Effects Of Magnetization and Heat Transfer on Electrically Conducting Magnetohydrodynamics Flow with Mixed Convection Flow Over Porous Duct

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Abstract

The effects of magnetic field and heat transfer problem on incompressible magnetohydrodynamics (MHD) mixed unsteady convective flow of an electrically conducting viscous fluid in a duct filled with a porous medium are investigated. The fluid flow is subjected to an applied uniform magnetic field vertical to the duct and uniform free stream of constant velocity and temperature. The system of nonlinear differential equations governing flow and heat transfer phenomena are derived and reduced into a set of ordinary differential equations using the auxiliary variable method. The reduced equations are solved numerically using Runge-Kutta method associated with slip boundary conditions. The graphical results for velocity, temperature and heat transfer rate are obtained using MATLAB software. It is observed that the velocity profiles increase with an increase of similarity parameter (f^1) while temperature gradient decreases rapidly with an increase of similarity parameter (θ^1) for different values of Grosh number, Magnetization and Prandtl number respectively.

Keywords: Porous duct, Magnetohydrodynamics, Magnetic field, incompressible, Flow

INTRODUCTION

The heat transfers and flow phenomenon in a porous media has momentously played a vital role in addressing problems associated with fluid dynamics (Hafeez et al., 2013). The fluid flow over porous materials has various applications in many engineering, biomedicine, heat exchange, fuel cell and many other related fields (Bhukta et al.,2015). These flows have been effectively derived by magnetic fields, electric fields and/ or combinations of the two (Buonomo et. Al.,2016; Dehghan et al.,2015). Among these effects, magnetohydrodynamics (MHD) effects become most promising candidate owing to its potential applications in fluid dynamics systems (Qian et.al ,2009; Jang et. Al.,2000). The MHD unsteady flows for electrical

conducting fluid flow subjected to the magnetic field have attracted the attention of many researchers due to its imminent applications in recent years (Abolbashari et. al.,2014; Daniel et. al 2017a), Daniel et. al.,2015a; Daniel et. al. ,2015b; Daniel et. al.,2016; Daniel et. Al.,2017b). Recently, enormous effort has been carried out by several researchers to explore boundary layer flow of unsteady and incompressible MHD flow over porous media (Bhukta et al.,2015; Veera et al.,2009; Suneetha et al.,2010). For instance, (Abdallah et.al.,2009) studied the effects of MHD, viscous, unsteady, incompressible and electrically conducting fluid flow over stretching materials and heat transfer problem in the presence of a magnetic field. The analytical results obtained was verified numerically. (Bhukta et.al 2015) reported the dissipative effect of MHD mixed convective flow of an electrically conducting fluid over stretching sheet in porous media subjected to a magnetic field in the presence of non-uniform heat source/sink. The differential equations derived were solved numerically and the results obtained reveals that the effects of electric field significantly improve the skin friction contributing to the flow instability Rabhi et al. (2017) investigated the effects of magnetic field, entropy generation and Nusselt number for MHD unsteady flow over porous duct using a modified axisymmetric lattice Boltzmann method. The simulation of MHD flow was carried out with LBM, and the obtained results indicated that the entropy generation intensified near wall of the duct. It also found out that magnetic field strongly affected the global irreversibility. Krishna et al. (2016) studied the unsteady MHD convective flow of second grate fluid over porous media in a rotating plate channel subjected to temperature dependent source. Analytical solution of velocity and temperature profile were obtained via Laplace transformation method and compared with numerical results. The results agree with already reported works. Ullah et al. (2016) presents the effects of the chemical reaction and thermal radiation on the MHD convective flow of Casson fluid over stretching surface through porous media. The obtained nonlinear differential equation is solved numerically using a Kellar box technique. It was revealed that the Casson fluid is better than the Newtonian fluid in controlling temperature and the nanoparticles.

In the last decades, the combined effects of heat transfer, chemical reaction and electric fields on electrically conducting MHD flow played significant role in chemical and manufacturing industries. Kandasamy et al. reported the effects of mass and heat transfer on Newtonian fluid of MHD mixed convection flow over a stretching sheet in the presence of chemical reaction (Kandasamy et. al, 2005). Perkidis et al (2006) theoretically studied the boundary layer flow of electrically conducting fluid flow through stretching sheet subjected to the chemical reaction. Damseh and Chamkha et al. (2010) reported the analytical and numerical solution of boundary layer flow of micropolar fluid through a stretching material. Raptis et al. (2006) investigated the effects of slip conduction chemical reaction on MHD electrically conducting fluid flow through porous materials.

On the other hand, flow over porous media drew attention of several researchers and industries owing to its several applications including, industrial machinery, disk drives, storage tank and so on, (Herero et. al.,1994). Heat transfer and MHD flow problem through a porous medium over a stretching surface are studied by Cortell et al. (2011), Chauhan et. al. (2011a) and Chauhan et al. (2011b). Abet et al analyzed the heat transfer on electrically conducting MHD flow of a second-grade fluid in a porous medium over stretching surface under the influence of heat source/sink, (Subhas et. Al.,2011). Bhukta et al. (2014)
studied the effects of mass and heat transfer on electrically conducting viscoelastic fluid in a boundary layer flow in a porous medium over a shrinking sheet in the presence of transverse magnetic field and heat source. Choudhry et al. (2014) analyzed the viscoelastic MHD flow over a porous plate in a porous medium subjected to chemical reaction and radiation under

influence of heat and mass transfer.

Strongly motivated by the earlier studies and widespread applications, it is of uttermost importance to explore the effects of magnetic field on incompressible magnetohydrodynamics (MHD) mixed unsteady convective flow of an electrically conducting viscous fluid in a duct filled with a porous medium and the heat transfer problem was investigated. The governing flow equations are derived and reduced into nonlinear partial differential equations associated with slip boundary conditions. These conditions are made dimensionless using a suitable similarity transformation. The system of non-dimensionless equations is solved numerically *via* iteration method. The numerical results obtained for different values of magnetic field, Kaman-Prandit, and Grosh numbers are presented graphically. The representation of the variation of velocity and temperature and differential characteristics boundary layer is discussed and shown graphically.

NOMENCLATURE   and   MEANING

a	*Stretching constant*	B	*Imposed or applied magnetic induction*
u,v	Kinematic component of velocity	g	Acceleration of free fall
T_W	Surface or wall temperature	σ	Electrical conductivity
P	Pressure	Ψ	Fluid stream function
C	Mass concentration	η	Similarity or transformation parameter
C	Specific heat capacity	T	Temperature
ρ	Density	θ	Temperature gradient
β	Coefficient of area expansion	v	Surface stretching parameter
K	Coefficient thermal expansion	M^2	Magnetization
H	Step size	z_1, z_2 ,z_3, ϕ	Auxiliaries variable
E	Function	'	Derivative
F	Dimensional function	x,y	Coordinate
N	Number of iterations	L	Length of porous media
Pr	Prandit number	Gr	Grosh number

THEORETICAL FORMULATION OF THE PROBLEM

The 2-Dimentional unsteady MHD convection flow of incompressible, viscous, and electrically conducting fluid flow over porous duct and heat transfer subjected to the magnetic field and constant stream velocity and temperature is considered. The imposed magnetic field is homogenous and perpendicular to the body surface. The transport properties of medium can be considered. The origin is kept fixed while the wall is stretched and the y-axis is perpendicular to the surface as seen in Figure. 1. The governing equations and the boundary conditions are as follows, (Hafeez et. al.,2013; Hafeez et. al.,2016)
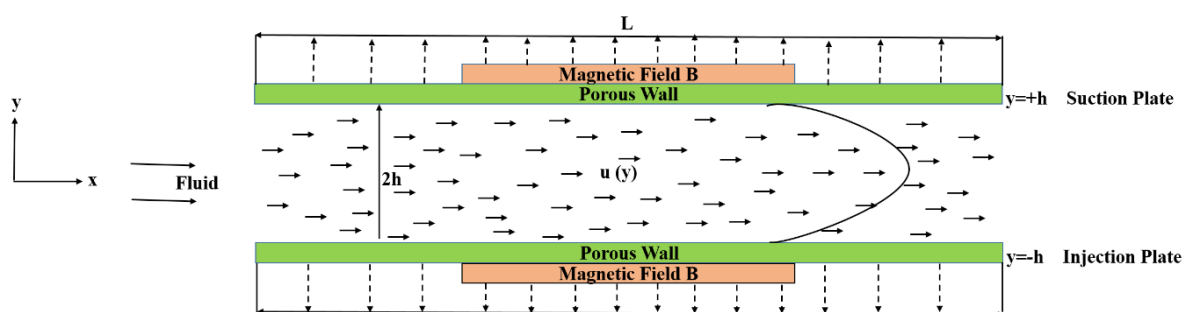Continuity equation

Figure.1 Fluid flow through porous duct

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + g\beta T - \frac{\sigma B^2}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + g\beta T - \frac{\sigma B^2}{\rho}v \tag{3}$$

The energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions associated with equation (4) are, Abdallah. Et. al (2009).

$$u(x,0) = ax, v(x,0) = 0, T(x,0) = T_w = constant, u(x,\infty) = 0, T(x,\infty) = 0 \tag{5}$$

where u and v are the velocity components along x and y coordinates respectively, T is temperature; β is coefficient of area expansion, ρ is the density, T_w is the surface temperature, and *a* is the stretching rate constant, g is the acceleration of free fall, T is the temperature, σ is the electric conductivity, B is the imposed (applied) magnetic induction and C is concentration of mass on boundary layer, (Abdallah. et. al., 2009).

SOLUTION OF THE PROBLEM
The velocity component along x and y coordinate in terms of stream function is given as (Hafeez et. al 2013)

$$u = \frac{\partial \Psi}{\partial y}, and\ v = -\frac{\partial \Psi}{\partial x} \tag{6}$$

By applying the transformation

$$\psi = x\sqrt{a\nu}f(\eta), \quad \eta = y\sqrt{\frac{a}{\nu}}, \quad \theta = \frac{T}{T_w} \tag{7}$$

where ψ is dimensionless stream function and η is similarity parameter. Consider the MHD flow along x- axis, the continuity equation (1) become

$$\frac{\partial u}{\partial x} = 0 \tag{8}$$

Substitution equation (7) into equation (2), we have

$$\frac{\partial \Psi}{\partial y}\frac{\partial}{\partial x}(\frac{\partial \Psi}{\partial y}) - \frac{\partial \Psi}{\partial x}\frac{\partial}{\partial y}(\frac{\partial \Psi}{\partial y}) = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \nu[\frac{\partial^2}{\partial x^2}(\frac{\partial \Psi}{\partial y}) + \frac{\partial^2}{\partial y^2}(\frac{\partial \Psi}{\partial y})] + g\beta T - \frac{\sigma B^2}{\rho}\frac{\partial \Psi}{\partial y} \tag{9}$$

Differentiating equation (7), we have

$$\frac{\partial \Psi}{\partial x} = \sqrt{a\nu}\ f(\eta), \quad \frac{\partial^2 \Psi}{\partial x^2} = 0, \quad \frac{\partial \Psi}{\partial y} = x\sqrt{a\nu}\ f'(\eta), \quad \frac{\partial^2 \Psi}{\partial y^2} = x\sqrt{a\nu}f''(\eta) \tag{10}$$

Putting equation (10) into equation (9), we get

$$x\sqrt{a\nu}f'\frac{\partial}{\partial x}(x\sqrt{a\nu}f') - \sqrt{a\nu}\ f\frac{\partial}{\partial y}(x\sqrt{a\nu}f')$$
$$= \nu[\frac{\partial^2}{\partial x^2}(x\sqrt{a\nu}f') + \frac{\partial^2}{\partial y^2}(x\sqrt{a\nu}f')] + g\beta T - \frac{\sigma B^2}{\rho}x\sqrt{a\nu}f' \tag{11}$$

We assumed that, the pressure flow along x coordinate is zero, thus equation (11) reduces to

$$f''' + f'f'' - f'^2 + g\beta T \frac{1}{x(a^2)} - \frac{\sigma B^2}{\rho} \frac{1}{a} f' = 0 \tag{12}$$

Now, equation (12) becomes

$$f''' + f'f'' - f'^2 + Gr\,\theta - M^2 f' = 0 \tag{13}$$

Equation (13) is nonlinear third order differential equation. Thus, the solution of the equation of motion and continuity equation is given by equation (14) subject to the boundary condition equation (14).

Where the prime represents differentiation with respect to η, $Gross\ number = Gr = g\beta T \frac{T_w}{x(a^2)}$, $Magnetic\ parameter = M^2 = \frac{\sigma B^2}{\rho} \frac{1}{a}$ and $T = \theta\,T_w$

The corresponding boundary conditions are, (Abdallah et. al., 2009).

$$f(0) = 0, \qquad f'(0) = 1, \ f'(\infty) = 0 \tag{14}$$

The exact solution of equation (13) with boundary conditions (14) is obtained by Abdallah et al. (2009) and Mahapatra et al (2012), as follows:

$$f(\eta) = 1 - e^{-\eta} \tag{15}$$

Similarly, for energy equation, we substitute equation (10) into equation (4), we have

$$x\sqrt{av}f' \frac{\partial(\theta T_w)}{\partial x} - \sqrt{av}\,f \frac{\partial(\theta T_w)}{\partial y} = \frac{k}{\rho C} \frac{\partial^2(\theta T_w)}{\partial y^2} \tag{16}$$

Assuming the temperature flow along x-coordinate is zero, equation (16) become

$$-\sqrt{av}\,f \frac{\partial(\theta T_w)}{\partial y} = \frac{k}{\rho C} \frac{\partial^2(\theta T_w)}{\partial y^2} \tag{17}$$

Now, equation (17) become

$$-\sqrt{av}\,f \frac{\partial\theta}{\partial y} = \frac{k}{\rho C} \frac{\partial^2\theta}{\partial y^2} \quad \Rightarrow \quad -\sqrt{av}\,f\,\theta' = \frac{k}{\rho C}\theta'' \tag{18}$$

The equation (18) can now be reduced to

$$\theta'' + \Pr f\,\theta' = 0 \tag{19}$$

Equation (19) is nonlinear differential equation with boundary conditions of

$$\theta(0) = 1, \theta(\infty) = 0 \tag{20}$$

The exact solution of (19) subject to the condition (20) is given by, (Abdallah. Et. Al.,2009).

$$\theta(\eta) = e^{-\eta} \tag{21}$$

NUMERICAL SOLUTION

The system of nonlinear differential equations (13) and (19) with boundary conditions (14) and (20) respectively, were solved numerically using the Runge-kutta method of order three. The third and second order system of nonlinear differential equation can be reduced to first order for the solution of initial value problem $y' = f(x,y), y(x_j) = y_i$ using any suitable step size which is always less than unity, (Richard et. al.,2011). For two different approximations to the solution are calculated values with the terminal point. So, in each of the following, three steps are required to be computed, (Richard et. al.,2011):

$$k_1 = f(x_n, y_n), \ k_2 = f(x_h + \frac{1}{2}h, y_n + \frac{1}{2}hk_1), \qquad k_3 = f(x_n + \frac{1}{2}h, y_n - hk_1, 2hk_2),$$
$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 4k_2 + k_3) \tag{22}$$

To solve the system of differential *equation*s (13) *and* (19) numerically, a system of equation reduction to first ordinary linear equation is employed by introducing three auxiliary variables in equation (13) and two auxiliary variables in (19), respectively. So. let
$$f = z_1, \ f' = z_2, \qquad f'' = z_3, \ f''' = z_4$$
Thus,
$$z_2 = z_1', z_3 = z_2', z_4 = z_3' \tag{23}$$

Substitution equation (23) into (13), we have
$$z_3' = -z_2 z_1 - z_1'^2 + Gr\theta - M^2 z_1 \tag{24}$$

Also, let
$$\theta' = \emptyset \implies \theta'' = \emptyset' \tag{25}$$

The equation (19) assume the form
$$\emptyset' + Prf\emptyset = 0$$
$$\emptyset' = -Prf\emptyset \tag{26}$$

Let the step size (h), Magnetization (M) and Gross number (Gr) be 0.2, 0.4 and 0.5, respectively. Now, the system of differential equations (23) to (24) are solved numerically using third order Runge-Kutter Method, using conditions $f(0) = 0, f'(0) = 1, then \ z_o = 0, z_1 = 1$. Similarly, equation (26) can be evaluated numerically for different value of Prandtl number (Pr) (i.e Pr= 0.2, 0.4, 0.6, 0.8 and 1.0). The initial value $f(0) = 1, x_o = y_o = 0, \theta_o = \emptyset_o = 1$. For Pr =0.0 the entire iterations are zero.

RESULTS AND DISCUSSION
The present study aimed at studying the effects of magnetic field and heat transfer problem on the electrically conducting MHD viscous flow over porous media. The effects are characterized by different values of non-dimensional parameters such as Prandtl number (Pr), Magnetization (M) and Gross number (Gr). All the numerical results were carried out for Pr= 0.2, 0.4, 0.6, 0.8 and 1.0 and M=0, 0.2, 0.4, 0.6, 0.8 and 1.0 at different values of Gr=1, 2, 3, 4 and 5.

Figures 2 to 9 illustrates the physical behavior of Pr, M and Gr on velocity ($\theta'(\eta)$ and $f'(\eta)$) profiles. Figure 2 shows the influence of Prandtl number (Pr) on the temperature profile. As expected, the Pr increases with decrease in temperature profile, suggesting that for large value of Pr, the heat will diffuse faster than the momentum. Moreover, it also observed that the thickness of thermal boundary decreases as Prandtl number (Pr) increases. Noticeably, higher Pr value substantially decreases the temperature owing to the fact that higher Pr leads to the low thermal conductivity of fluid, which largely reduces conduction resulting in temperature fall. A similar trend of results was observed by (Ullah et al.,2016) and (Khan et al.,2019). Notably, it is significant to identify that the MHD temperature increases as the heat(energy) increase due to the fact that the conduction impact of the MHD improves in the presence of thermal expansion parameter.
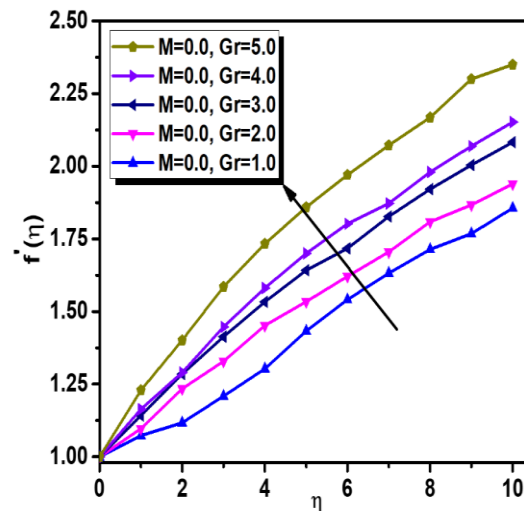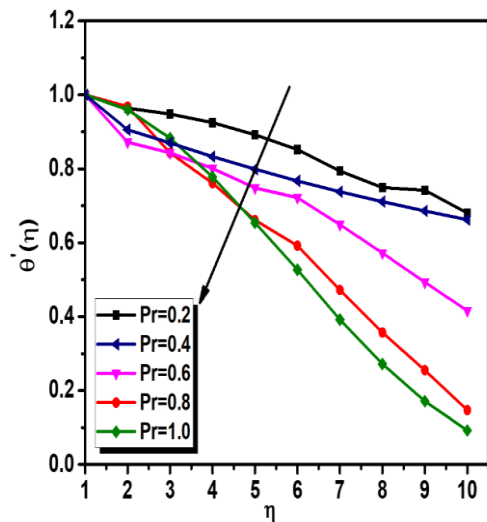
Figure 2. Effects of Prandtl number (Pr) on temperature. Figure 3. Effects of Gross number (Gr) on velocity at constant Magnetization (M=0.0)

Figures 3 to 8 presents the velocity profile displaying the effects of the Gross number parameter and Magnetization parameter. The discussion shows that the magnitude of all velocity profile presents the positive values. It has also been observed that the profile increases with an increase in Gross number parameter at constant magnetization (M), suggesting that Gr improves the fluid flows resulting in an increase in velocity profile. This result is in agreement with previously reported work
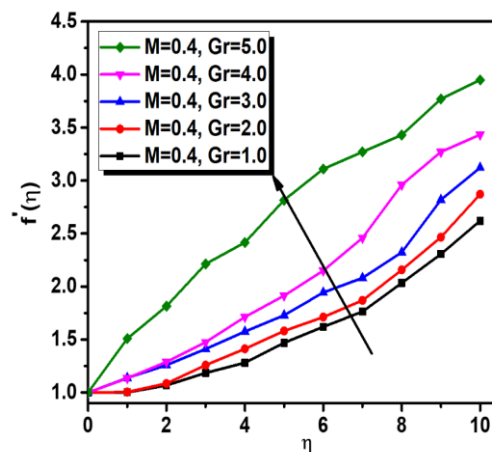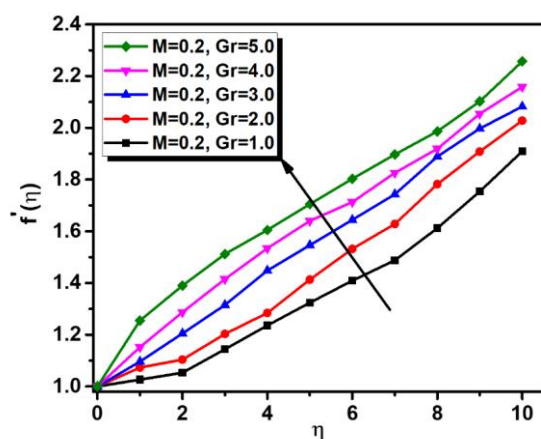(Ullah et. Al.,2016).

Figure 4. Effects of Gross number (Gr) on velocity at constant Magnetization (M=0.2).

Figure 5. Effects of Gross number (Gr) on velocity at constant Magnetization (M=0.4)
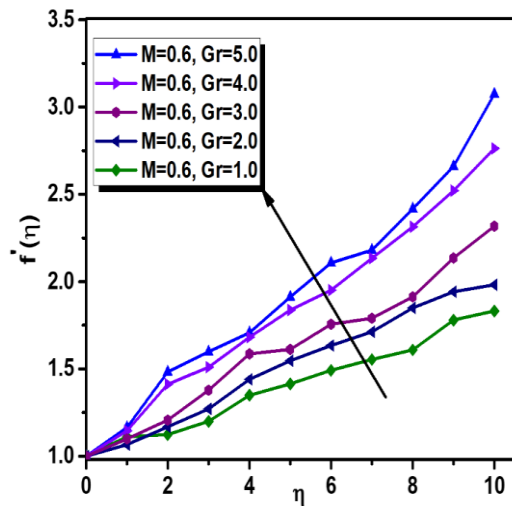
Figure 6. Effects of Gross number (Gr) on velocity at constant Magnetization (M=0.6)
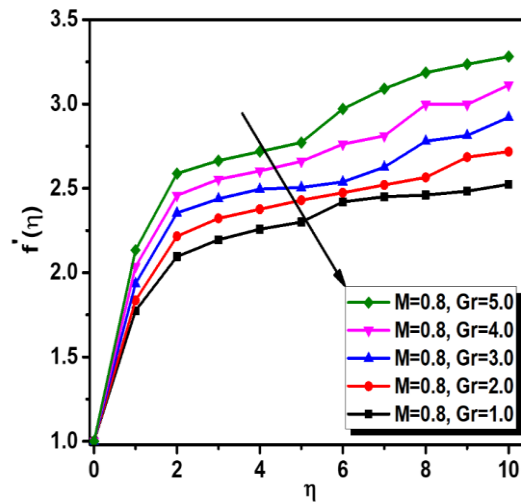
Figure 7. Effects of Gross number (Gr) on velocity at constant Magnetization (M=0.8)
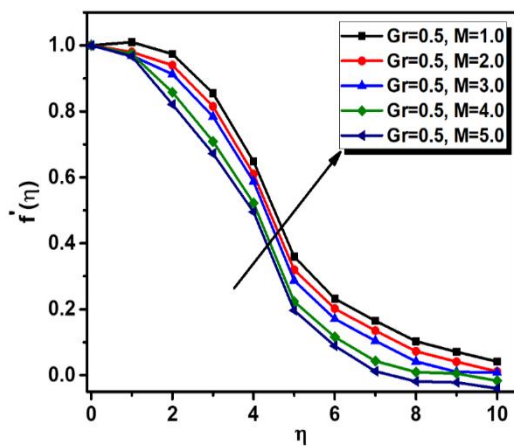

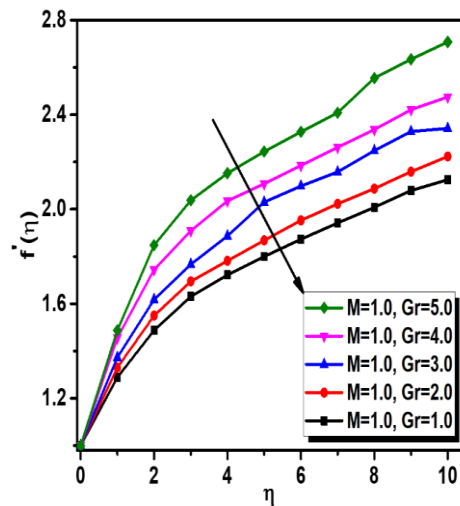Figure 8. Effects of Gross number (Gr) on velocity at constant Magnetization (M=1.0)
Figure 9. Effects of Magnetization on velocity at constant Gross number (Gr=0.5)

Figure 8. Effects of Gross number (Gr) on velocity at constant Magnetization (M=1.0)
Furthermore, the effect of magnetization (M) is prominent on the velocity for electrically conducting fluid flow (as shown in Figure 9). It is interesting to note that magnetization (M) decrease the fluid velocity, indicating the presence of Lorentz force. Obviously, Lorentz force is opposite the fluid direction which decrease the velocity. Moreover, higher value of magnetization (M) increase resistive forces that resist the fluid flow, resulting in decreasing fluid velocity. This result is consistent with the existing literature, (Ullah et. al 2016; Khan et al 2019).

Finally, it was observed that the velocity profiles ($f'(\eta)$) increase with an increase in the similarity parameter(η) while the temperature gradient profile ($\theta'(\eta)$) decrease with increasing similarity parameter(η). These results are in good agreement with previous literature (Ullah et al.,2016; Bhukta et. al 2014; Khan et al 2019). It's good to note that, the

existence of porous duct significantly improves the velocity because it acts as an insulator to the vertical surface, suggesting energy loss is prevented owing to convection as reported by Bhukta et al.,(2014). Furthermore, it is also reported that the rate of heat transfer plays a vital role in the presence of duct and magnetic field for MHD viscous fluid flow, (Bhukta et. al.,2014).

CONCLUSION

In conclusion, the impart of unsteady MHD flow with mixed convection over porous duct by the combined effects of magnetic field and heat transfer on the electrically conducting fluid flow are investigated. The governing flow and heat transfer equations are derived and converted into sets of nonlinear ordinary differential equations using similarity transformations and then solved numerically using a Runge-Kutter method with an implicit finite difference. On the basis of this findings with various parameters we draw the following main conclusions:

a) Velocity profiles increase with an increase in the similarity parameter while the temperature gradient profiles decrease with similarity parameter
b) Prandtl number and Gross number have an increasing effect on velocity profiles
c) Effect of Magnetization is prominent and has a decreasing effect on velocity profile
d) MHD temperature increases as the heat(energy) increase
e) There is a lush velocity increase similarity

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REFERENCES

Abdallah I. A. (2009). "Homotopy Analytical Solution of MHD Fluid Flow and Heat Transfer Problem". *Appl. Math. & Inform. Sci, 3, 223-233.*

Abolbashari. M.H., Freidoonimehr N., Nazari F., Rashidi M.M. (2014) 'Entropy analysis for an unsteady MHD flow past a stretching permeable surface in nano-fluid". *Powder Technol., 267, 256-267*

Bhukta D, Dash G.C, Mishra., S.R. and Baag S. (2015). "Dissipation effect on MHD mixed convection flow over a stretching sheet through porous medium with non-uniform heat source/sink". *Engineering Physics and Mathematics, 1, 1-9*

Buonomo B., Lauriat G., Manca O. and, Nardini S. (2016). "Numerical investigation on laminar slot-jet impinging in a confined porous medium in local thermal non-equilibrium". *Int. J. Heat Mass Transfer, 98, 484*

Chauhan DS, Agarwal R (2011a). "MHD flow through a porous medium adjacent to a stretching sheet: numerical and an approximate solution". *Eur. Phys. J. Plus, 126, 11047–53.*

Chauhan DS, Rastogi Priyanka (2011b). "Heat transfer and entropy generation in MHD flow through a porous medium past a stretching sheet". *IJET, 3, 1–13.*

Choudhury R. and Kumar Das S. (2014). "Visco-Elastic MHD Free Convective Flow through Porous Media in Presence of Radiation and Chemical Reaction with Heat and Mass Transfer". *J. Appl. Fluid Mech., 7, 603-609.*

Cortell R (2007). "MHD flow and mass transfer of an electrically conducting fluid of second-grade in a porous medium over a stretching sheet with chemically reactive species". *Chem. Eng Process, 46, 721–8.*

D. Bhukta, G. C. Dash, and S. R. Mishra (2014). "Heat and Mass Transfer on MHD Flow of a

Viscoelastic Fluid through Porous Media over a Shrinking Sheet". *Int. Scho. Res. Notices, 11*

Damseh RA, Al-Odat MQ, Chamkha AJ and Shannak BA (200). "Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface". *Int. J. Therm. Sci,* 48,1658–1663.

Daniel Y.S (2016). "Laminar convective boundary layer slip flow over a flat plate using homotopy analysis method". *J. Insti. Engineers (India): Series E, 97,* 115-121

Daniel Y.S., Z.A. Aziz, Z. Ismail, F. Salah (2017a). "Effects of slip and convective conditions on MHD flow of magnetohydrodynamics over a porous nonlinear stretching/shrinking sheet". *Aust. J. Mech. Eng., 1-17*

Daniel. Y.S (2015a). "Presence of heat generation/absorption on boundary layer slip flow of magnetohydrodynamics over a porous stretching sheet". *Amer. J. Heat Mass Transf, 2, 15 (2015a)*

Daniel. Y.S (2015b). "Steady MHD laminar flows and heat transfer adjacent to porous stretching sheets using HAM". *Amer. J. Heat Mass Transf, 2, 146-159*

Daniel. Y.S (2017b). "MHD laminar flows and heat transfer adjacent to permeable stretching sheets with partial slip condition". *J. advan. Mech. Eng, 1-15*.

Dehghan M., Valipour M. S. and Saedodin S. (2015). "Temperature-dependent conductivity in forced convection of heat exchangers filled with porous media: A perturbation solution" *Energy Convers. Manage.,* 91, 259-266.

Dolat Khan, Arshad Khan, Ilyas Khan, Farhad Ali, Faizan ul Karim and I. Tlili (2019). "Effects of Relative Magnetic Field, Chemical reaction, Heat generation and Newtonian Heating on convection flow of Casson Fluid over a moving vertical plate embedded in a Porous Medium". *Scientific Report*, 4, 400.

H afeez H.Y and C.E. Ndikilar (2016). "Flow of viscous fluid between two parallel porous plates with bottom injection and top suction". *Progress in Physics, 4, 1-8*.

Hafeez H.Y. Orou, J.B.C and Ojo O.A (2013). "Nonlinear effects in flow in porous duct". *Progress in Physics, 4, 74-77*.

Herrero J, Humphrey JAC and Gilralt F (1994). "Comparative analysis of coupled flow and heat transfer between co-rotating disks in rotating and fixed cylindrical enclosures". *ASME J Heat Transfer, 300, 111–21*.

Imran Ullah, Ilyas Khan and Sharidan Shafie (2016). "MHD Natural Convection Flow of CassonNanofluid over Nonlinearly Stretching Sheet Through Porous Medium with Chemical Reaction and Thermal Radiation". *Nanoscale Res. Lett, 11, 527*.

Jang J. and Lee S. S. (2000). "Theoretical and experimental study of MHD (magnetohydrodynamic) micropump". *Sens. Actuators, A, 80, 84-89*.

Kandasamy R, Periasamy K and KSP (2005). "Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects". *Int J Heat Mass Transf, 48, 4557–4561*.

Magyari E, and Chamkha AJ (2010). "Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface". *Int. J. Therm. Sci, 49, 1821–1828*.

Mahapatra TR, Mondal S, Pal D (2012). "Heat transfer due to magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface in presence of thermal radiation and suction/injection". *ISRN Thermodyn,9, 465864*.

Qian S. and Bau H. H. (2009). "Magneto-hydrodynamics based microfluidics". *Mech. Res. Commun., 36, 10-21*.

Raja Rabhi, Abir Yahya, Bayssain Amami and Hacen Dhahri (2017). "Influence of magnetohydrodynamic viscous flow on entropy generation within porous micro duct using the Lattice Boltzmann Method". *RSC Adv., 7, 30673*.

Raptis A, Perdikis C (2006). "Viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field". *Int J Non-Linear Mech, 41, 527–529.*

Richard L. Burden and J. Douglas Faires (2011). "Numerical Analysis", *Ninth Edition, ISBN-13: 978-0-538-73351-9 Brooks/Cole20 Channel Center Street Boston, MA02210USA Cengage Learning*

Subhas Abel M, Mahesha N, Sharanagouda, Malipatil B (2011). "Heat transfer due to MHD slip flow of a second-grade liquid over a stretching sheet through a porous medium with non-uniform heat source/sink". *Chem. Eng.Commun, 198,191–213.*

Suneetha S V, Veera Krishna M and Siva Pradad R (2010). "Hall Effect on Unsteady Rotating Hydro Dynamic Flow of an Incompressible Second Grade Fluid in a Porous Half Space". *J. Pure and Appl. Physics 22, 143-156.*

Veera Krishna M and Subba Reddy G (2016). "Unsteady MHD convective flow of Second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source". *IOP Conf. Series: Mat. Sci and Eng, 149, 012216.*

Veera Krishna M and Suneetha S V (2009). "Hall Effects on Unsteady MHD Rotating Flow of an Incompressible Viscous Fluid through a Porous Medium". *J. Pure and Appl. Physics, 21, 143-156.*