

Determining The Role of Thermal Radiation on Hydromagnetic Flow in A Vertical Porous Superhydrophobic Microchannel

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Abstract

The focus of this article is to scrutinize the performances of thermal radiation and superhydrophobicity on a free convection of an electrically conducting fluid across an upstanding microchannel affected by a transverse magnetic field. The plates were alternatively heated and incorporated with suction/injection effect. The approximate expressions of the formulated differential equations have been calculated. It is interesting to report that the heat gradient and fluid motion are significantly propelled for mounting values of thermal radiation parameter in both case I: super-hydrophobic surface (SHS) is heated and case II: no slip surface (NSS) is heated. However, this effect brought down the fluid temperature at the NSS, which is attributable to the direction of the heat flow. On the other hand, the velocity deteriorates for mounting values of magnetic field factor by the Lorentz force attracting the molecules, consequently leading to the fluid deterioration.

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Introduction

Suction/injection exists when a plane channel surface has pores that permit fluid passage. Works like Mishra *et al.* (2002), Chaudhary and Jain (2007) and Khalid *et al.* (2015) demonstrate the concept. In boundary layer regulation, especially in engineering, aerodynamics, space research, and medicine, suction/injection factors are very essential. These advantages instigated our interest to dig deeper into the impact of wall porosity in various types of industrial working fluids. With these concerns in mind, Jha *et al.* (2022) used the Laplace transform approach to study the impact of wall porosity on steady natural convection in an upstanding channel driven by point or line heat generation or absorption effects. A numerical investigation on free convection in an upstanding porous plates generated by bi-thermal sources was presented by Saeid (2006). When there is a suction effect, raising the top element's temperature improves heat transfer to the lower element. Uwanta *et al.* (2014) demonstrated the effect of suction and injection on hydro-magnetic free convection on an upstanding plate in a different experiment. It has been noted that fluid flow in the channel can be handled via suction or injection. Jha *et al.* (2018) also looked at the influence of a time-dependent hydro magnetic flow due to wall porosity effect. Suction may be utilized to evacuate reacting substances during chemical reactions, while injection can be used to introduce reactants. The implication of suction and injection on a time-dependent oscillatory and hydro-magnetic flow was investigated by Falade *et al.* (2017). It was revealed that injection on the hot plate increases frictional force at the channel walls. Jha *et al.* (2019) also investigated the consequences of suction and injection on magnetohydrodynamics natural convection flow. It was found that elevating velocity also reduced injection parameters. As shear stress increases, so do the suction characteristics.

In transverse channel flows of incompressible liquids, the Lorentz force, rather than pump pressure, is used to accelerate the channel liquid more deeply or faster. An MHD micro-pump is a mechanism that integrates this sort of magneto-hydrodynamic (MHD) drive (Jha and Gwandu 2020). The study of MHD has aroused much interest in recent decades owing to its usefulness in several MHD applications, such as MHD generators, cooling baths with cooling metallic plates, electric transformers, and MHD injectors. Chemical energy technology, which includes the utilization of MHD pumps to transfer electrically conductive fluids, is currently employed in some nuclear power plants. Apart from these purposes, when the fluid is electrically conductive, an applied magnetic field can drastically boost natural convection movement (Jha *et al.* 2015). With these concerns in mind, Ojmeri *et al.* (2023) recently investigated the hydro magnetic flow of an electrically conductive Casson fluid driven by radiation factor in an upstanding porous channel. Hamza *et al.* (2022) emphasized on the impact of MHD flow of a chemically reacting fluid that is convectively heated in a vertical channel imagined with a porous medium. While the unsteady state situation was investigated using a numerical scheme, the steady-state component was obtained using the homotopy perturbation approach. The actions of thermal diffusion, thermal dissipation, and chemical reaction on steady two-dimensional MHD free convection flow over an inclined plate in the coexistence of porosity and suction/injection effects were resolved by Taid and Ahmed (2022) using the perturbation series scheme. Siva *et al.* (2021) outlined a detailed reaction to the hydro magnetic influence on a heat transfer analysis of electroosmotic flow in a rotating microfluidic channel. Choudhary (2012) investigated heat and mass transfer problem for viscoelastic hydro-magnetic boundary layer flow toward an upstanding flat plate. The scrutinization of an inclined magnetic field controlled by time-dependent free convection of a dusty reactive

fluid restricted to two immeasurable flat plates immersed with porosity effect were carried out by Sandeep and Sugunamma (2013).

Microfluidic research is significant in the micro-electro-mechanical systems (MEMS) industry since it reflects the constantly growing requirement for size minimization and the functional examination of equipment for best results. Thakre *et al.* (2008) and Tamilselvan *et al.* (2018) conducted studies that shed more light on this. Scientists, technicians, and engineers are eager to see how the unique combination of hydro-magnetic flows in a super-hydrophobic (SHO) micro-channel performs. SHO layers have a tendency to minimize drag in a flow due to the substantial slip that occurs at liquid/solid interfaces, thus making them an extremely helpful parameter for determining the degree to which drag reduction is based on the slip size in industries such as oil and gas, semiconductor fabrication facilities, and companies that put together miniature machinery. Recent studies that looked at the effect of fluid flows on micro-geometry in various physical situations have been published. To this end, Ojmeri and Onwubuya (2023) recently explored the dynamics of viscous dissipative MHD fluid in the coexistence of porosity effect and mixed convection through a heated super-hydrophobic microchannel. Hamza *et al.* (2023) investigated the consequence of Arrhenius-controlled heat transfer enhancement due to an induced magnetic field in a microchannel. Wang and Ng (2014) echoed the role of isothermal and iso-flux heating conditions on free convection in a slit micro-channel. Jha and Gwandu (2017) extended Wang and Ng's (2014) work by conducting a theoretical study of hydro-magnetic free convection in an upward slit micro-channel having super-hydrophobic slip and temperature jump. Later, Jha and Gwandu (2019) used non-linear Boussinesq approximation methods to examine natural convection of an incompressible fluid in an upstanding micro-channel coated with super-hydrophobic surface. According to the computational results, increasing the temperature jump coefficient lowers the energy when the super-hydrophobic surface is heated and an increase in temperature when the no-slip surface is heated. In another paper, Jha and Gwandu (2020) offered another study of natural convection airflow incorporated with suction/injection effect porous across a super-hydrophobic microchannel that is heated separately. Ojmeri and Hamza (2022) proposed an analysis of a chemically reactive fluid contemplated with heat generation/absorption effect fluid in a microchannel employing the homotopy perturbation procedure. Ramanuja *et al.* (2020) evaluated natural convection in an isothermally heated microchannel with one surface having super-hydrophobic slip and a temperature jump. Hatte and Pitchumani (2021) extensively and carefully described the action of heat transfer flow within a cylinder having no wetting surfaces using a fractional rough surface characterization. The method investigates the fluid interaction's dynamic stability in the asperities of air-infused super-hydrophobic surfaces. It was revealed from their results that, against what most people think, super-hydrophobicity, which is indicated by the highest contact angles, does not always lead to peak convective heat transfer coefficient and that, under some fluid flow situations, hydrophobic surfaces can give high thermal operation.

The significance of thermal radiation and wall porosity across a heated vertical micro-channel with one wall having super-hydrophobic surface (SHS) condition have not been examined in any of the above-mentioned literature, which instigated the interest for this paper. Thus, prompted by the above knowledge gap, the ultimate aim of this article is to extend the work of Jha and Gwandu (2020) by researching the performance of thermal radiation on steady MHD free convection over a heated vertical super-hydrophobic microchannel affected by wall porosity, temperature jump and unequal wall heating. The results of this work can give a broader usefulness into micro-devices made with micro-fabrication techniques, micro- and bio-fluidics, solar power technology, gas turbines, nuclear power plant, micro-electro-

mechanical systems (MEMS) and in biomedical sciences, etc. Incorporating SHS as one of the plate surface is to minimize the retention of moisture on the inward surface of the channel. Since the channel is not vacuum, the presence of air is certain and this, together with moisture forms rust on metal surfaces. In micro-electronic devices and other similar equipment, rusted surfaces impede the conduction of electricity of those devices and this promote the malfunctioning of the system. This present study can help to eradicate that challenge.

Mathematical structure of the flow configuration

Envision an electrically conducting fluid traveling slowly upward along a vertical parallel plate microchannel by convection, each plate being heated separately. As a consequence of a particular micro-engineering treatment, one of the surfaces is exceedingly difficult to wet (super-hydrophobic). The opposite side (the no-slip surface) had not been altered. The super-hydrophobic surface is kept at a position $y_0 = 0$ while the no-slip surface is at $y_0 = L$ as shown by figure 1. Different temperature jump and slip conditions were used for the different plates because the main interest is in the super-hydrophobicity of a surface, not the characteristics of the flow. A magnetic field of uniform intensity $(0, B_0, 0)$ is assumed to be introduced transversely to the direction of flow. The component u is the fluid vertical velocity and S is the suction/injection velocity. Heat flow along the channel was ignored. Following Jha and Gwandu (2020), under the Boussinesq buoyancy approximation and assuming that the fluid is viscous, electrically conducting, and in the coexistence of thermal radiation and suction/injection, the controlling equations for the current problem in dimensional form can be as follows:

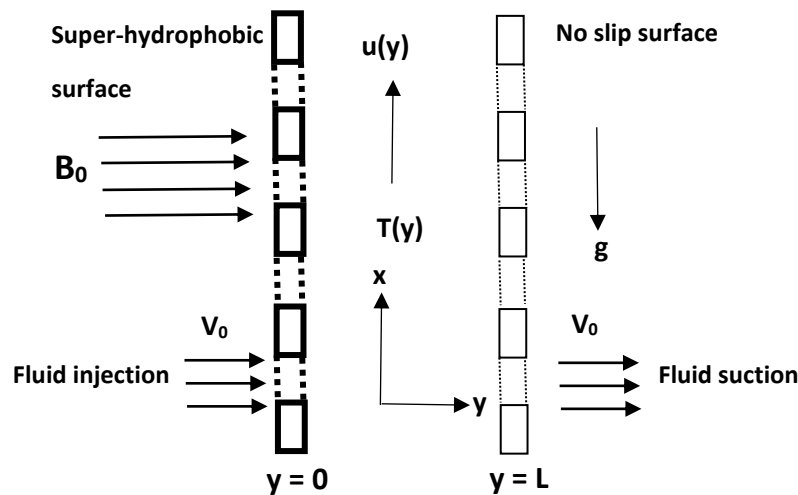


Figure 1: Physical configuration of the flow domain

$$V_0 \frac{du'}{dy'} = \nu \frac{d^2u'}{dy'^2} + g\beta(T' - T_0) - \frac{\sigma B_0^2 u'}{\rho} \quad (1)$$

$$V_0 \frac{dT'}{dy'} = \frac{k}{\rho c_p} \frac{d^2T'}{dy'^2} \quad (2)$$

The appropriate boundary conditions of the model are:

$$\left. \begin{aligned} u(y') &= \lambda' \frac{du'}{dy'} \\ T(y') &= T_h + \gamma' \frac{dT'}{dy'} \end{aligned} \right\} \quad \text{at} \quad y' = 0$$

$$\left. \begin{aligned} u(y') &= 0 \\ T(y') &= T_0 \end{aligned} \right\} \quad \text{at} \quad y' = L \quad (3)$$

Applying the Rosseland approximation, the radiative heat flux qr is indicated by

$$qr = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (4)$$

Where σ^* and k^* are the Stefan Boltzmann constant and Rosseland mean absorption parameters, respectively. Consider that the temperature difference inside the flow is sufficiently negligible so that T^4 may be considered as a linear function of temperature in a Taylor series about T_0 and ignoring higher terms one can get

$$T^4 \cong 4T_0^3T - 3T_0^4 \tag{5}$$

Using equations (4) and (5) equation (3) becomes

$$\frac{k}{\rho c_p} \left(1 + \frac{16\sigma^* T^3}{3kk^*} \right) \frac{\partial^2 T}{\partial y^2} - S \frac{\partial T}{\partial y} = 0 \tag{6}$$

Where $R = 16\sigma^* T^3 / 3kk^*$ is taken as the radiation parameter, so the above equation becomes

$$\frac{k}{\rho c_p} (1 + R) \frac{\partial^2 T}{\partial y^2} - S \frac{\partial T}{\partial y} = 0 \tag{7}$$

And where the dimensionless parameters used are:

$$y = \frac{y'}{b}, u = \frac{u'}{U_0}, \theta = \frac{T - T_0}{T_1 - T_0}, x = \frac{x'v}{Uh^2}, (Y, \gamma, \Gamma) = \frac{Y', \gamma', \Gamma'}{h}, M^2 = \frac{\sigma\beta_0^2 h^2}{\rho\nu}, Pr = \frac{\rho c_p \nu}{k} \tag{8}$$

Where $U_0 = \rho g \beta (T - T_0) b^2 / \mu$ is dimensionless velocity.

Incorporating equation (8) in equation (1), (2) and (3), the governing equations now becomes, following Abbas *et al.* (2020):

$$\frac{d^2 u}{dy^2} - S \frac{du}{dy} - M^2 u + \theta = 0 \tag{5}$$

$$(1 + R) \frac{d^2 \theta}{dy^2} - S Pr \frac{d\theta}{dy} = 0 \tag{6}$$

The boundary conditions in dimensionless forms are:

$$\left. \begin{aligned} \theta(0) &= 1 + \gamma \frac{d\theta}{dy}, \quad u(0) = \lambda \frac{du}{dy} \\ \theta(1) &= 0, \quad u(1) = 0 \end{aligned} \right\} \tag{7}$$

Where R is the thermal radiation, M is the magnetic field intensity, S is the suction/injection parameter, γ is the temperature jump coefficient and λ is the velocity slip condition.

Solution of the problem

The governing equations are system of ordinary differential equations with constant coefficients. The system of linear ordinary differential equations has been determined in closed form by the theory of simultaneous ordinary differential equations. The steady state temperature and velocity has been derived as follows:

$$\theta(y) = W_1 + W_2 e^{-m_2 y} \tag{8}$$

$$U(y) = Z_1 e^{m_3 y} + Z_2 e^{-m_4 y} + Z_3 + Z_4 e^{-m_2 y} \tag{9}$$

The rate of heat transfer rate and the drag force coefficients are computed as follows:

$$Nu_0 = \frac{d\theta}{dy} = -m_2 W_2 \tag{10}$$

$$Nu_{u_1} = \frac{d\theta}{dy} = -m_2 W_2 e^{-m_2} \tag{11}$$

$$\tau_0 = \frac{du}{dy} = m_3 Z_1 - m_4 Z_2 - m_2 Z_4 \tag{12}$$

$$\tau_1 = \frac{du}{dy} = m_3 Z_1 e^{m_3} - m_4 Z_2 e^{-m_4} + m_2 Z_4 e^{-m_2} \tag{13}$$

Where the constants $W_1, W_2, Z_1, Z_2, Z_3,$ and Z_4 are indicated in the appendix.

Results and discussion

The current article is devoted to the analysis of thermal radiation effects on steady hydro-magnetic flow of an electrically conducting fluid, moving vertically through an isothermally

heated porous micro-channel, with one surface coated with super-hydrophobic slip condition and temperature jump, while the other has no slip. Analytical expressions for energy, momentum, heat transfer coefficient and skin friction have been derived. Various graphs were sketched to demonstrate and explain the actions of the main regulating parameters on the fluid flow and thermal profile for case I (SHS) heated and case II (NSS) heated.

The fluctuation of thermal radiation parameter on the dimensionless temperature distribution for fixed value of ($\lambda = \gamma = 1, S=M=0.5$) is displayed in figure 2. It clearly reveals that, from these figures, the temperature of the fluid is scaled up with an increase in the thermal radiation in case I while it reduces it in case II. This was due to the trapped warm air packets at the SHS and the direction of the heat flow.

With the help of figure 3, the action of thermal radiation on the velocity gradient is described. It can be seen that thermal radiation effect favors the fluid flow in the both cases.

Figure 4 indicate the impact of MHD on the velocity distribution. The graph shows a reduction in the upward velocity (especially the peak velocity) with increase in magnetic number (when both λ and γ are each equal to 1). In other words, it was evident that the magnetic parameter impedes the fluid motion by attracting the molecules, consequently, leading to its deterioration.

The functions of thermal radiation on the rate of heat transfer are plotted in figures 5a and b. In figure 5a, the heat transfer rate is observed to demonstrate a decaying tendency at the SHS surface while a reverse case is noticed at the no slip surface ($y=1$) as shown in figure 5b.

Figure 6 plots the implication of thermal radiation on the skin friction. It was plain that rising values of thermal radiation parameter is seen to encourage the sheer stress at SHS, whereas a counter attribute occurs at the NSS.

The consequence of fluctuating magnetic number on the skin friction is portrayed in figure 7. It is evident that the action of MHD is to strengthen the sheer stress at the SHS ($y = 0$), while the opposite phenomenon happens at the NSS ($y=1$).

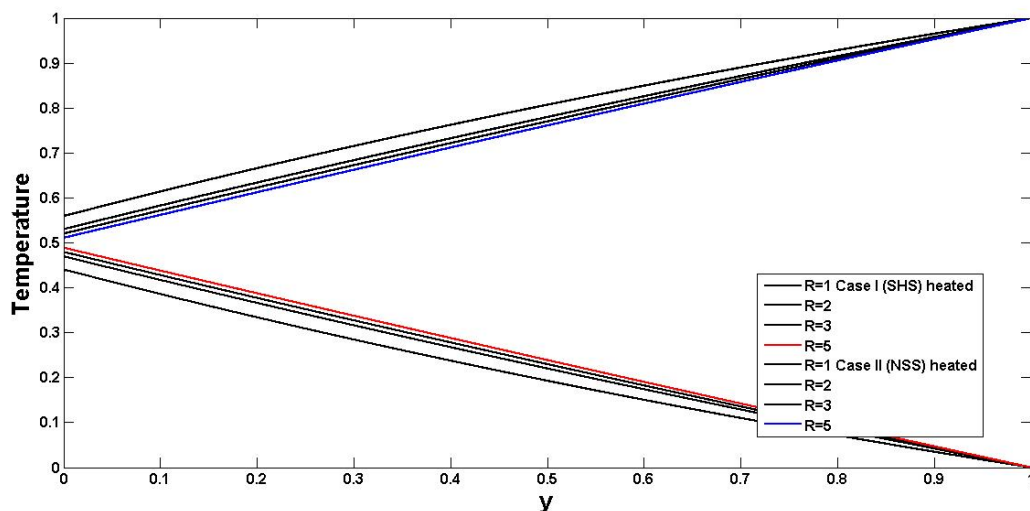


Figure 2 Impact of thermal radiation on temperature gradient

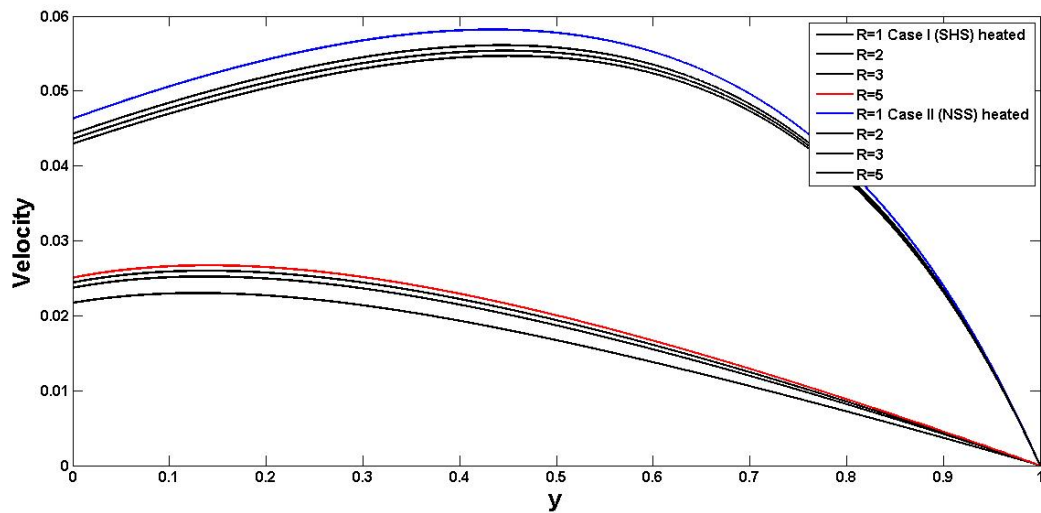


Figure 3 Impact of thermal radiation on velocity gradient

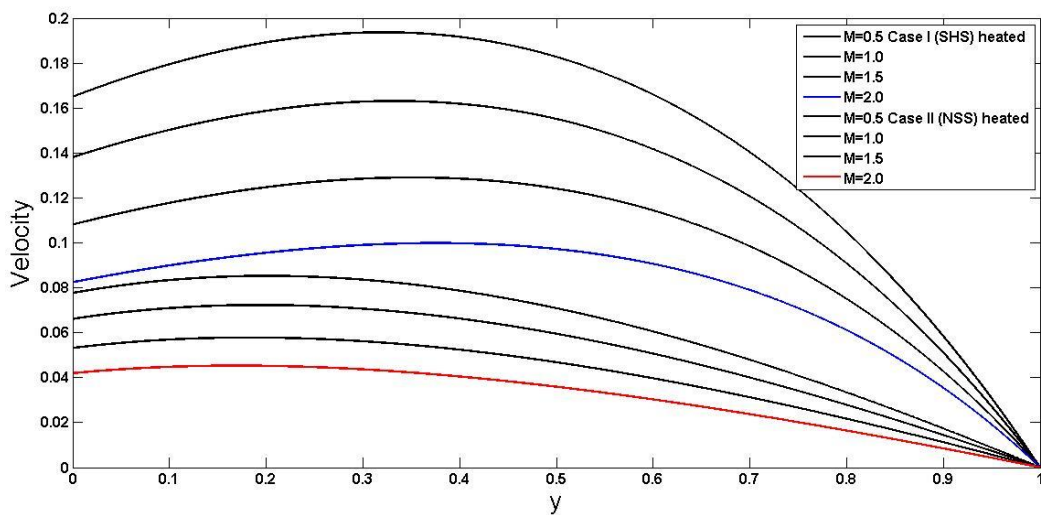


Figure 4 Impact of magnetic field on velocity gradient

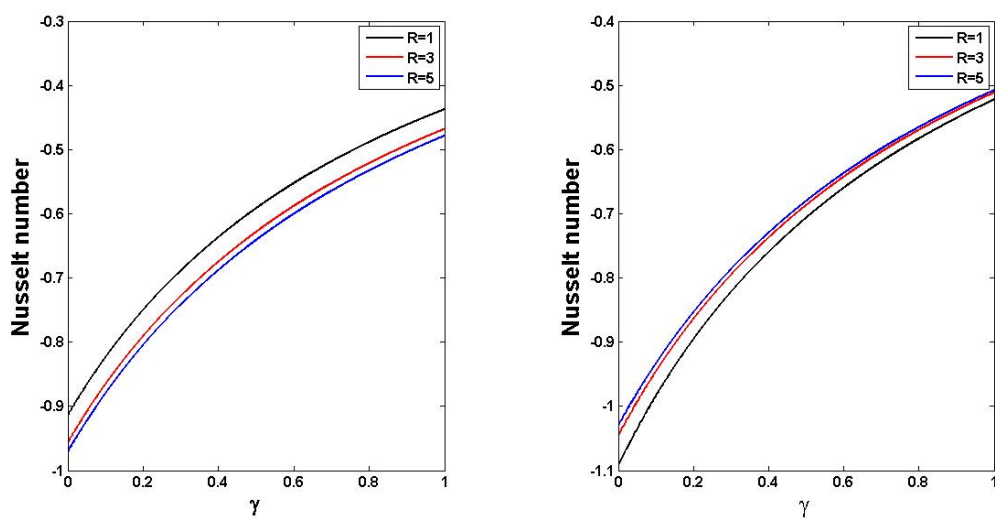


Figure 5 Impact of thermal radiation on Nusselt number

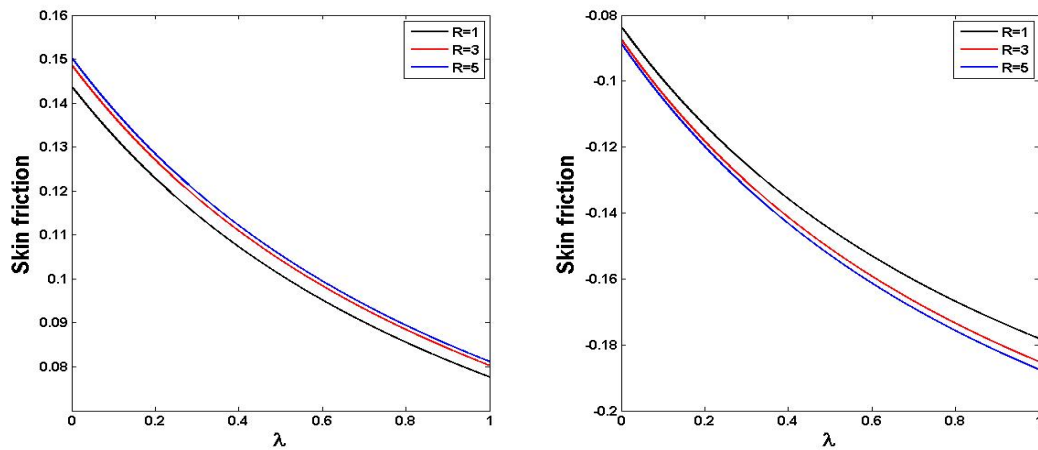


Figure 6 Impact of thermal radiation on Skin friction

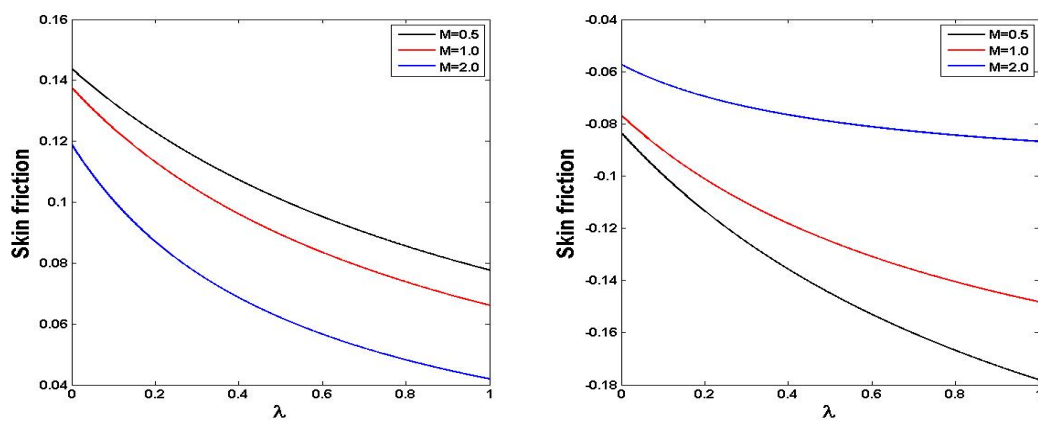


Figure 7 Impact of magnetic field on Skin friction

Validation of results

Table 1 Showcases the numerical comparison between the work of Jha and Gwandu (2020) and the present study for the both cases for constant values of $\lambda = \gamma = 1$, $S=M=0.5$ and setting R to be equal to 0. The comparison establishes an excellent agreement.

Y	Jha and Gwandu (2020)		Present study		
	Case I	θ	U	θ	U
0.1		0.4037	0.0779	0.4037	0.0779
0.2		0.3522	0.0795	0.3522	0.0795
0.3		0.3025	0.0778	0.3025	0.0778
0.4		0.2546	0.0732	0.2546	0.0732
0.5		0.2083	0.0658	0.2083	0.0658
	Case II				
0.1		0.5963	0.1849	0.5963	0.1849
0.2		0.6478	0.1949	0.6478	0.1949
0.3		0.6975	0.1992	0.6975	0.1992
0.4		0.7454	0.1971	0.7454	0.1971
0.5		0.7917	0.1877	0.7917	0.1877

Conclusion

This paper inspected the joint influences of thermal radiation and wall porosity in steady fully developed hydro magnetic flow of an electrically conducting fluid, moving vertically through an isothermally heated parallel plates micro-channel with one plate exhibiting superhydrophobic slip and temperature jump, while the other did not. Since the governing equations are system of ordinary differential equations with constant coefficients, a closed form solution has been obtained by the theory of simultaneous ordinary differential equations. The impacts of controlling parameters such as thermal radiation, suction/injection and magnetic field intensity on the fluid flow, temperature, drag force and heat transfer amount were presented and discussed fully with the help of illustrated plots.

The summary of the outcomes from this research is highlighted below:

- (i) The fluid temperature is noticeably propelled for increasing values of thermal radiation in case I while the reverse happens in case II. This is attributable to the direction of the heat flow.
- (ii) Thermal radiation effect is to promote the fluid velocity in the both cases
- (iii) The velocity of the fluid is lowered with an increase in the magnetic field number while the skin friction depicts a diminishing tendency when the SHS is heated, a contrast behavior was recorded when the NSS is heated.
- (iv) The rate of heat transfer is minimized at the SHS in favor of thermal radiation parameter, while a contrary behavior is recorded at the NSS. In contrast, the respective counter attribute is portrayed in the shear stress.

References

- Abbas Z., Naveed M., Hussain M. and Nadeem S, (2020), Analysis of entropy generation for MHD flow of viscous fluid embedded in a vertical porous channel with thermal radiation, *Alexandria Eng. J.*, vol **59**, pp 3395-3
- Chaudhary R. C. and Jain A. (2007). Combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium. *Rom J Phys.* 52: 505-524.
- Chaudhary, D. (2012). Heat and Mass Transfer for visco-elastic MHD Boundary Layer Flow Past a Vertical Plate, *Theo. Appl. Mech.*, **33**, 281-309.
- Falade, J. A., Ukaegbu, J., Egere, A. C., and Adesanya, S. (2017) MHD Oscillatory Flow Through a Porous Channel Saturated with Porous Medium, *Alexandria Eng. J.*, 56(1), 147-152.
- Hamza, M. M. Ojemer Godwin and Ahmad, S. K. (2023). Theoretical study of Arrhenius-controlled heat transfer flow on natural convection affected by an induced magnetic field in a microchannel, *Engineering Reports, Wiley*, DOI:10.1002/eng2.12642
- Hamza, M. M., Shuaibu A. and Ahmad, S. K. (2022). Unsteady MHD free convection flow of an exothermic fluid in a convectively heated vertical channel filled with porous medium, *Scientific Reports*, **12**, 11989.
- Hatte, S., and Pitchumani, R. (2021). Analysis of convection heat transfer on multiscale rough super-hydrophobic and liquid infused surfaces, 1-29. <https://www.sciencedirect.com/science/article/am/pii/S13858947210184>
- Jha, B. K., Altine, M. M. and Hussaini A. M. (2022). Role of suction/injection on free convective flow in a vertical channel in the presence of point/line heat source/sink, *ASME J. Heat Transfer-trans. ASME*, 144, 062602
- Jha, B. K., Azeez, L. A. and Oni M. O. (2019). Unsteady hydro-magnetic free convection flow with suction/injection. *J Taibah Univ*, 13: 136-145.

- Jha, B. K and Gwandu, B. J. (2017). MHD free convection flow in a vertical slit micro-channel with super-hydrophobic slip and temperature jump: Heating by constant wall temperature, *Journal of Alexandria Engineering*, (57)3,2541- 2549.
- Jha, B. K. and Gwandu, B. J. (2019). MHD free convection flow in a vertical slit micro-channel with super-hydrophobic slip and temperature jump: non-linear Boussinesq approximation approach, *SN Appl. Sci.*, DOI: 10.1007/S42452-019-0617-Y.
- Jha, B. K., and Gwandu, B. J. (2020). MHD free convection flow in a vertical porous super-hydrophobic microchannel, *Proceedings of the Inst. of Mech engineers; part E: J Proc Mech Eng*, **235**(2).
- Jha, B. K., Isah, B. Y., and Uwanta, I. J. (2018). Combined Effect of Suction/Injection on MHD Free-Convection Flow in a Vertical Channel with Thermal Radiation, *Ain Shams Eng. J.*, **9**(4), 1069–1088.
- Khalid, A., Khan, I., Khan A. and Shafie S. (2015). Unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium. *Eng Sci Technol*, **18**, 309–317.
- Mishra, A. K., Paul, T. and Singh, A. K. (2002). Mixed convention flow in a porous medium bounded by two vertical walls. *Forsch Ingenieurwes*, **67**, 198–205.
- Nayak, A., Dash, G. C. and Panda, S. (2013). Unsteady MHD flow of a visco-elastic fluid along vertical porous surface with chemical reaction. *Proc Natl Acad Sci, India, Sect A Phys Sci.*, **83**: 153–161.
- Ojemer, G. and Hamza, M. M. (2022). Heat transfer analysis of Arrhenius-controlled free convective hydromagnetic flow with heat generation/absorption effect in a micro-channel, *Alexandria Eng. J.*, **61**, 12797-12811.
- Ojemer, G., Omokhuale E., Hamza M. M., Onwubuya I. O. and Shuaibu A. (2023). A Computational Analysis on Steady MHD Casson Fluid Flow Across a Vertical Porous Channel Affected by Thermal Radiation Effect. *International Journal of Science for Global Sustainability*, **9**(1), <https://doi.org/10.57233/ijsgs.v9i1.393>
- Ojemer, G. and Onwubuya I. O. (2023). Exploring the dynamics of viscous dissipative fluid past a super-hydrophobic microchannel in the coexistence of mixed convection and porous medium, *Saudi Journal of engineering and technology*, **8**(4),71-80.
- Ramanuja, M., Krishna, G. G., Sree, H. K., Radhika, V. N. (2020). Free convection in a vertical slit microchannel with super-hydrophobic slip and temperature jump conditions, *Int. J. Heat Tech.*, **38**(3), 738-744.
- Saeid, N. H. (2006). Natural Convection from Two Thermal Sources in a Vertical Porous Layer, *ASME J. Heat Transfer-Trans. ASME*, **128**(1), 104–109.
- Sandeep, N. and Sugunamma, V. (2013). Effect of an Inclined Magnetic Field on Unsteady free Convection flow of a Dusty Viscous fluid between two Infinite flat Plates filled by a Porous medium, *Int. J. Appl. Math. Mod.* **1**(1), 16-33.
- Siva, T., Jaangili, S. and Kumbhakar, B. (2021). Heat transfer analysis of MHD and electroosmotic flow of non-Newtonian fluid in a rotating microfluidic channel: an exact solution, *Appl. Math. Mech*, **42**, 1047-1062.
- Taid, B. K. and Ahmed, N. B. (2022). MHD free convection flow across an inclined porous plate in the presence of heat source, solet effect and chemical reaction affected by viscous dissipation ohmic heating, *Bio-interface Res. Applied Chem*, **12**(5), 6280-6296.
- Tamilselvan, V., Jayabarathi, T., Raghunathan, T., Yang, X. (2018). Optimal capacitor placement in radial distribution system using flower pollination algorithm. *Alexandria Eng. J.*, **57**: 2775–2786

- Thakre K., Mohanty K. B. and Chatterjee A. (2018). Reduction of circuit devices in symmetrical voltage source multilevel inverter based on series connection of basic unit cells. *Alexandria Eng J* ,57: 2703–2712.
- Uwanta, I. J., and Hamza, M. M. (2014). Effect of Suction/Injection on Unsteady hydro-magnetic Convective Flow of Reactive Viscous Fluid Between Vertical Porous Plates with Thermal Diffusion, *Int. Scholarly Res. Not.*, 2014, pp. 1–14.
- Wang, C. Y. and Ng, C-O. (2014). Natural convection in a vertical slit micro-channel with super-hydrophobic slip and temperature jump. *ASME J Heat Transfer*, 136:034502.