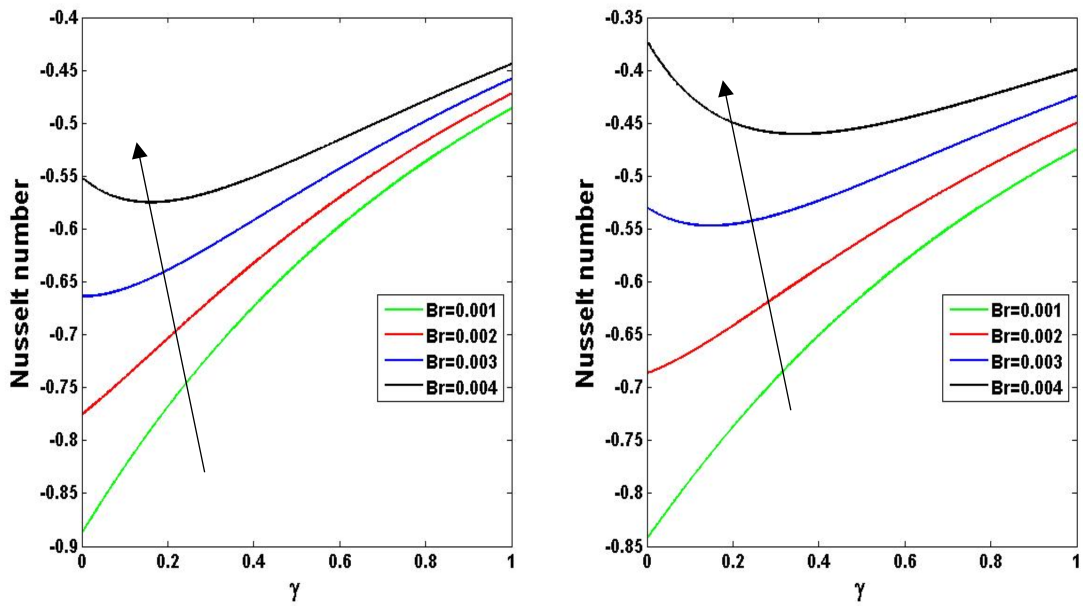

Figure 6: Variation for Nusselt number for Br at (a) y =0 and (b) y=1

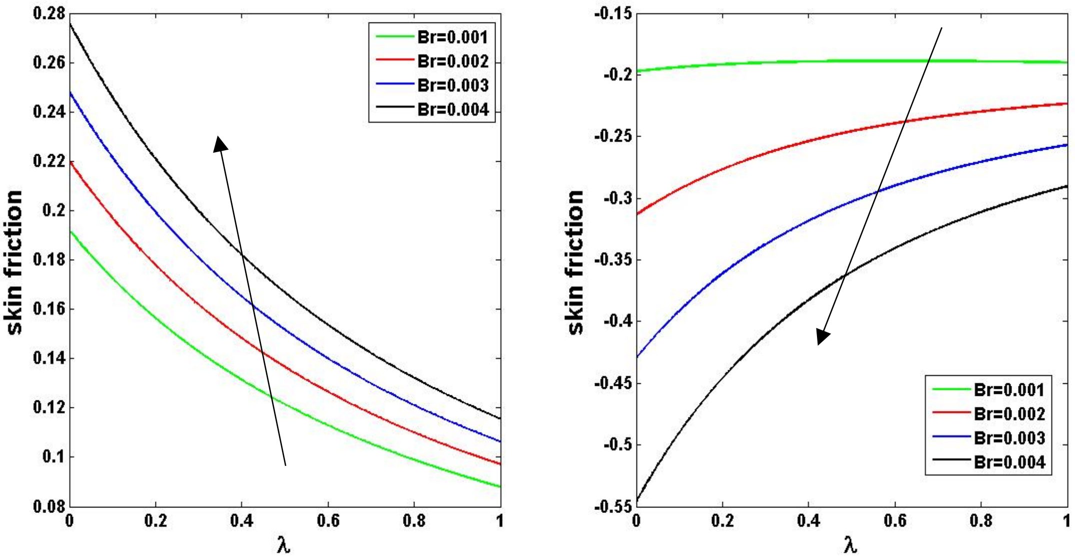

Figure 7: Variation for skin friction for Br at (a) y =0 and (b) y=1

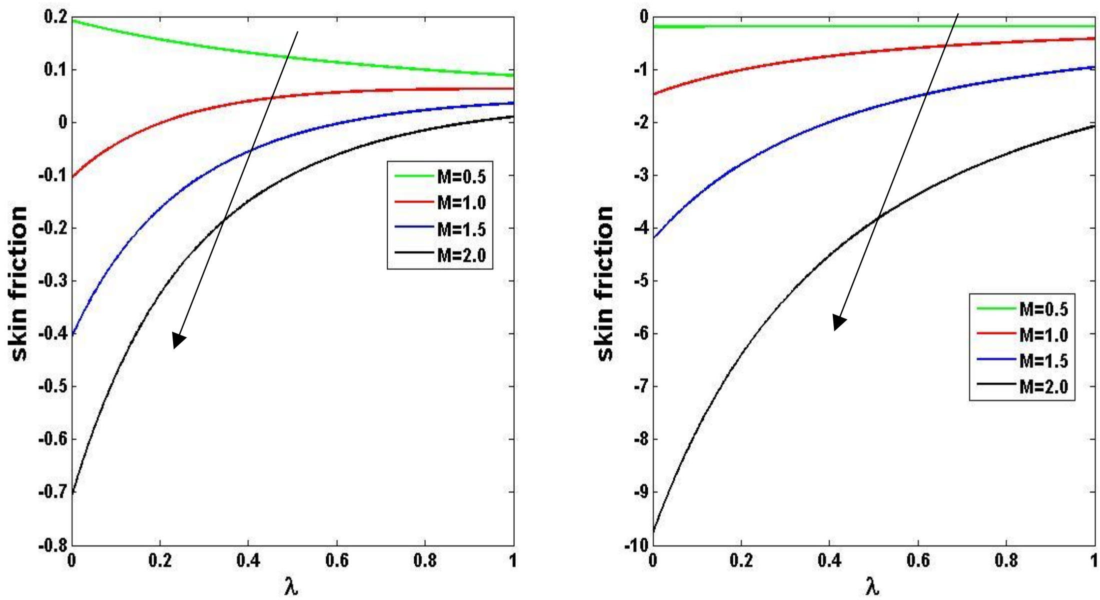

Figure 8: Variation for skin friction for M at (a) y =0 and (b) y=1

Validation of Results

The work of Jha and Gwandu (2017) is successfully recovered by setting Br to zero, thereby establishing an excellent relationship between this current investigation and their work. Table 1 describes the numerical computations of the comparison between the work of Jha and Gwandu (2017) and the current investigation.

Table 1: Computations of comparison between the work of Jha and Gwandu (2017) with the present work for temperature and velocity distributions for $\lambda = \gamma = 1$, $M = 0.5$ and setting Br to be zero.

	Jha and Gwandu (2017)		Present work	
	θ (Y)	U(Y)	θ(Y)	U(Y)
Y				
0.1	0.4500	0.0843	0.4500	0.0843
0.2	0.4000	0.0856	0.4000	0.0856
0.3	0.3500	0.0831	0.3500	0.0831
0.4	0.3000	0.0772	0.3000	0.0772
0.5	0.2500	0.0686	0.2500	0.0686

CONCLUSION

This present article examined the consequences of MHD and viscous dissipation on the steady flow of a viscous, electrically-conducting fluid traveling vertically across an isothermally heated parallel plate micro-channel, with one surface exhibiting super-hydrophobic slip and a temperature jump and the other not. A semi-analytical method (perturbation series approach) was employed to generate the steady-state solutions for temperature, velocity, rate of heat transfers, and sheer stress. The influence of pertinent parameters dictating the flow configuration is discussed in detail using various plots. Viscous dissipation is vital in most lubrication industries. This study can therefore find relevance in science, engineering, and industrial technologies such as cooling of electrical appliances, geothermal energy, porous solids drying, thermal insulation, gas drainage, plasma physics, gas turbines, fossil fuel combustion, and food processing industries, to mention a few.

The following is an overview of the significant findings from this study:
(i) The maximum velocity attained by heating the super-hydrophobic surface is smaller than that recorded by heating the no-slip surface when there is a temperature jump and no super-hydrophobic slip or both. The opposite is true if there is a super-hydrophobic slide but no temperature surge. When neither exists, the maximum velocities are equal.
(ii) As the Brinkman number parameter goes up, the velocity of the fluid and the heat gradient grow noticeably, but the local skin friction goes down at $y = 1$ and up at $y = 0$.
(iii) The velocity gradient diminishes upon raising the levels of the magnetic parameter due to the Lorentz forces, which manifest under the MHD effect.
(iv) By uplifting the values of Br, the heat transfer rate shows similar increasing trends at both the no-slip and the super-hydrophobic surfaces.
(v) In the future, this research will be expanded to consider the unsteady state (time-dependent) case, the heat-generating or absorbing effect, or a different physical geometry.

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