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Solutions of Schrödinger Equation Based on Modified Exponential Screened Plus Yukawa Potential (MESPYP) within the Frame of Parametric Nikiforov-Uvarov Method

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Abstract
In this paper, we deduced an approximate bound state solutions of Schrödinger equation based on Modified Exponential Screened Plus Yukawa Potential (MESPYP), with the help Greene–Aldrich Approximation to determine the centrifugal term. The analytical solution was obtained based on Nikiforov–Uvarov (NU) method. The bound state solutions of five diatomic molecules which are mercury hydride (HgH), zinc hydride (ZnH), cadmium hydride (CdH), hydrogen bromide (HBr) and hydrogen fluoride (HF) molecules were also calculated numerically. The estimated energy eigenvalues for these molecules were deduced using the resulting energy eigenequation and total unnormalized wavefunction is expressed in term of Jacobi polynomial. The numerical solution shows that the bound state solutions increase with increase in the quantum state.

Keywords: Nikiforov–Uvarov method, Modified Exponential Screened Plus Yukawa Potential, Schrodinger Equation and Greene–Aldrich Approximation

INTRODUCTION
In most cases, quantum mechanics study the relativistic and nonrelativistic wave equations which stimulate the interest of many scholars and researchers (Okon, *et al.,* 2021a). The equation of Schrödinger is nonrelativistic wave equation, while Dirac and Klein-Gordon equations are relativistic wave equation describing spin-half and spinless particles, (Antia, *et al.,* 2019a; Omugbe, *et al.,* 2020a and Onate, *et al.,* 2020a) respectively. The total function provides implicitly the information about the quantum mechanical system (Onate, *et al.,*

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2020a). For decade many researchers have adopted many methods in providing solutions to both relativistic and nonrelativistic wave equations. Among the approach adopted by researcher are Nikiforov–Uvarov method (Parmar *et al.,* 2021a and Purohit *et, al.,* 2021a), exact proper quantization (Purohit *et, al.,* 2021a), asymptotic iteration method, (Omugbe *et al.,* 2020a). Wentzel–Kramers–Brillouin (WKB) approach (Omugbe *et al.,* 2020a) and many others (Okon *et at.,* 2021d; Okon *et al.,* 2022b; Onate *et al.,* 2021b and Antia *et al.,* 2019b).These techniques have been used to solve some quantum mechanical potentials, like Hulthen, Yukawa, Poschl–Teller, Tietz–Wei, Tietz–Hua, exponential-type potentials, hyperbolic potentials, Kratzer potentials, screened-Kratzer potential, Mobius square, Hellmann, coulomb, Cornel, Killingbeck, Woods–Saxon, Deng-Fan, Hylleraas, Eckart, pseudo harmonic, Poschl–Teller, modified Yukawa potential, supersymmetric quantum mechanics approach and many others (Farout *et al.,* 2020c; Omugbe *et al.,* 2020c and Onate *et al.,* 2021b).

The Yukawa potential is also called the screened coulomb potential which is a short-range potential with many applications such as in particle physics, high energy and molecular physics which is basically used in describing the interaction within the atoms of diatomic molecules (Onate *et al.,* 2021a). On the same note, plenty research has been carried out on thermodynamic properties of some considerable potentials. Omugbe *et al.,* (2020b), studied spin and pseudospin solutions of Dirac equation and its thermodynamic properties using hyperbolic Hulthen plus hyperbolic exponential inversely quadratic potential where they obtained both numerical bound state solutions for spin and pseudospin symmetries based on nonrelativistic limit, they also obtain nonrelativistic energy eigen equation presented in a close form to study partition function and other thermodynamic properties. According to Okon *et al.,* (2021d); Purohit *et al.,* (2021a) and Omugbe *et al.,* (2021a), in their studieds, they used Mobius Square plus screened kratzer potential based on Nikiforov–Uvarov (NU) method to estimate thermodynamic properties and bound state solutions of Schrödinger equation. Farout *et al*., (2020a), in their studies, they were able to describe the energy spectra and thermodynamic properties of hyperbolic poshchl-teller potential. They also solved Dirac equation using modified factorization method to obtain both relativistic and nonrelativistic for vibrational energy spectra and thermodynamic properties on the basis of some diatomic molecules such as Hydrogen Chloride (HCl), Chlorine (Cl_2), Carbon(II)Oxide and Lithium Hydride (LiH). Manga *et al.,* (2021a), made use of the modified Yukawa potential to described the relationship between pseudovector and pseudoscalar on the neutron – neutron, proton – proton and proton – neutron coupling constant. Similarly, the used of suitable potential to reduces screened – kratzer and kratzer potential as special case in quantum approximations. Onate *et al.,* (2021b) and Manga *et al*., (2021b), they study bound state and thermal properties of modified Tietz–Hua potential using supersymmetric quantum mechanics approach where they looked into vibrational energy spectra and thermodynamic properties as applicable to some diatomic properties. Purohit *et al.,* (2020b), in their studied they estimate the value of eigen solution and with various properties of screened cosine Kratzer potential in D-dimension based on relativistic and nonrelativistic treatment. They extend their study to rotational and vibrational energies for few heterogeneous diatomic molecules. Manga *et al.,* (2021c), adopted coupling constant to calculate the effective radius and scattering length between proton-neutron and neutron – neutron based on Modified Yukawa Potential. So researchers work on potential model to study the partition function as well as information theoretic measures like Tsallis, Renyi, Shannon and Fisher information entropies.

Recently, researchers have studied potential model under the influence of Aharonov–Bohm flux and external magnetic field. Purohit *et al.,* (2021a), studied the thermomagnetic properties of the screened-Kratzer potential under the influence of Aharonov–Bohm flux and external

magnetic field where the obtained thermomagnetic properties as well as persistent current, magnetization and magnetic susceptibility. Okon *et al.,* (2022b) investigated the effect of Aharonov–Bohm and external magnetic field on Hellmann plus screened-Kratzer potential where the energy equation was presented in a closed form and applied to study thermomagnetic properties as applicable to some diatomic molecules. On the same note, the authors obtained a normalized wave function which is expressed in terms of Jacobi polynomials as well as the wave function and probability density plots for some selected diatomic molecules. They also obtained other thermomagnetic properties including partition function, vibrational mean energy, vibrational heat capacity, magnetization, persistent current and magnetic susceptibility.

In this study, we deduced bound state solutions of Schrödinger equation based on modified exponential screened plus Yukawa potential (MESPYP) within the frame of parametric Nikiforov-Uvarov method. This new potential is found to be suitable for the purpose of this studies.

NIKIFOROV-UVAROV METHOD.

This NU-Method is widely used in solving second order linear differential equation by reducing it to a generalized equation of hypergeometric type as follows (Omugbe *et al*., 2020c).

$$\psi''(s)+\frac{\bar{\tau}(s)}{\sigma(s)}\psi'(s)+\frac{\bar{\sigma}(s)}{\sigma^2(s)}\psi(s)=0 \qquad .. (1)$$

Where $\sigma(s)$ and $\sigma^2(s)$ are polynomials at most second degree, and $\bar{\tau}(s)$ is a first-degree polynomial.

The simplified Parametric Nikiforov Uvarov Method by is given as (Omugbe *et al*., 2020a and Farout *et al.,* 2020a).

$$\psi''(s)+\frac{C_1-C_2S}{S(1-C_3S)}\psi'(s)+\frac{1}{S^2(1-C_3S)^2}(-X_1S^2+X_2S-X_3)\psi(s)=0 \qquad \ldots (2)$$

The total wave function is given as

$$\psi(s)=N_{nl}S^{C_{12}}(1-C_3S)^{-C_{12}-\left({}^{C_{13}}/_{C_3}\right)}P_n^{\left(C_{10}-1,\frac{C_{11}}{C_3}-C_{10}-1\right)}(1-2S) \qquad \ldots (3)$$

Where the parametric constants can be obtained as follows

$$C_4=\frac{1}{2}(1-C_1) \qquad \ldots (4)$$

$$C_5=\frac{1}{2}(C_2-2C_3) \qquad \ldots (5)$$

$$C_6=C_5^2+X_1 \qquad \ldots (6)$$

$$C_7=2C_4C_5-X_2 \qquad \ldots (7)$$

$$C_8=C_4^2+X_3 \qquad \ldots (8)$$

$$C_9=C_3C_7+C_3^2C_8+C_6 \qquad \ldots (9)$$

$$C_{10}=C_1+2C_4+2\sqrt{C_8} \qquad \ldots (10)$$

$$C_{11}=C_2-2C_5+2(\sqrt{C_9}+C_2\sqrt{C_8}) \qquad \ldots (11)$$

$$C_{12} = C_4 + \sqrt{C_8} \quad \ldots (12)$$

$$C_{13} = C_5 - (\sqrt{C_9} + C_3\sqrt{C_8}) \quad \ldots (13)$$

A special case for Laguerre polynomial where $C_3 = 0$

$$limP_n^{\left(C_{10}-1,\frac{C_{11}}{C_3}-C_{10}-1\right)}(1-2C_3S) = L_n^{C_{10}-1}(C_{11}S) \quad \ldots (14)$$

And

$$\lim_{C_3\to 0}(1-C_3S)^{-C_{12}-\frac{C_{13}}{C_3}} = e^{C_{13}S} \quad \ldots (15)$$

Therefore, the solution of total wave function as in equation (3) is given as (Omugbe *et al*., 2020b).

$$\psi(s) = S^{C_{12}}e^{C_{13}S}L_n^{C_{10}-1}(C_{11}S) \quad \ldots (16)$$

Radial Solution of Schrodinger Equation with Modified Exponential Screened Plus Yukawa Potential

The modified exponential screened plus Yukawa potential is as follows (Pamar *et al.*,2020)

$$V(r) = D_0\left(\frac{1+e^{-2\alpha r}}{1-e^{-2\alpha r}}\right) + \frac{D_1e^{-2\alpha r}}{r} \quad \ldots (17)$$

Where $D_0 = \frac{D_e}{2} > 0, D_e$ is the dissociation energy that describes the depth of the potential well, α is the screening parameter which characterizes the strength of the potential. D_1 is a real constant which also serve as a control parameter for the potential model, while r is the internuclear distance between the atoms of diatomic molecules (Pamar *et al.*,2020)

The Schrodinger wave equation is given as

$$\frac{d^2R}{dr^2} + \left(\frac{2\mu}{\hbar^2}\left(E - V(r)\right) - \frac{\lambda}{r^2}\right)R(r) = 0 \quad \ldots (18)$$

Substitute equation (17) into equation (18), we have

$$\frac{d^2R}{dr^2} + \left(\frac{2\mu}{\hbar^2}\left(E - D_0\left(\frac{1+e^{-2\alpha r}}{1-e^{-2\alpha r}}\right) - \frac{D_1e^{-2\alpha r}}{r}\right) - \frac{l(l+1)}{r^2}\right)R(r) = 0 \quad \ldots (19)$$

Applying the Greene–Aldrich approximation to the centrifugal term given as (Antia *et al*., 2019a)

$$\frac{1}{r^2} = \frac{4\alpha^2e^{-2\alpha r}}{(1-e^{-2\alpha r})^2} \Longrightarrow \frac{1}{r} = \frac{2\propto e^{-2\alpha r}}{(1-e^{-2\alpha r})} \quad \ldots (20)$$

By substituting equation (20) into (19) we have

$$\frac{d^2R}{dr^2} + \left(\frac{2\mu}{\hbar^2}\left(E - D_0\left(\frac{1+e^{-2\alpha r}}{1-e^{-2\alpha r}}\right) - \frac{2\propto D_1e^{-2\alpha r}}{1-e^{-2\alpha r}}\right) - \frac{4\alpha^2e^{-2\alpha r}l(l+1)}{(1-e^{-2\alpha r})^2}\right)R(r) = 0 \quad \ldots (21)$$

Let $s = e^{-2\alpha r}$, substitute for s into equation (21), we have.

$$\frac{d^2R(s)}{ds^2} + \frac{(1-s)}{s(1-s)}\frac{dR(s)}{ds} + \frac{1}{s^2(1-s)^2}\{[(-\xi^2 + A + B)s^2 + (2\xi^2 - B - \lambda)s] + (-\xi^2 - A)\}R(s) = 0 \quad \ldots (22)$$

Where

$$\xi^2 = -\frac{\mu E}{2\alpha^2\hbar^2}, A = \frac{\mu D_0}{2\alpha^2\hbar^2}, C = \frac{\mu D_1}{\propto\hbar^2} \quad \ldots (23)$$

Comparing equation (22) with equation (2)

$$X_1 = -\xi^2 + A + B,\ X_2 = 2\xi^2 - B - \lambda,\ X_3 = 2\xi^2 + A \quad \ldots (24)$$

The parametric constants can be calculated as?

$C_1 = C_2 = C_3 = 1$ … (25)

$C_4 = 0$ … (26)

$C_5 = -\frac{1}{2}$ … (27)

$C_6 = \frac{1}{4} + \xi^2 - A - B$ … (28)

$C_7 = -2\xi^2 + B + \lambda$ … (29)

$C_8 = \xi^2 + A$ … (30)

$C_9 = \lambda + \frac{1}{4}$ …. (31)

$C_{10} = 1 + 2\sqrt{\xi^2}$+A … (32)

$C_{11} = 2 + 2(\sqrt{\lambda + \frac{1}{4}} + \sqrt{\xi^2}$+A …. (33)

$C_{12} = \sqrt{\xi^2}$+A …. (34)

$C_{13} = -\frac{1}{2} - \sqrt{\lambda + \frac{1}{4}} - \sqrt{\xi^2}$+A … (35)

The eigen energy equation for the proposed potential as follows (Onate *et al.,* 2021b).

$E_{nL} = -\frac{2\propto^2\hbar^2}{\mu}\{\frac{n^2+n+\frac{1}{2}+\left(n+\frac{1}{2}\right)\sqrt{4l(l+1)+1}+l(l+1)+\frac{\mu D_1}{\propto\hbar^2}+\frac{\mu D_e}{2\propto^2\hbar^2}}{2n+1+\sqrt{4l(l+1)+1}}\}^2 + \frac{D_e}{2}$ … (36)

The total unnormalized wave function is given as follows (Okon *et al.,* 2022b).

$\psi_{nL}(s) = N_{nL}s^{\sqrt{\frac{\mu D_o}{2\propto^2\hbar^2}-\frac{\mu E_{nl}}{2\propto^2\hbar^2}}}(1-s)^{\frac{1}{2}+\sqrt{\frac{1}{4}+\lambda}}p_n^{\{(2\sqrt{\frac{\mu D_o}{2\propto^2\hbar^2}-\frac{\mu E_{nl}}{2\propto^2\hbar^2}})(2\sqrt{\frac{1}{4}+\lambda})\}}(1-2s)$ … (37)

simplifying equation (37) we have

$\psi_{nL}(s) = N_{nL}s^{X_1}(1-s)^{X_2}p_n^{\{(2X_1)(2X_2-1)\}}(1-2s)$ … (38)

Where $X_1 = \sqrt{\frac{\mu D_o}{2\propto^2\hbar^2} - \frac{\mu E_{nl}}{2\propto^2\hbar^2}}$, $X_2 = \frac{1}{2} + \sqrt{\frac{1}{4}+\lambda}$ … (39)

NUMERICAL COMPUTATION OF ENERGY EIGENVALUES

By applying equation (36) the numerical bound state solutions were carried out for fixed principal quantum number (n) with varying orbital angular quantum number $l = 0, 1, 2\ and\ 3$.

Table 1: Result of spectroscopic constants for five selected diatomic molecules Okon *et, al.*, (2022)

Molecules	$\mu\ (amu)$	$D_e(MeV)$	$\alpha\ (cm^{-1})$
HgH	1.0031	0.4635	0.3129
ZnH	0.9928	0.9504	0.2513
CdH	0.9992	0.7663	0.2183
Hbr	0.9958	3.9169	0.2261
Hf	0.9573	6.6456	0.7711

Table2: Numerical solution for HgH, ZnH, CdH, Hbr and Hf

n	L	$E_{n(HgH)}(eV)$	$E_{n(ZnH)}(eV)$	$E_{n(CdH)}(eV)$	$E_{n(Hbr)}(eV)$	$E_{n(Hf)}(eV)$
0.0	0.0	-8.02414473	-56.8580585	-284.288200	-3468.71640	-146.298099
1.0	0.0	-0.97025904	-4.48428850	-19.3566766	-220.107139	-17.99971677
2.0	0.0	-0.59289067	-1.74733844	-5.21989155	-46.3555263	-17.32764777
3.0	0.0	-0.52939119	-1.45275103	-2.93440083	-17.2265713	-33.03782158
4.0	0.0	-0.51203863	-1.69713538	-2.49248259	-9.47809033	-68.30742615
5.0	0.0	-0.50580566	-2.32959198	-2.63994448	-7.04964095	-132.9084166
6.0	0.0	-0.50313375	-3.42750815	-3.16795448	-6.54715819	-239.4327103
7.0	0.0	-0.50183694	-5.13245041	-4.08312017	-7.06670360	-402.8857622
8.0	0.0	-0.50114679	-7.62079011	-5.45893705	-8.37296740	-640.6138172
9.0	0.0	-0.50075241	-11.0965740	-7.39973433	-10.4571814	-972.2862533
10.0	0.0	-0.50051390	-15.7893962	-10.0299115	-13.4051430	-1419.890311
11.0	0.0	-0.50036285	-21.9536583	-13.4902001	-17.3513724	-2007.729264

Table3: Numerical solution for HgH, ZnH, CdH, Hbr and Hf

n	L	$E_{n(HgH)}(eV)$	$E_{n(ZnH)}(eV)$	$E_{n(CdH)}(eV)$	$E_{n(Hbr)}(eV)$	$E_{n(Hf)}(eV)$
0.0	1.0	-1.36010257	-4.48428850	-19.3566766	-4.34956461	-17.99971677
1.0	1.0	-1.04382474	-1.74733844	-5.21989155	-3.10224342	-17.32764777
2.0	1.0	-1.35883436	-1.45275103	-2.93440083	-2.52232863	-33.03782158
3.0	1.0	-2.16959108	-1.69713538	-2.49248259	-2.23099571	-68.30742615
4.0	1.0	-3.68769285	-2.32959198	-2.63994448	-2.09975830	-132.9084166
5.0	1.0	-6.21410425	-3.42750815	-3.16795448	-2.06815083	-239.4327103
6.0	1.0	-10.1130452	-5.13245041	-4.08312017	-2.10079918	-402.8857622
7.0	1.0	-15.8074697	-7.62079011	-5.45893705	-2.17507637	-640.6138172
8.0	1.0	-23.7779918	-11.0965740	-7.39973433	-2.27637276	-972.2862533
9.0	1.0	-34.5625698	-15.7893962	-10.0299115	-2.39535561	-1419.890311
10.0	1.0	-48.7563977	-21.9536583	-13.4902001	-2.52609040	-2007.729264
11.0	1.0	-67.0118635	-29.8682818	-17.9362043	-2.66478188	-2762.421731

Table4: Numerical solution for HgH, ZnH, CdH, Hbr and Hf

n	L	$E_{n(HgH)}(eV)$	$E_{n(ZnH)}(eV)$	$E_{n(CdH)}(eV)$	$E_{n(Hbr)}(eV)$	$E_{n(Hf)}(eV)$
0.0	2.0	-1.05353151	-1.74733844	-6169.92737	-46.3555263	-17.32764777
1.0	2.0	-1.40681681	-1.45275103	-1975.92180	-17.2265713	-33.03782158
2.0	2.0	-2.26828472	-1.69713538	-830.004949	-9.47809033	-68.30742615
3.0	2.0	-3.86012368	-2.32959198	-418.702749	-7.04964095	-132.9084166
4.0	2.0	-6.48914994	-3.42750815	-242.852795	-6.54715819	-239.4327103
5.0	2.0	-10.5247114	-5.13245041	-158.163419	-7.06670360	-402.8857622
6.0	2.0	-16.3947325	-7.62079011	-113.981079	-8.37296740	-640.6138172
7.0	2.0	-24.5847544	-11.0965740	-89.8878503	-10.4571814	-972.2862533
8.0	2.0	-35.6376482	-15.7893962	-76.7738479	-13.4051430	-1419.890311
9.0	2.0	-50.1535156	-21.9536583	-70.2627122	-17.3513724	-2007.729264
10.0	2.0	-68.7896498	-29.8682818	-68.1462792	-22.4612258	-2762.4217
11.0	2.0	-92.2605193	-39.8365838	-69.2866663	-28.9232397	-3712.901221

Table5: Numerical solution for HgH, ZnH, CdH, Hbr and Hf

n	L	$E_{n(HgH)}(eV)$	$E_{n(ZnH)}(eV)$	$E_{n(CdH)}(eV)$	$E_{n(Hbr)}(eV)$	$E_{n(Hf)}(eV)$
0.0	3.0	-1.40681681	-1.45275103	-2.93440083	-17.2265713	-33.03782158
1.0	3.0	-2.26828472	-1.69713538	-2.49248259	-9.47809033	-68.30742615
2.0	3.0	-3.86012368	-2.32959198	-2.63994448	-7.04964095	-132.9084166
3.0	3.0	-6.48914994	-3.42750815	-3.16795448	-6.54715819	-239.4327103
4.0	3.0	-10.5247114	-5.13245041	-4.08312017	-7.06670360	-402.8857622
5.0	3.0	-16.3947325	-7.62079011	-5.45893705	-8.37296740	-640.6138172
6.0	3.0	-24.5847544	-11.0965740	-7.39973433	-10.4571814	-972.2862533
7.0	3.0	-35.6376482	-15.7893962	-10.0299115	-13.4051430	-1419.890311
8.0	3.0	-50.1535156	-21.9536583	-13.4902001	-17.3513724	-2007.729264
9.0	3.0	-68.7896498	-29.8682818	-17.9362043	-22.4612258	-2762.4217
10.0	3.0	-92.2605193	-39.8365838	-23.5377770	-28.9232397	-3712.901221
11.0	3.0	-121.337759	-52.1862206	-30.4787316	-36.9455981	-4890.416298

Table 6: Numerical solution for unnormalized wave function for HgH, ZnH, CdH, Hbr and Hf at $L = 0$

n	$\Psi_{nl(HgH)}$	$\Psi_{nl(ZnH)}$	$\Psi_{nl(CdH)}$	$\Psi_{nl(Hbr)}$	$\Psi_{nl(Hf)}$
0.0	7.2445	2.32E-19	1.5033	1.7311	2464.2
1.0	14.4891	4.65E-19	3.0066	3.4623	4928.5
2.0	21.7336	6.97E-19	4.5099	5.1935	7392.8
3.0	28.9782	9.30E-19	6.0132	6.9247	9857.1
4.0	36.2227	1.16E-18	7.5166	8.6559	12321.
5.0	43.4673	1.39E-18	9.0199	10.387	14785.
6.0	50.7118	1.63E-18	10.523	12.118	17250.
7.0	57.9564	1.86E-18	12.026	13.849	19714.
8.0	65.2009	2.09E-18	13.529	15.580	22178.
9.0	72.4455	2.32E-18	15.033	17.311	24642.
10.0	79.6900	2.56E-18	16.536	19.043	27107.
$\Psi_{nl(Total)}$	478.1404	1.53E-17	99.219	114.25	16264

Table 7: Numerical solution for unnormalized wave function for HgH, ZnH, CdH, Hbr and Hf at $L = 1$

n	$\Psi_{nl(HgH)}$	$\Psi_{nl(ZnH)}$	$\Psi_{nl(CdH)}$	$\Psi_{nl(Hbr)}$	$\Psi_{nl(Hf)}$
0.0	11.78102	2.900415	1.273501	1.555785	42335.31923
1.0	23.56205	5.80083	2.547001	3.11157	84670.63845
2.0	35.34307	8.701245	3.820502	4.667355	127005.9577
3.0	47.12409	11.60166	5.094003	6.22314	169341.2769
4.0	58.90512	14.50208	6.367504	7.778925	211676.5961
5.0	70.68614	17.40249	7.641004	9.33471	254011.9154
6.0	82.46716	20.30291	8.914505	10.89049	296347.2346
7.0	94.24819	23.20332	10.18801	12.44628	338682.5538
8.0	106.0292	26.10374	11.46151	14.00206	381017.873
9.0	117.8102	29.00415	12.73501	15.55785	423353.1923
10.0	129.5913	31.90457	14.00851	17.11363	465688.5115
$\Psi_{nl(Total)}$	777.5475	191.4274	84.05105	102.6818	2794131.069

Table 8: Numerical solution for unnormalized wave function for HgH, ZnH, CdH, Hbr and Hf at $L = 2$

n	$\Psi_{nl(HgH)}$	$\Psi_{nl(ZnH)}$	$\Psi_{nl(CdH)}$	$\Psi_{nl(Hbr)}$	$\Psi_{nl(Hf)}$
0.0	19.1582	3.13082	1.078813322	1.39815741	727299.051
1.0	38.3164	6.26165	2.157626643	2.79631481	1454598.1
2.0	57.4746	9.39247	3.236439965	4.19447222	2181897.15
3.0	76.6328	12.5233	4.315253286	5.59262962	2909196.2
4.0	95.791	15.6541	5.394066608	6.99078703	3636495.25
5.0	114.949	18.7849	6.47287993	8.38894443	4363794.3
6.0	134.107	21.9158	7.551693251	9.78710184	5091093.35
7.0	153.266	25.0466	8.630506573	11.1852592	5818392.41
8.0	172.424	28.1774	9.709319894	12.5834167	6545691.46
9.0	191.582	31.3082	10.78813322	13.9815741	7272990.51
10.0	210.74	34.4391	11.86694654	15.3797315	8000289.56
$\Psi_{nl(Total)}$	1264.44	206.634	71.20167923	92.2783888	48001737.3

Table 9: Numerical solution for unnormalized wave function for HgH, ZnH, CdH, Hbr and Hf at $L = 3$

n	$\Psi_{nl(HgH)}$	$\Psi_{nl(ZnH)}$	$\Psi_{nl(CdH)}$	$\Psi_{nl(Hbr)}$	$\Psi_{nl(Hf)}$
0.0	31.15488	3.379538	0.913889	1.256500206	12494624.32
1.0	62.30975	6.759076	1.827778	2.513000411	24989248.64
2.0	93.46463	10.13861	2.741667	3.769500617	37483872.95
3.0	124.6195	13.51815	3.655556	5.026000822	49978497.27
4.0	155.7744	16.89769	4.569444	6.282501028	62473121.59
5.0	186.9293	20.27723	5.483333	7.539001234	74967745.91
6.0	218.0841	23.65676	6.397222	8.795501439	87462370.23
7.0	249.239	27.0363	7.311111	10.05200164	99956994.54
8.0	280.3939	30.41584	8.225	11.30850185	112451618.9
9.0	311.5488	33.79538	9.138889	12.56500206	124946243.2
10.0	342.7036	37.17492	10.05278	13.82150226	137440867.5
$\Psi_{nl(Total)}$	2056.222	223.0495	60.31667	82.92901357	824645205

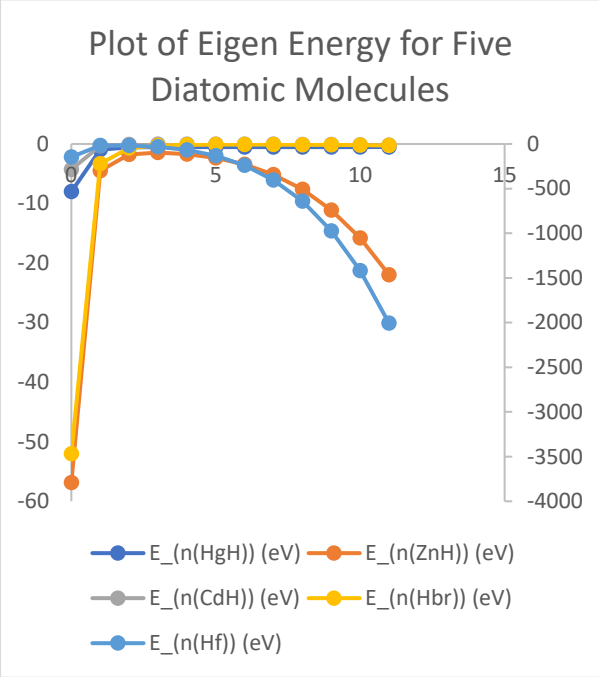

Figure 1: The Plot of Eigen Energy for Diatomic Molecules at L=0

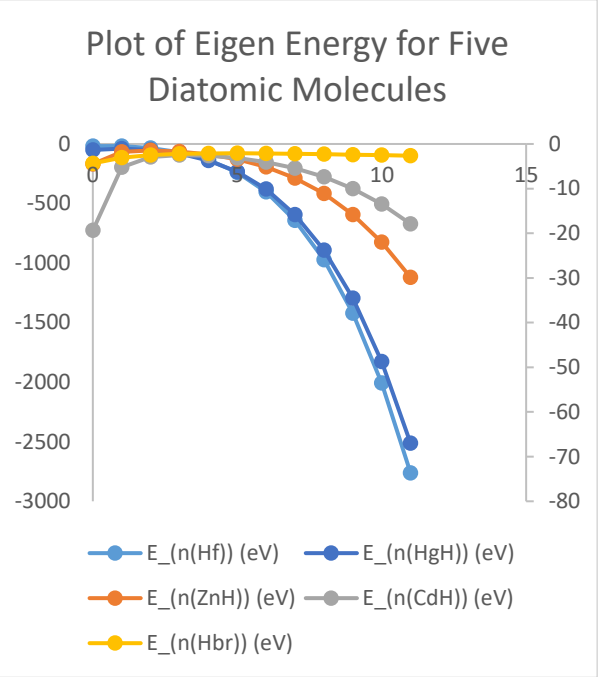

Figure 2: The Plot of Eigen Energy for Five Five Diatomic Molecules at L=1

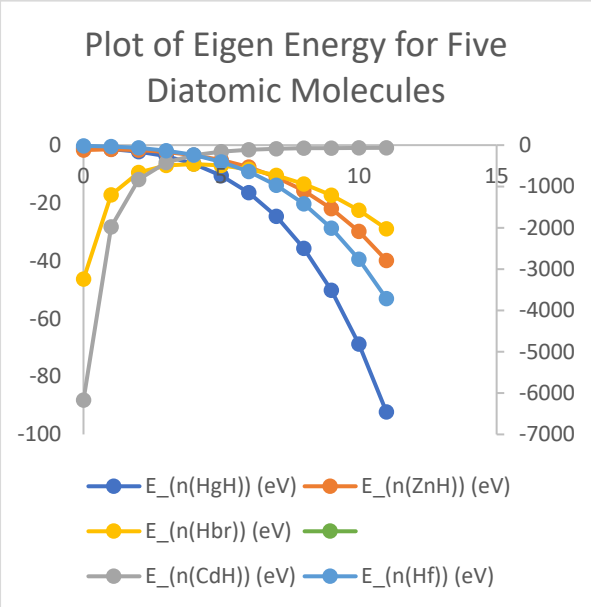

Figure 1: The Plot of Eigen Energy for Diatomic Molecules at L=2

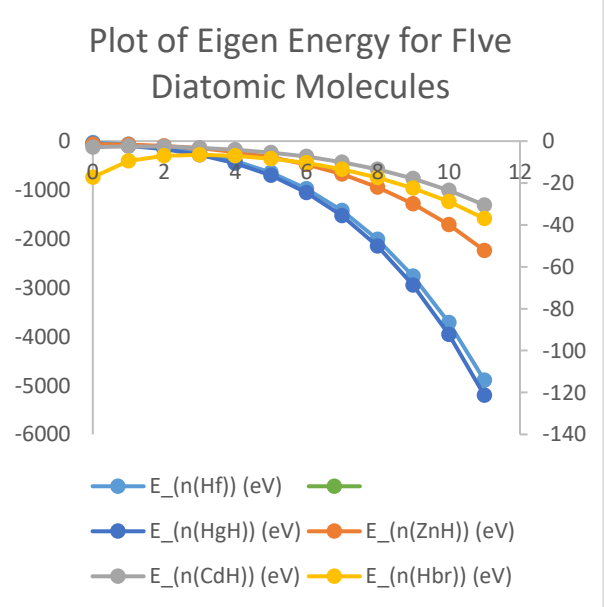

Figure 2: The Plot of Eigen Energy for Five Five Diatomic Molecules at L=3

DISCUSSION OF RESULTS

The numerical solution of bound state energy selected across five diatomic molecules on the basis of modified exponential screened plus yukawa potential (MESPYP) within the frame of parametric nikiforov-uvarov method, which are HgH, ZnH, CdH, Hbr and Hf were analyzed. The result of the five selected diatomic molecules spectroscopic constant were shown in Table 1. Reduce mass (μ) and dissociation energy (De) were calculated numerically across the five diatomic molecules. Table 2, table 3, table 4 and table 5, shows the result of the numerical solution across the five diatomic molecules at an orbital angular quantum number of $l = 0, 1, 2\ and\ 3$, with screening parameter of $0.3129, 0.2513, 0.2183, 0.2261\ and\ 0.7711$, respectively. The results show a trend between the screening parameter, orbital angular quantum number, which signify increased or decrease associated from the aforementioned.

Reduce mass (μ) and dissociation energy (De) were also calculated numerically across the five diatomic molecules.

Table (6-9) is the results of the numerical solution of the unnormalized wave function for HgH, ZnH, CdH, Hbr and Hf at an orbital angular quantum number of $l = 0, 1, 2\ and\ 3$. The total wavefunction($\boldsymbol{\Psi_{nl}}$) for HgH, ZnH, CdH, Hbr and Hf is given within the range of (1264.44 - 223.0495), (777.5475 - 1.53E-17), (99.2191 – 17.2016), (114.25 – 82.929) and 82.4645E+8 – 1.6264E+8) respectively. The estimated wave increases with an increased in the quantum number as shown in figure (1-4).

CONCLUSION

In this paper, we study an approximate bound state solutions of Schrödinger wave equation with (MESPYP) potential model using parametric Nikiforov–Uvarov method, where we obtain energy eigenequation and total unnormalized wave function expressed in terms of associated Jacobi polynomial.

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