

Effect of Understaffing and Overstaffing Constraints on Recruitment and Wastage Manpower Planning Model

¹S.A Ogumeyo* and ²K. I. Ekeh

^{1,2}Department of Mathematics,
Delta state University of Science and Technology,
Ozoro,
Nigeria.

Email: simonogumeyo64@gmail.com

Abstract

The problem of overstaffing and understaffing is a key issue facing every business organization. This paper examined the effect of overstaffing and understaffing constraints on recruitment and wastage in a manpower planning model. Apart from the usual constraints of overstaffing and understaffing associated with the objective function of manpower planning models, we have incorporated two extra constraints. The two added constraints are (a) that the number of overstaffing employees be non-negative periodically, and (b) the organization started at full capacity at the initial period which makes the model different from existing models. It is observed that after the addition of n -extra nonnegative employee's constraints in the proposed Dynamic Programming (DP) model, the number of wastage staff is equal to the number of staff recruited when the organization is operating at full capacity and that the model can be used to evaluate the inventory of the work-force periodically without waiting till the end of the total periods under consideration. Thus, our proposed model has advantage of sensitivity analysis compare to existing manpower planning models.

Keywords: Manpower, overstaffing, understaffing, recruitment

INTRODUCTION

The problem of assessing future manpower requirements in terms of number, type of skills and competence as well as formulating plans to meet those requirements has remained a challenge to researchers in human resources management. Manpower planning is defined by Bulla and Scot (1994) as a process of ensuring that the human resource requirements of an organization are identified and plans are made for satisfying those requirements. The principal objective of manpower planning is to model the migration of staff from one grade to another in discrete time which could be as a result of recruitment, promotion or retirement, (see Robbin and Harrison (2007), Mehlmann (1980) and Gregoriades (2000)).

The challenges of manpower planning according Ezendu (2009), includes lateness to work, maternity leaves, leaves of absence, how fast people can work, wastage etc. Pinder (1995) remarked that manpower demand is unpredictable due to seasonal fluctuations and random arrival of projects in organizations. Hence many managers resort to the option of periodic recruitment and retrenchment in order to satisfy manpower requirement at each period. Due to uncertainty of manpower demand, researchers over the years have resolved to use different techniques to model manpower systems. For example, a manpower planning model which

*Author for Correspondence

determines optimum workforce – size in a civil and military establishment is developed in Sterman, (2000). Hiring and retaining the right employees is one of the biggest challenges of human resource management. Common problems related to understaffing and overstaffing and how to avoid them are contained in Rubatt (2021).

Taha (2007) also developed a manpower planning model which determines when recruitment and retrenchment should be carried out in an organization in order to checkmate incident of understaffing and overstaffing. A manpower planning model which incorporates global constraints such as production capacity/demand rate and allowable time of operation to reflect the reality of activities in production organization is developed in Akinyele (2007) while Ogumeyo and Ekoko (2008) developed a manpower planning model which determines optimal recruitment policies by using a dynamic programming technique.

Hall (2009) argued that at a steady state in manpower planning model, wastage must be equal to recruitment, which could be achieved by allowing the model to allocate wastage above the first two periods. The major problem in manpower planning is how to strike a balance between having too many staff (i.e. overstaffing) and not having adequate staff (i.e. understaffing) in a business organization. Finding the balance between having too few and too many employees can be tricky. While too many employees drastically increase overhead costs, too few can limit business expansion, (Valier 2023). According to Ogumeyo (2010) and Rubatt (2021), the two extremes (i.e. overstaffing and under staffing) both have negative effects on any business organization. While overstaffing leads to economic law of diminishing returns, understaffing results in low productivity and decrease in revenue generation.

Many research works in literature on manpower planning deal with minimization of manpower cost without addressing the effects of periodic overstaffing and understaffing constraints in their mathematical formulations. For example, Ogumeyo (2010) optimum workforce – size model use dynamic programming approach which considers only overstaffing to establish a recruitment schedule for a category of workers in which understaffing was not allowed. The schedule seeks to minimize the total recruitment and overstaffing cost subject to the restriction that it meets the entire manpower requirement on time. The dynamic programming model in linear programming form for manpower recruitment by Ogumeyo and Ekoko (2011) manpower model incorporates recruitment and overstaffing costs in its formulation and disallowed understaffing. In Ogumeyo and Okogun (2023) recruitment and wastage manpower planning model, the total number of overstaffing and understaffing employees' constraints are considered without considering their periodic effects. This paper is aimed at examining periodic effects of overstaffing and understaffing in a manpower planning problem which is an extension of the manpower planning models in Rao (1990), Ogumeyo (2010), Ogumeyo and Ekoko (2011) and Ogumeyo and Okogun (2023) which consider the effects of overstaffing and understaffing constraints only at the end period. The proposed model has an advantage over existing models due to its sensitivity analysis.

MATERIALS AND METHODS

The following symbols are used in the model description and formulation:

x_j = number of staff that are on wastage in period j .

y_j = number of staff that are recruited in period j .

c_j = average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the system.

$c'_j =$ average salary per recruited staff in period j .

$h =$ initial number of staff on ground in the organization at the beginning of the time horizon.

$H =$ total number of staff at the end of the time horizon under consideration.

Variables and Parameter Description:

Let $y_j(t + \delta)$ be the number of staff recruited at time $(t + \delta)$ of period j where δ is the very small time difference between recruitment and assumption of duty so that the recruited staff arrive at time $(t + \delta)$ for work. Let $x_j(t + \delta)$ and $c_j(t + \delta)$ be the number of staff on wastage and the average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the organization. Let $c'_j(t + \delta)$ be the average salary per recruited staff at time $(t + \delta)$ of period j when the recruitment was done. As $\delta \rightarrow 0$, the above notations become $x_j(t)$, $y_j(t)$, $c_j(t)$ and $c'_j(t)$ or simply x_j , y_j , c_j and c'_j . Given h , H , $c_j(t)$ and $c'_j(t)$ of a manpower planning problem, it is required to determine the optimal quantities x_j and y_j so that the accruable net revenue is a maximum.

As we are dealing here with a dynamic situation, we divide the time span of interest into time intervals, which we shall assume to be sufficiently short so that we can consider $x_j(t)$, $y_j(t)$, $c_j(t)$ and $c'_j(t)$ to be constant during the time intervals but discontinuous from one time interval to the next.

The problem of the manpower planning is to maximize the periodic additional revenue accruable to the organization from the wastage staff wage bill less the periodic salary of recruited staff i.e. $\sum_{j=1}^n (c_j x_j - c'_j y_j)$.

Model Formulation

The following are the assumptions of the proposed model:

- (a) Recruitment and wastage at a particular grade are considered
- (b) Periodic recruitment (c'_j) and wastage (c_j) costs are known and fixed.
- (c) Number of staff of the organization at initial and end of time-horizon interval are known.
- (d) Both overstaffing and understaffing are considered.

The objective function of the proposed model can be written as:

$$\text{Maximize } z = \sum_{j=1}^n (c_j x_j - c'_j y_j) \dots\dots\dots (1)$$

There are two sets of staffing constraints and two sets of non-negativity constraints in this manpower planning problem.

(i) The overstaffing constraints:

The constraints of overstaffing state that the total number of overstaffing staff of the first i periods should not exceed the available vacancies $(H - h)$ in the establishment, i.e.

$$\sum_{j=1}^i (y_j - x_j) = - \sum_{j=1}^i x_j + \sum_{j=1}^i y_j \leq H - h, \quad i = 1(1)n \dots\dots\dots (2)$$

Where $(y_j - x_j) > 0$ is the number of staff by which the organization is overstaffed in period j . The LHS of equation (2) can also be called the net increase in manpower in the first i periods.

(ii) The understaffing constraints: The constraints of understaffing represent the number of staff by which the organization is understaffed for the first $(i - 1)$ periods plus wastages at period i and this should not exceed h the number of staff originally in the organization. If it does, it means the organization has only material resources which is not the case in practical situation as existence of an organization is based on the contribution of human and material resources. Mathematically this is expressed as:

$$\sum_{j=1}^{i-1} (x_j - y_j) + x_i = \sum_{j=1}^i x_j - \sum_{j=1}^{i-1} y_j \leq h, \quad i = 1(1)n \dots\dots\dots(3)$$

Where $(x_j - y_j) > 0$ is the number of staff by which the organization is understaffed in period j . The L.H.S of equations (3) can also be called the net increase in manpower subtracted from wastage staff in the first $(i - 1)$ periods plus the wastage manpower in period i . Note that the second summation in equation (3) does not exist for $i = 1$.

(iii) Non-negativity constraints: The non-negativity constraints are

$$x_j, y_j \geq 0, \quad j = 1(1)n \dots\dots\dots(4)$$

Equation (3.4) stated above constitutes the total manpower planning cost from all the n periods while equations (1)-(4) constitute a DP problem which is stated thus:

Primal LP Problem

$$\left. \begin{array}{l} \text{Maximize } z = \sum_{j=1}^n (c_j x_j - c'_j y_j) \\ \text{s.t.} \\ - \sum_{j=1}^i x_j + \sum_{j=1}^i y_j \leq H - h, \quad i = 1(1)n \\ \text{and } \sum_{j=1}^i x_j - \sum_{j=1}^{i-1} y_j \leq h, \quad i = 1(1)n \\ x_j, y_j \geq 0, \quad j = 1(1)n \end{array} \right\} \dots\dots\dots(5)$$

The system (5) is the DP model of the manpower planning problem which makes use of both recruitment and wastage factors. The DP model in system (5) has $2n$ linear constraints, $2n$ non-negativity constraints in $2n$ variables. Further simplification of (5) yields the system in (6).

$$\left. \begin{array}{l} \text{Max } z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n - c'_1 y_1 - c'_2 y_2 - \dots - c'_n y_n \\ \text{s.t.} \\ -x_1 \qquad \qquad \qquad + y_1 \qquad \qquad \qquad \leq H - h \\ -x_1 - x_2 \qquad \qquad + y_1 + y_2 \qquad \qquad \leq H - h \\ -x_1 - x_2 - x_3 \qquad + y_1 + y_2 + y_3 \qquad \leq H - h \\ \dots\dots\dots \\ -x_1 - x_2 - x_3 - \dots - x_n + y_1 + y_2 + y_3 + \dots + y_n \leq H - h \\ x_1 \qquad \qquad \qquad \leq h \\ x_1 + x_2 \qquad \qquad - y_1 \qquad \qquad \leq h \\ x_1 + x_2 + x_3 \qquad - y_1 - y_2 \qquad \leq h \\ \dots\dots\dots \\ x_1 + x_2 + x_3 + \dots + x_n - y_1 - y_2 - y_3 - \dots - y_{n-1} \leq h \\ x_j, y_j \geq 0, \quad j = 1(1)n \end{array} \right\} \dots\dots\dots(6)$$

The matrix skeleton of the system (6) is shown in Fig.1

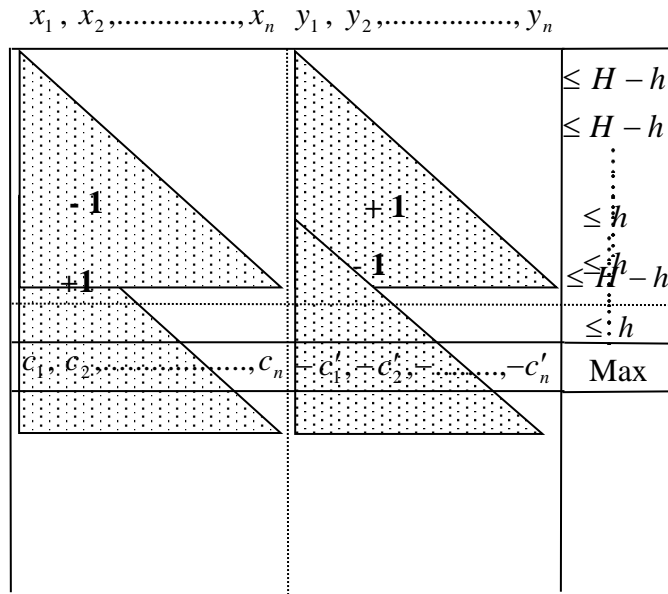


Fig. 1: Matrix skeleton of the primal LP model coefficient arrays.

The matrix array consists of triangular blocks, which are typical of dynamic situations. The coefficients within the blocks are either +1 or -1 depending on the block. The triangular block in the lower right-hand corner is smaller by one row and one column than the other three. The DP model of this section for manpower planning based on recruitment and wastage factors is sparse and consequently has the advantage of less computational time when using the computer. Let d_1, d_2, \dots, d_n be the first n dual variables for the first n constraints in system (6) and e_1, e_2, \dots, e_n be the last n dual variables for dual DP model of the manpower planning problem:

Dual DP Problem

$$\text{Minimize } w = (H - h) \sum_{i=1}^n d_i + h \sum_{i=1}^n e_i \dots\dots\dots(7)$$

s.t.

$$-\sum_{i=k}^n d_i + \sum_{i=k}^n e_i \geq c_k, \quad k = 1(1)n \dots\dots\dots(8)$$

$$\sum_{i=k}^n d_i - \sum_{i=k+1}^n e_i \geq -c'_k, \quad k = 1(1)n \dots\dots\dots(9)$$

$$d_i, e_i \geq 0, \quad i = 1(1)n \dots\dots\dots(10)$$

It is understood that the second summation in equation (3.9) does not exist if $k = n$. The corresponding matrix skeleton of the dual DP problem in equations (7)–(10) is shown in Fig.



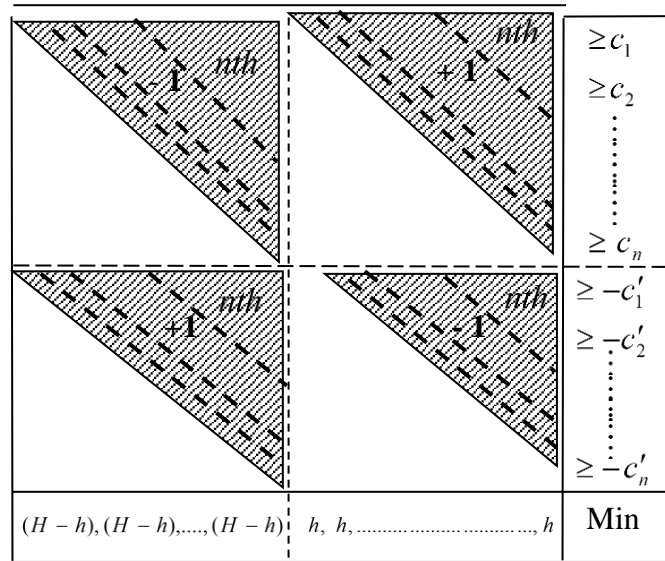


Fig. 2: Matrix skeleton of the dual DP model coefficient arrays.

We define new variables D_k and E_k as follows:

$$D_k = \sum_{i=k}^n d_i, \quad k = 1(1)n \dots\dots\dots(11)$$

$$E_k = \sum_{i=k}^n e_i, \quad k = 1(1)n \dots\dots\dots(12)$$

Since by the dual DP problem, d_i and e_i are nonnegative, D_k and E_k must be nonnegative. However, non-negativity of D_k and E_k does not imply that $d_i \geq 0$ and $e_i \geq 0, \forall i$. In view of the definition of D_k and E_k , we see that non-negativity of d_i and e_i will be ensured if we augment the dual LP problem, expressed in terms of D_k and E_k by the constraints:

$$D_k \geq D_{k+1}, \quad k = 1(1) n-1 \dots\dots\dots(13)$$

$$E_k \geq E_{k+1}, \quad k = 1(1) n-1 \dots\dots\dots(14)$$

Note that the constraints in equations (13) and (14) do not exist when $k = n$ because $D_{n+1} = E_{n+1} = 0$. Hence, we have $2(n-1)$ additional constraints in equations (13) and (14). In general caution must be exercised, whenever a change of variable is made in a linear programming problem. We must make sure that the original variables will turn out to be nonnegative in the optimal solution, that this may occasionally require the addition of constraints to the original system. In this case, the $2(n-1)$ additional constraints involved in equations (13) and (14) are the price we have to pay for changing variables according to equations (11) and (12).

Imposition of Additional Constraints on the Manpower Planning Model

The two requirements to be added to the proposed manpower planning model in (6) are (1) that the number of overstaffing employees be nonnegative periodically, i.e. $(x_j - y_j) \geq 0, j = 1(1)n$ (2) initially the organization is working at full capacity (i.e. $h = H$). We can augment the DP problem in system (6) with these additional constraints as follows.

$$\begin{array}{l}
 \text{Max } z = c_1x_1 + c_2x_2 + \dots + c_nx_n - c'_1y_1 - c'_2y_2 - \dots - c'_ny_n \quad (a) \\
 \text{s.t.} \\
 -x_1 \qquad \qquad \qquad + y_1 \qquad \qquad \qquad \leq 0 \quad (b_1) \\
 -x_1 - x_2 \qquad \qquad \qquad + y_1 + y_2 \qquad \qquad \leq 0 \quad (b_2) \\
 -x_1 - x_2 - x_3 \qquad \qquad \qquad + y_1 + y_2 + y_3 \qquad \qquad \leq 0 \quad (b_3) \\
 \dots\dots\dots \\
 -x_1 - x_2 - x_3 - \dots - x_n + y_1 + y_2 + y_3 + \dots + y_n \leq 0 \quad (b_n) \\
 x_1 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \leq H \quad (c_1) \\
 x_1 + x_2 \qquad \qquad \qquad - y_1 \qquad \qquad \qquad \leq H \quad (c_2) \\
 x_1 + x_2 + x_3 \qquad \qquad \qquad - y_1 - y_2 \qquad \qquad \leq H \quad (c_3) \\
 \dots\dots\dots \\
 x_1 + x_2 + x_3 + \dots + x_n - y_1 - y_2 - y_3 - \dots - y_{n-1} \leq H \quad (c_n) \\
 -x_1 \qquad \qquad \qquad + y_1 \qquad \qquad \qquad \geq 0 \quad (d_1) \\
 \quad -x_2 \qquad \qquad \qquad + y_2 \qquad \qquad \qquad \geq 0 \quad (d_2) \\
 \quad \quad -x_3 \qquad \qquad \qquad + y_3 \qquad \qquad \qquad \geq 0 \quad (d_3) \\
 \dots\dots\dots \\
 \qquad \qquad \qquad -x_n \qquad \qquad \qquad + y_n \qquad \geq 0 \quad (d_n) \\
 x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n \geq 0 \qquad \qquad \qquad (e)
 \end{array} \quad (15)$$

In system (15), equation (a) is called the objective function, which is the measure of effectiveness. Equation 15(b₁) - (c_n) constitute the set of linear constraints while equation (e) is the set of non-negativity constraints. Specifically equations 15(d₁)-(d_n) constitute the n periodic nonnegative excess recruitment. The manpower problem which is expressed in (a) to (e) is compared to the primal DP in (6) as follows: The objective functions and non-negativity constraints are the same in both DP problems. The organization started with full capacity (H) in the latter DP problem. By the addition of n extra nonnegative excess employee's constraints to the latter problem, the latter LP problem has a total of 3n linear constraints (as against 2n in the former) in 2n variables. Hence we state and prove a theorem which is based on the latter DP problem (a)-(e) in system (15).

Theorem 1: Given that the manpower system is initially at full capacity (*i.e.* $h = H$) and that the periodic excess employees is nonnegative ($y_j - x_j \geq 0, j = 1(1)n$) then

- (i) $x_j = y_j$
- (ii) $0 \leq x_j \leq H$ and $0 \leq y_j \leq H$
- (iii) The DP problem in (a)-(e) can be reduced to have only n-variables, $x_j, (j = 1(1)n)$ which are the number of staff on wastage in period j .

Proof: The proof of the theorem is as follows:
From equations (3.15b₁) and (3.15d₁) we have

$$-x_1 + y_1 = 0 \dots\dots\dots(16)$$

Substituting equation (16) into equation (15b₂) and considering equation (15d₂), we have

$$-x_2 + y_2 = 0 \dots\dots\dots(17)$$

i.e. $x_2 = y_2$

Similarly,

$$-x_n + y_n = 0 \dots\dots\dots(18)$$

i.e. $x_n = y_n$

From equation (15c₁)

$$x_1 \leq H \dots\dots\dots(19)$$

By substituting equation (15) into equation (15c₂), we have

$$x_2 \leq H \dots\dots\dots(20)$$

Similarly, substituting (16) and (17) into (15c₃), we have

$$x_3 \leq H \dots\dots\dots(21)$$

Similarly by substituting equations (16) – (18) in equation (15c_n), we have

$$x_n \leq H \dots\dots\dots(22)$$

Since $x_j = y_j$ for $j = 1, 2, \dots, n$, the objective function can be expressed only in terms of the x_j variables and the LP problem in (15a)-(15e) is now reduced to:

$$\left. \begin{array}{l} \text{Max } z = (c_1 - c'_1)x_1 + (c_2 - c'_2)x_2 + \dots + (c_n - c'_n)x_n \\ \text{s.t.} \\ \quad x_1 \leq H \\ \quad x_2 \leq H \\ \quad \vdots \\ \quad x_n \leq H \\ \quad x_1, x_2, x_3, \dots, x_n \geq 0 \end{array} \right\} \dots\dots\dots(23)$$

This completes the required proof.

DISCUSSION

Given that the manpower system is initially at full capacity (*i.e.* $h = H$) and that the periodic excess employees is nonnegative ($y_j - x_j \geq 0, j = 1(1)n$) then the number of recruited staff and the number of staff on wastage will be equal (*i.e.* $x_j = y_j$) as shown in equation (17) and the DP problem in 15(a)-(e) can be reduced to have only n-variables, $x_j, (j = 1(1)n)$ which are the number of staff on wastage in period j as shown in the proof of theorem 1. The addition of the two extra constraints in the model enables the human resource managers to evaluate the inventory of the work-force periodically unlike the existing models in Rao (1990), Ogumeyo and Ekoko (2008) and (2011) and Ogumeyo and Okogun (2023) which involve waiting till the end of the total periods under consideration. Thus, our proposed model has advantage of sensitivity analysis compare to these existing manpower planning models.

CONCLUSION

In this paper a two -factor DP manpower planning model which consist of recruitment and wastage has been discussed. Apart from the usual overstaffing and understaffing constraints associated with dynamic programming model for manpower planning, additional constraints have been imposed on the objective function which makes it different from existing models. The effects of additional constraints have been discussed in this paper with a theorem stated and proved.

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