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Comparative Analysis of Fuzzy Regression and Fuzzy ARIMA Models for Forecasting Real Gross Domestic Product of Nigeria

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Abstract
Over the years, a number of studies have been conducted in terms of forecasting the real gross domestic product (GDP) of Nigeria. The GDP growth rate measures the annual growth rate in percentage of the monetary value of all finished goods and services made within a country. It is an important indicator of economic growth. This paper presents Fuzzy ARIMA and Regression to determine the interval of possibility for predicting the real GDP. The Fuzzy ARIMA (FARIMA) and Fuzzy Regression (FR) methods requires small size data as compared to the classical time series. Comparison between FARIMA and Fuzzy Regression with a threshold level of zero ($h = 0$) is performed by calibrating the models on existing data. The minimum values of the total spreads for FARIMA and FR are 0.791 and 4.077 respectively. In addition, the MAPE values for FARIMA is smaller than that of FR. Furthermore, the results indicate that the FARIMA gives a narrower interval of possibility for prediction than the FR.

Keywords: Forecasting, Fuzzy ARIMA, Fuzzy Regression, Interval of Possibility, Real GDP

INTRODUCTION
Gross Domestic Product forecasting has attracted the attention of many scholars because of its importance in measuring the performance of any economy. Thus, forecasting the Nigeria's Gross Domestic Product (GDP) could not be exceptional from this claim as over the years, a number of studies have been conducted in terms of forecasting the real GDP of Nigeria using different statistical techniques. The Box and Jenkins (1970) methodology, popularly Autoregressive Integrated Moving Average (ARIMA) and Regression Analysis are the common techniques found in the literature. Based on the ARIMA concept, real GDP can be expressed as a linear function of its own past values and associated random noise. Similarly, GDP could be evaluated as the linear function of key macroeconomic indicators using the regression analysis approach.

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Gross Domestic Product is the value of the goods and services produced by the nation's economy less the value of the goods and services used up in production (Dynan and Sheiner, 2018).The studies of Divya and Devi (2014), Jabaru and Jimoh (2020), Anthony and Emediong (2021), Dynan and Sheiner (2018) and Oyeyemi and Awujola (2014) are few undertakings that forecasted the real GDP. However, the methodologies in these classical approaches are constrained due to underlying model assumptions. One such assumption is the linearity as well as crisp relationship between response and explanatory variables in addition to randomness of the residuals. Such forceful assumptions may lead to the loss of some vital information, Pandit *et al*. (2021). Residuals, which are the deviations between the observed and estimated values are sometimes due to indefiniteness of the structure of the system or imprecise observations, Kahraman *et al*. (2006). These limitations among others makes unfit the ARIMA and the regression models. Hence, Fuzzy ARIMA (FARIMA) and Fuzzy Regression (FR) models are the ideal scientific approaches in this regard. These models make it possible to forecast the best and worst possible values of response variable based on predetermined values of the regressors.

The FR model predicts GDP against related factors. In other words, it becomes possible to forecast the best and worst possible interval based on predetermined or anticipated values of the related explanatory factors (Bakawu et al., 2023). A number of studies have been conducted to demonstrate the applications of fuzzy regression. More recent applications of fuzzy regression were proposed in Malyaretz et al. (2018), Lee et al. (2020), Taheri et al. (2020) and Attanayake (2021).

Additionally, in FARIMA models, instead of using crisp parameters, fuzzy parameters, in the form of triangular fuzzy numbers are used, Torbat et al. (2017). Consequently, the use of the fuzzy parameter reduces the need for large historical data unlike ARIMA models which requires at least 50 observations; preferably more than 100 observations. Hence, when the sample period is shorter, the prediction using Fuzzy ARIMA is better than other models, Mehdi et al. (2019). Recently, the FARIMA models have been widely applied to forecasting problems. Xie et al. (2021) developed a fuzzy ARIMA correction model for transport volume forecast capable of long term prediction. Reyes et al. (2021) proposed a new hybrid fuzzy time series model based on Fuzzy Time Series and Fuzzy ARIMA that achieved better in-sample and out-sample accuracy tests. Torbat et al. (2018) using FARIMA forecasted the Iran's steel consumption with improved accuracy. By using quadratic approach, Wang et al. (2009) demonstrated the application of FARIMA models on simulated time series data set.

It is apparent that the application of fuzzy regression and FARIMA in the recent times has attracted attention across diverse fields of human endeavours. This paper aimed to determine a narrower interval of possibility for predicting the real GDP of Nigeria by calibrating FARIMA and Fuzzy Regression models on existing data. The rest of the paper is structured as follows: In section 2, Materials and Method is presented, followed by Results and Discussion in section 3. Finally, conclusion is provided in section 4.

MATERIALS AND METHODS

Fuzzy Regression and Fuzzy Autoregressive Integrated Moving Average Models

Fuzzy linear regression is a fuzzy type of classical regression analysis in which some elements of the model are represented by fuzzy numbers (Alsoltany and Alnaqash, 2015). The relationship between the response and explanatory variables as reported in Tanaka et al. (1982) is presented as follows:

$$\tilde{Y} = \tilde{A}_0 + \tilde{A}_1 x_1 + \tilde{A}_2 x_2 + . \ . \ . + \tilde{A}_k x_k \tag{1}$$

In matrix form;

$$\hat{Y} = \tilde{A}X \tag{2}$$

Where:
$\tilde{Y}$ is the fuzzy output,
$X = (x_{0i}, x_{1i}, x_{2i}, \ . \ . \ ., x_{ki})^T$, p-dimensional crisp input vector,
$\tilde{A} = (\tilde{A}_0, \ \tilde{A}_1, \tilde{A}_{2,}, \ . \ . \ ., \tilde{A}_k)^T$, fuzzy vector of coefficients presented in the form of a symmetric triangular fuzzy number denoted by $\tilde{A}_k = [c_k, w_k]$, respectively, c_k and w_k are its centre and width, while $x_{0i} = 1$, $i = 1, \dots n$.

The FARIMA utilises same formulation as the FR, except the explanatory variables are lagged values of the response variable and the associated residuals. Hence, the following is the generalised FARIMA (p, d, q) model:

$$\widetilde{\omega}_p(L)Y_t^* = \ \tilde{\tau}_q(L)\varepsilon_t \tag{3}$$

$$Y_t^* = \Delta^d(Y_t - \mu) \tag{4}$$

The extended form of equation (3) is given in equation (5):

$$Y_t^* = \widetilde{\omega}_0 + \widetilde{\omega}_1 Y_{t-1}^* + \widetilde{\omega}_2 Y_{t-2}^* + \cdots + \widetilde{\omega}_p Y_{t-p}^* + \varepsilon_t - \tilde{\tau}_1\varepsilon_{t-1} - \tilde{\tau}_2\varepsilon_{t-2} - \ \cdots - \tilde{\tau}_q\varepsilon_{t-q} \tag{5}$$

Where, equation (4) is the ARIMA process of the time series Y_t, t is the time, $\Delta = 1 - L$, the difference operator, L is a lag operator; generally, $L^n Y_t = Y_{t-n}$, Y_t are observations, while $\widetilde{\omega}_0, \widetilde{\omega}_1, \widetilde{\omega}_2, \cdots \widetilde{\omega}_p$ and $\tilde{\tau}_1, \tilde{\tau}_2 \ \cdots \tilde{\tau}_q$ are fuzzy numbers.

The structure of the FARIMA (p, d, q) is built on the ARIMA process of the time series Y_t. Thus, p is the order of the Autoregressive term, q is the order of the Moving Average term, while d is the differencing order needed to achieve stationarity of the time series Y_t. The autocorrelation function (ACF) and partial autocorrelation function (PACF) are primary tools used to develop the structure of the FARIMA model. The sample ACF plot and the sample PACF plot are compared to the theoretical behaviour of these plots when the order is known. Additionally, Equation (5) is modified as shown in equation (6):

$$Y_t^* = \tilde{A}_0 + \tilde{A}_1 Y_{t-1}^* + \tilde{A}_2 Y_{t-2}^* + \cdots + \tilde{A}_p Y_{t-p}^* + \varepsilon_t - \tilde{A}_{p+1}\varepsilon_{t-1} - \tilde{A}_{p+2}\varepsilon_{t-2} - \ \cdots - \tilde{A}_{p+q}\varepsilon_{t-q} \tag{6}$$

where,
$\tilde{A}_0, \tilde{A}_1, \tilde{A}_2 \cdots \tilde{A}_p, \tilde{A}_{p+1}, \tilde{A}_{p+2}, \cdots \tilde{A}_{p+q}$ are fuzzy parameters.

Determination of the Fuzzy Parameters
A symmetrical fuzzy number A_j denoted as $\tilde{A}_j = [c_j, w_j]$ is defined as
$\mu_{A_j(a_j)} = L((a_j - c_j)/w_j), \ w_j > 0,$
where c_j is a centre, w_j is a width and $L(a_j)$ is a shape function of fuzzy number defined by:

i. $L(a_j) = L(-a_j)$,
ii. $L(0) = 1$,
iii. $L(a_j)$ is strictly decreasing function for $a_j \geq 0$,
iv. $\{a_j | \ L(a_j) \geq 0\}$ is a closed interval.

For each type of A_j, the membership functions are assumed triangular. By definition, it can be expressed as:

$$\mu_{\tilde{A}_j}(a_j) = \begin{cases} 1 - \frac{|c_j - a_j|}{w_j} & if\ c_j - w_j \leq a_j \leq c_j + w_j \\ 0 & otherwise \end{cases} \tag{7}$$

Where $w_j > 0$.

According to the extension principle (Zadeh, 1975), the membership function of the fuzzy number $\tilde{Y}$ and Y_t^* respectively are given in equations (8) and (9):

$$\mu_Y(y) = \begin{cases} Max(0, 1 - \frac{|y - \sum_{j=0}^{p} c_j x_{ij}|}{\sum_{j=0}^{p} w_j x_{ij}}) & if\ x_{ij} \neq 0 \\ 1 & if\ x_{ij} = 0, y \neq 0 \\ 0 & if\ x_{ij} = 0, y = 0 \end{cases} \tag{8}$$

The spread of $\tilde{y}$ is $\sum_{j=0}^{p} w_j x_{ij}$ and the middle value of $\tilde{y}$ is $\sum_{j=0}^{p} c_j x_{ij}$.

$$\mu_{\widetilde{Y^*}}(Y_t^*) = \begin{cases} 1 - \frac{|Y_t^* - \sum_{j=0}^{p} c_j Y_{t-j}^* - \varepsilon_t + \sum_{j=p+1}^{p+q} c_j \varepsilon_{t+p-j}|}{\sum_{j=0}^{p} w_j |Y_{t-j}^*| + \sum_{j=p+1}^{p+q} w_j |\varepsilon_{t+p-j}|} & for\ Y_t^* \neq 0,\ \varepsilon_t \neq 0 \\ 0 & otherwise \end{cases} \tag{9}$$

Linear Programming Formulation

Objective Function: We seek to find the coefficients $\tilde{A}_k = [c_k, w_k]$ that minimize the spread of the fuzzy output for all data sets. Mathematically, for the FR, this becomes:

$$Min\ S = \sum_{i=1}^{n} \sum_{k=0}^{K} w_{\boldsymbol{k}} |x_{\boldsymbol{ik}}| \tag{10}$$

Similarly, for a FARIMA problem with coefficients $\tilde{A}_j = [c_j, w_j]$, the objective function is given as equation (11):

$$Min\ S = \sum_{t=1}^{n} \sum_{j=0}^{p} w_j |\varphi_{jj}| |Y_{t-j}^*| + \sum_{t=1}^{n} \sum_{j=p+1}^{p+q} w_j |\rho_{j-p}| |\varepsilon_{t+p-j}| \tag{11}$$

Constraints: The constraints require that each observation y_i ($or\ y_t$ in the case of FARIMA) has a threshold value h in the interval $(0, 1)$ which is specified by the user of belonging to $\tilde{y}(y)$ (Taghizadeh et al. 2011). This implies,

$$\tilde{y}(y_i) \geq h,\ \ i = 1, 2, \ldots, n \tag{12}$$

After separately substituting equations (8) and (9) into equation (12), the simplified resulting LP models along the respective objective functions are obtained as model 1 and model 2 for the FR and FARIMA respectively.

Model 2: FR

$$\begin{aligned} & Min\ S = \sum_{i=1}^{n} \sum_{k=0}^{K} w_k |x_{ik}| \\ s.t\quad & \sum_{k=0}^{K} c_k x_{ik} - (1-h) \sum_{k=0}^{K} w_k |x_{ik}| \leq y_i,\ \forall i = 1, \ldots, n \\ & \sum_{k=0}^{K} c_k x_{ik} + (1-h) \sum_{k=0}^{K} w_k |x_{ik}| \geq y_i,\ \forall i = 1, \ldots, n \\ & w_k \geq 0, \forall i = 1, \ldots, n;\ x_{i0} = 1 \end{aligned} \tag{13}$$

Where, w_k and c_k for $k = 0, 1, \ldots, K$ are the FR unknown variables vectors

Model 1: FARIMA

$$\begin{aligned} & Min\ S = \sum_{t=1}^{n} \sum_{j=0}^{p} w_j |\varphi_{jj}| |Y_{t-j}^*| + \sum_{t=1}^{n} \sum_{j=p+1}^{p+q} w_j |\rho_{j-p}| |\varepsilon_{t+p-j}| \\ s.t\quad & \sum_{j=0}^{p} c_j Y_{t-j}^* + \varepsilon_t - \sum_{j=p+1}^{p+q} c_j \varepsilon_{t+p-j} + (1-h) \sum_{j=0}^{p} w_j |Y_{t-j}^*| + \sum_{j=p+1}^{p+q} w_j |\varepsilon_{t+p-j}| \geq Y_t, \forall t = 1, \ldots, n \\ & \sum_{j=0}^{p} c_j Y_{t-j}^* + \varepsilon_t - \sum_{j=p+1}^{p+q} c_j \varepsilon_{t+p-j} - (1-h) \sum_{j=0}^{p} w_j |Y_{t-j}^*| + \sum_{j=p+1}^{p+q} w_j |\varepsilon_{t+p-j}| \leq Y_t, \forall t = 1, \ldots, n \\ & w_j \geq 0, \forall j = 0, \ldots, p+q. \end{aligned}$$

(14)

Similarly, w_j, c_j, for $j = 0, 1, \ldots, p$. are unknown variables vectors, ρ_{j-p} and φ_{jj} are the autocorrelation coefficient of time lag $j - p$ and partial autocorrelation coefficient of time lag j respectively.

Based on the results in equations (13) and (14), the relation in equations (1) and (6) can be rewritten in possibilistic form as follows:

$$\tilde{Y} = (c_0, w_0) + (c_1, w_1)x_{1i} + (c_2, w_2)x_{2i} + \cdots + (c_k, w_p)x_{ki} \quad (15b)$$

$$Y_t^* = (c_0, w_0) + (c_1, w_1)Y_{t-1}^* + (c_2, w_2)Y_{t-2}^* + \cdots + (c_p, w_p)Y_{t-p}^* + \varepsilon_t - (c_{p+1}, w_{p+1})\varepsilon_{t-1} - (c_{p+2}, w_{p+2})\varepsilon_{t-2} - \cdots - (c_{p+q}, w_{p+q})\varepsilon_{t-q} \quad (15a)$$

$i = 1, 2, \ldots, n$, where n is the number of observations.

The interval prediction models, that is equations (15a) and (15b) makes it possible to forecast the best and worst possible values of $\tilde{Y}$ based on predetermined values of $X = (x_{1i}, x_{2i}, \ldots, x_{ki})$ when FR model is considered or the lagged values of Y_t^* in the case of FARIMA model.

RESULTS AND DISCUSSION

In this section, we solved the LP problems (equations (13) and (14)) with threshold level of $h = 0$ in order to determine the minimal fuzziness of the models. The data related to GDP, unemployment rate, inflation rate and FDI are obtained from Ogosi et al. (2022). The empirical results are in three phases as follows:

Phase I: The fuzzy parameters are obtained by solving models 1 and 2 using Tora Optimization Software (Taha, 2011). The central values and widths of each fuzzy parameter in equations (1) and (3) for $h = 0$ were obtained and presented in Tables 1 and 2 along with the corresponding upper bound (UB) and lower bound (LB) respectively.

Table 1. Central and widths values for fuzzy parameters for FR

Fuzzy parameters	Centre	Width	UB	LB
A_o	4.021	0.116	4.137	3.906
A_1	1.005	0.000	1.005	1.005
A_2	-0.145	0.022	-0.124	-0.167
A_3	0.376	0.000	0.376	0.376

Table 2. Central and widths values for fuzzy parameters for FARIMA

Fuzzy parameters	Centre	Width	UB	LB
A_o	0.074	0.000	0.074	0.074
A_1	0.990	0.007	0.997	0.983

The estimated fuzzy linear regression model for the real GDP $(\tilde{y})$ of Nigeria against the three macroeconomic factors and FARIMA model are provided in equations (16a) and (16b) respectively:

$$GDP\ (\tilde{y}_t) = (4.021, 0.116) + (1.005, 0.000)x_1 + (-0.145, 0.022)x_2 + (0.376, 0.000)x_3 \quad (16a)$$

$$GDP\ (\tilde{y}_t) = (0.074, 0.000) + (0.990, 0.007)y_{t-1} \quad (16b)$$

Equation (16a) implies real GDP can be suitably predicted when the unemployment indicator is exactly 1.005, index of inflation is between -0.167 and -0.124, and foreign direct investment is exactly 0.376 (see Table 1, columns 4 and 5). Whereas, equation (16b) indicates that real GDP can be predicted when the previous GPD is between 0.983 and 0.997 (see Table 2, columns 4 and 5) respectively. Additionally, the minimum values of the total spreads for FARIMA and FR are 0.791 and 4.077 respectively.

Phase II: Prediction of bounds: Using equations (16a) and (16b), the best and worst possible real GDP for the considered time range were predicted and the results are shown in Tables 3 and 4. Figures 3 and 4 represents graphical plot of the predicted UB and LB of the FR and FARIMA models along with the actual real GPD respectively.

Table 3. Prediction results of FR model

Year	GDP	FR UB	FR LB	Year	Actual GDP	FR UB	FR LB
1991	4.283	4.507	4.228	2006	4.602	4.847	4.576
1992	4.293	4.484	4.182	2007	4.633	4.896	4.633
1993	4.299	4.545	4.238	2008	4.663	4.900	4.623
1994	4.301	4.608	4.301	2009	4.698	4.925	4.647
1995	4.309	4.309	3.998	2010	4.737	4.869	4.588
1996	4.326	4.422	4.128	2011	4.760	4.944	4.668
1997	4.338	4.477	4.205	2012	4.778	4.898	4.620
1998	4.349	4.395	4.121	2013	4.801	4.873	4.602
1999	4.351	4.618	4.351	2014	4.827	4.940	4.669
2000	4.375	4.635	4.367	2015	4.839	4.839	4.567
2001	4.403	4.588	4.302	2016	4.832	5.086	4.804
2002	4.462	4.689	4.410	2017	4.836	5.120	4.836
2003	4.501	4.694	4.414	2018	4.844	5.037	4.759
2004	4.544	4.676	4.394	2019	4.854	5.114	4.837
2005	4.574	4.820	4.535				

Table 4. Prediction results of FARIMA model

Year	Actual GDP	FARIMA LB	FARIMA UB	Year	Actual GDP	FARIMA LB	FARIMA UB
1992	4.293	4.285	4.342	2006	4.602	4.571	4.633
1993	4.299	4.295	4.352	2007	4.633	4.599	4.660
1994	4.301	4.301	4.358	2008	4.663	4.630	4.691
1995	4.309	4.303	4.360	2009	4.698	4.659	4.721
1996	4.326	4.311	4.368	2010	4.737	4.693	4.756
1997	4.338	4.328	4.385	2011	4.760	4.732	4.795
1998	4.349	4.339	4.397	2012	4.778	4.754	4.818
1999	4.351	4.350	4.408	2013	4.801	4.772	4.836
2000	4.375	4.352	4.410	2014	4.827	4.795	4.859
2001	4.403	4.376	4.434	2015	4.839	4.820	4.885
2002	4.462	4.403	4.462	2016	4.832	4.832	4.897
2003	4.501	4.461	4.521	2017	4.836	4.825	4.890
2004	4.544	4.500	4.560	2018	4.844	4.829	4.894
2005	4.574	4.542	4.603	2019	4.854	4.837	4.902

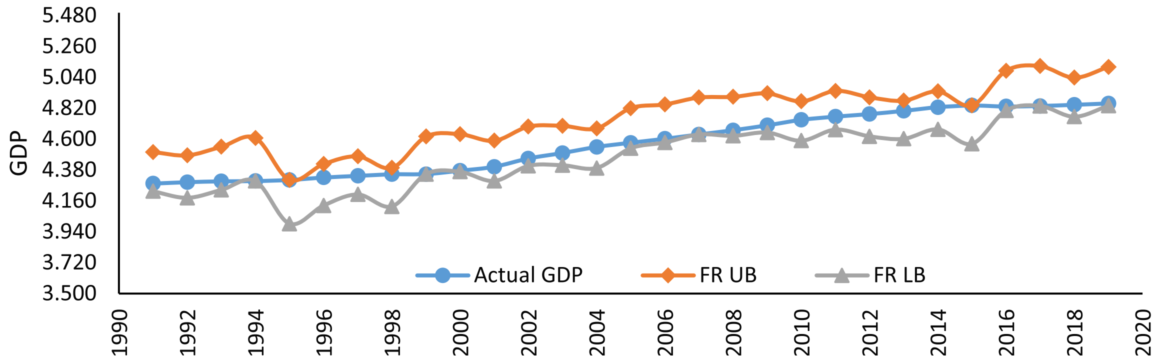

Figure 3: Actual real GDP along with UB and LB resulting from FR model

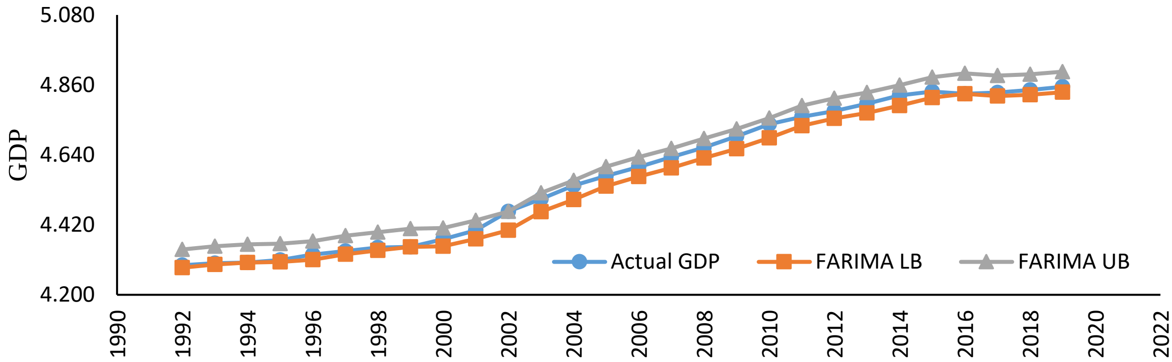

Figure 4: Actual real GDP along with UB and LB resulting from FARIMA model

Phase III: Bound assessment: From Table 3 and Figure 3, as well as Table 4 and Figure 4, it can be observed that the actual GDP values are located within the predicted bounds. However, in Table 3 and Figure 3, the interval of possibility is narrower as compared to Table 4 and Figure 4. Additionally, model adequacy assessment based on Mean Absolute Percentage (MAPE) suggests the fitness of the established models as both bounds are within ten percent error, which is an indication of high accuracy (Akincilar et al. (2011); Bakawu et al. (2020)). Furthermore, the MAPE values for FR UB and FR LB are 4.047% and 2.131%, while the FARIMA UB and FARIMA LB are 0.490% and 0.838% respectively. This shows that the MAPE values for equation (16b) is smaller than that of equation (16a). Hence, equation (16b) could be the suitable model for predicting the future real GDP of Nigeria. This adequacy assessment, tallies with the accuracy of fuzzy regression reported in Malyaretz et al., (2018).

CONCLUSION

This study employed a methodology based on Fuzzy Autoregressive Integrated Moving Average and Fuzzy Linear Regression capable of predicting the real GDP of Nigeria, assuming that residuals are due to system fuzziness. Based on the empirical results, FR and FARIMA models were established with threshold value of 0 that can adequately estimate the real GDP of Nigeria. Consequently, considering the criteria of interval of possibility and MAPE, FARIMA is found to be the most suitable model for predicting the real GDP.

Future research is focused on hybridising FARIMA with other available tools. The results of the hybrid method are evaluated on the basis of some performance metrics.

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Appendix

Data on Macroeconomic Variables: GPD(Y), Unemployment (X1), Inflation (X2), and FDI(X3)

Year	GDP	UNEMPLOYMENT	INFLATION	FDI	Year	GDP	UNEMPLOYMENT	INFLATION	FDI
2019	4.854	0.908	1.057	0.519	2004	4.544	0.579	1.176	0.272
2018	4.844	0.916	1.082	0.301	2003	4.501	0.582	1.147	0.303
2017	4.836	0.924	1.218	0.544	2002	4.462	0.582	1.110	0.276
2016	4.832	0.849	1.195	0.648	2001	4.403	0.577	1.276	0.076
2015	4.839	0.634	0.955	0.486	2000	4.375	0.577	0.841	0.057
2014	4.827	0.659	0.906	0.671	1999	4.351	0.579	0.821	0.000
2013	4.801	0.568	0.928	0.745	1998	4.349	0.575	1.000	-0.523
2012	4.778	0.573	1.087	0.849	1997	4.338	0.575	0.931	-0.328
2011	4.760	0.576	1.035	0.946	1996	4.326	0.576	1.466	-0.301
2010	4.737	0.576	1.137	0.780	1995	4.309	0.575	1.862	-0.469
2009	4.698	0.571	1.099	0.932	1994	4.301	0.575	1.756	0.292
2008	4.663	0.549	1.064	0.913	1993	4.299	0.573	1.757	0.130
2007	4.633	0.553	0.732	0.781	1992	4.293	0.565	1.649	-0.046
2006	4.602	0.562	0.915	0.686	1991	4.283	0.561	1.114	-0.149
2005	4.574	0.573	1.252	0.697					

Source: Ogosi et al. (2022)