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Determination of Periodic Optimal Recruitment and Wastage Schedule using Dynamic Programming Approach

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Abstract
In this paper we propose a dynamic programming (DP) model that can assist organizations in determining future manpower requirement in terms of number, level of skills, and competence as well as formulating plans to meet those requirements. The model which incorporates both wastage and recruitment costs in its formulation is derived with an algorithm for obtaining planned periodic wastages and recruitments which can result in maximum net accruable revenue. We applied data obtained from an institution of higher learning to illustrate our proposed DP model based on wastage and recruitment costs and the results reveals that there should be no staff wastage in periods 2, 5, 9 and 10, and no recruitment should take in periods 2, 5, 8, 9 and 12 if the total accruable revenue from human resources to the organization is to be maximized. We also observed that the dual objective function value and that of the primal are equal which is in agreement with the duality theorem for symmetric duals.

Keywords: dynamic programming, manpower, overstaffing recruitment, wastage

INTRODUCTION
Dynamic programming is a mathematical technique which deals with the optimization of multistage decision problems. Gupta and Hira (2005) stated that dynamic programming deals with problems in which the decision variables vary with time thereby making the objective function to vary from one stage to another. Dynamic programming (DP) is a mathematical technique in which a given problem is decomposed into a number of sub-problems called stages whereby lower dimensional optimization takes place and the objective in such problems is to find a combination of decisions that will optimize some appropriate measure of effectiveness, Wagner (2001). According to Hillier and Lieberman (2001), Gupta and Hira (2005) and Kothari (2008), dynamic programming is applicable to many practical problems such as selection of advertising media to promote goods and services, replacement of worn-out equipment, inventory control problems, capital budgeting for allocating scarce resources to new adventures, shortest route problems and manpower planning which requires series of interrelated decisions.

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On the other hand manpower which is also known as human resources are people who contribute to the production of goods and services in an organization, Urhoma (2009). Bontis et al (1999) defines manpower as the human factor in an organization, the combine intelligence, skills and expertise that gives the organization its distinctive character which according to Armstrong (2004) constitutes a key element of the organization's market worth. Bulla and Scot (1994) and Setlhare (2007) further explain that manpower planning involves assessing future manpower requirement in terms of numbers, level of skills, and competence as well as formulating plans to meet those requirements.

The two major questions usually asked in manpower planning according to Setlhare (2007) are: (i) How many people are needed? and (ii) what sort of people are needed? The principal objective of manpower planning is to model the migration of staff from one grade to another in discrete time which could be as a result of recruitment, promotion or retirement, Robbin and Harrison (2007). According to Cole (2005), there are two sources of manpower supply namely; external and internal supply. External supply has to do with recruitment of staff from outside the organization while internal manpower supply sources include transfer and redeployment of employees within the organization. Promotion is a process whereby a staff in an organization is moved from a lower grade to a higher one, Nirmala and Jeeva, (2010). Recruitment and promotion costs are costs incurred in the process of recruitment and promotion of staff due to expansion of an organization while wastage refers to staff that leave an organization for various reasons such as resignation, retirement, retrenchment, dismissal, death etc. according to the Leeson (1982).

Dynamic programming approach to manpower planning has been extensively discussed by many researchers, (see Mehlmann (1980) and Rao (1990). The Rao's model involves using a backward recursive approach to determine optimal recruitment policies. Ogumeyo and Ekoko (2011), extended Rao's model by applying dynamic programming to determine optimal recruitment policies using a forward recursive approach. Nirmala and Jeeva (2010) also extended the Rao (1990) manpower model by incorporating promotion policies which considers optimal manpower recruitment and promotion policies for a two-grade system. In Nirmala and Jeeva (2010), dynamic programming model was developed with the objective of minimizing the manpower system cost during the recruitment and promotion periods which are determined by changes that take place in the system. These dynamic programming models based on only recruitment cost factor are founded on the assumption that the number of staff required in period j (R_j) can be estimated with known unit overstaffing cost.

In this paper we extend the manpower planning models presented in Mehlmann (1980), Rao (1990) and Nirmala and Jeeva (2010) by formulating a dynamic programming (DP) model that incorporates periodic wastage costs in addition to periodic recruitment costs as a way of addressing this shortcoming of these existing models. Though wastage cost was defined in these models as one of the components of total manpower cost it was not incorporated in the model formulation.

We first state the assumptions and notations as follows:

METHODOLOGY

The following are the assumptions of the DP model in LP form for manpower planning based on recruitment and wastage: (a) Recruitment and wastage at a particular grade are considered (b) Periodic recruitment $\left(c'_j\right)$ and wastage $\left(c_j\right)$ costs are known and fixed. (c) Number of staff of the organization at initial and end of time-horizon interval are known. (d) Both overstaffing and understaffing are considered. Where x_j = number of staff that are on wastage in period j, y_j = number of staff that are recruited in period j, c_j = average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the system, c'_j = average salary per recruited staff in period j, h = initial number of staff on ground in the organization at the beginning of the time horizon and H = total number of staff at the end of the time horizon under consideration.

MODEL FORMULATION

Let $y_j(t+\delta)$ be the number of staff recruited at time $(t+\delta)$ of period j where δ is the very small time difference between recruitment and assumption of duty so that the recruited staff arrive at time $(t+\delta)$ for work. Let $x_j(t+\delta)$ *and* $c_j(t+\delta)$ be the number of staff on wastage and the average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the organization. Let $c'_j(t+\delta)$ be the average salary per recruited staff at time $(t+\delta)$ of period j when the recruitment was done. As $\delta \to 0$, the above notations become $x_j(t)$, $y_j(t)$, $c_j(t)$ *and* $c'_j(t)$ or simply x_j , y_j, c_j *and* c'_j. Given h, H, $c_j(t)$ and $c'_j(t)$ of a manpower planning problem, it is required to determine the optimal quantities x_j *and* y_j so that the accruable net revenue is a maximum.

As we are dealing here with a dynamic situation, we divide the time span of interest into time intervals, which we shall assume to be sufficiently short so that we can consider $x_j(t)$, $y_j(t)$, $c_j(t)$ *and* $c'_j(t)$ to be constant during the time intervals but discontinuous from one time interval to the next.

The problem of the manpower planning is to maximize the periodic additional revenue accruable to the organization from the wastage staff wage bill less the periodic salary of recruited staff i.e. $\sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right)$.

The objective function can be written as:

$$Maximize\ z = \sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right) \quad (1)$$

There are two sets of staffing constraints and two sets of non-negativity constraints in this manpower planning problem.

(i) The overstaffing constraints:

The constraints of overstaffing state that the total number of overstaffing staff of the first i periods should not exceed the available vacancies $(H-h)$ in the establishment, i.e.

$$\sum_{j=1}^{i}\left(y_j - x_j\right) = -\sum_{j=1}^{i} x_j + \sum_{j=1}^{i} y_j \le H - h,\quad i = 1(1)n \quad (2)$$

Where $(y_j - x_j) > 0$ is the number of staff by which the organization is overstaffed in period j

The LHS of equation (2) can also be called the net increase in manpower in the first i periods.

(ii) The understaffing constraints:

The constraints of understaffing represent the number of staff by which the organization is understaffed for the first $(i-1)$ periods plus wastages at period i and this should not exceed h the number of staff originally in the organization. If it does, it means the organization has only material resources which is not the case in practical situation as existence of an organization is based on the contribution of human and material resources. Mathematically this is expressed as:

$$\sum_{j=1}^{i-1}\left(x_j - y_j\right) + x_i = \sum_{j=1}^{i} x_j - \sum_{j=1}^{i-1} y_j \le h, \quad i = 1(1)n \tag{3}$$

Where $(x_j - y_j) > 0$ is the number of staff by which the organization is understaffed in period j. The L.H.S of equations (3) can also be called the net increase in manpower subtracted from wastage staff in the first $(i-1)$ periods plus the wastage manpower in period i.

Note that the second summation in equation (3) does not exist for $i = 1$. (iii) Non-negativity constraints: The non-negativity constraints are

$$x_j,\ y_j \ge 0, \quad j = 1(1)n \tag{4}$$

Equation (1) stated above constitutes the total manpower planning cost from all the n periods while equations (1)-(4) constitute a DP problem which is stated thus:

Primal LP Problem

$$\left.\begin{aligned}
&Maximize \quad z = \sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right) \\
&s.t. \\
&\qquad -\sum_{j=1}^{i} x_j + \sum_{j=1}^{i} y_j \le H - h, \quad i = 1(1)n \\
&and \quad \sum_{j=1}^{i} x_j - \sum_{j=1}^{i-1} y_j \le h, \quad i = 1(1)n \\
&\qquad x_j,\ y_j \ge 0, \quad j = 1(1)n
\end{aligned}\right\} \tag{5}$$

Dual DP Problem

$$Minimize\ w = (H - h)\sum_{i=1}^{n} d_i + h\sum_{i=1}^{n} e_i \tag{6}$$

s.t.

$$-\sum_{i=k}^{n} d_i + \sum_{i=k}^{n} e_i \ge c_k, \quad k = 1(1)n \tag{7}$$

$$\sum_{i=k}^{n} d_i - \sum_{i=k+1}^{n} e_i \ge -c'_k, \quad k = 1(1)n \tag{8}$$

$$d_i,\ e_i \ge 0, \quad i = 1(1)n \tag{9}$$

It is understood that the second summation in equation (8) does not exist if $k = n$. we define new variables D_k *and* E_k as follows:

$$D_k = \sum_{i=k}^{n} d_i, \quad k = 1(1)n \tag{10}$$

$$E_k = \sum_{i=k}^{n} e_i, \quad k = 1(1)n \tag{11}$$

Since by the dual DP problem, d_i *and* e_i are nonnegative, D_k *and* E_k must be nonnegative. However, non-negativity of D_k *and* E_k does not imply that $d_i \ge 0$ *and* $e_i \ge 0, \ \forall\ i$. In view of the definition of D_k *and* E_k, we see that nonnegativity of d_i *and* e_i will be ensured if we augment the dual LP problem, expressed in terms of D_k *and* E_k by the constraints:

$$D_k \geq D_{k+1}\ , \quad k = 1(1)\ n-1 \tag{12}$$

$$E_k \geq E_{k+1}\ , \quad k = 1(1)\ n-1 \tag{13}$$

Note that the constraints in equations (12) and (13) do not exist when $k = n$ because $D_{n+1} = E_{n+1} = 0$. Hence, we have $2(n-1)$ additional constraints in equations (12) and (13)

Substituting for $\sum_{i=1}^{n} d_i$ *and* $\sum_{i=1}^{n} e_i$ in the dual DP problem in equations (6)-(9) and incorporating constraints (12) and (13), we obtain the dual in system (14).

$$\left.\begin{aligned}
&Min\ \ w = (H-h)D_1 + hE_1 \\
&s.t. \\
&\quad D_k \geq E_{k+1} - c'_k\ ,\ k = 1(1)n \\
&\quad D_k \geq D_{k+1}\ ,\ k = 1(1)n-1 \\
&\quad D_k \geq 0 \quad ,\ k = 1(1)n \\
&\quad E_k \geq D_k + c_k\ ,\ k = 1(1)n \\
&\quad E_k \geq E_{k+1}\ ,\ k = 1(1)n-1 \\
&\quad E_k \geq 0 \quad ,\ k = 1(1)n
\end{aligned}\right\} \tag{14}$$

This is the dual DP problem starting with period 1 while D_1 *and* E_1 are the smallest values in their solution set. The dual DP problem in system (14) has n sub-problems each involving the pairs $(D_k\ ,\ E_k)$, $k = 1(1)n$. In particular the sub-problem involving D_1 *and* E_1 is:

$$\left.\begin{aligned}
&Min\ \ w = (H-h)D_1 + hE_1 \\
&s.t. \\
&\quad D_1 \geq D_2 - c'_1 \\
&\quad D_1 \geq D_2 \\
&\quad D_1 \geq 0 \\
&\quad E_1 \geq D_1 + c_1 \\
&\quad E_1 \geq E_2 \\
&\quad E_1 \geq 0
\end{aligned}\right\} \tag{15}$$

The constraints in system (15) imply that the subproblem in (15) cannot be solved except we know D_2 *and* E_2. This suggests that the procedure should be by backward recursive approach to first obtain D_2 *and* E_2. Continuing this way, we certainly have to solve for n suboptimal solutions for the pairs $(D_k\ ,\ E_k)$, $k = 1(1)n$. We start from the last (*nth*) pair $(D_n\ ,\ E_n)$ with subproblem given as:

$$\left.\begin{array}{l} Min\ \ w=(H-h)D_1+hE_1 \\ s.t. \\ \quad D_1 \ge D_2 - c_1' \\ \quad D_1 \ge D_2 \\ \quad D_1 \ge 0 \\ \quad E_1 \ge D_1 + c_1 \\ \quad E_1 \ge E_2 \\ \quad E_1 \ge 0 \end{array}\right\} \qquad (16)$$

We now proceed to use the backward recursive approach to determine $(D_1\ ,\ E_1)$.

Backward Recursive Approach for the Determination of D1 and E1 The backward recursive approach is used to determine the n optimal pairs $(D_n, E_n), (D_{n-1}, E_{n-1}), \cdots, (D_1, E_1)$ as suboptimal solutions to the n sub-problems. The procedure culminates in the algorithm that follows: We start from the last sub-problem in system (16)

nth Sub-problem

The constraints are:

$\left.\begin{array}{l} D_n \ge -c_n' \\ D_n \ge 0 \end{array}\right\} \Rightarrow$ solution set is $D_n \ge 0$, minimizing w, means D_n must be the smallest value in the solution set.

i.e. $D_n = \max(-c_n', 0)$ (17)

$D_n = \max(-c_n', 0) = 0$

$\left.\begin{array}{l} E_n \ge D_n + c_n \\ E_n \ge 0 \end{array}\right\}$ solution set is $E_n \ge (D_n + c_n)$ and $E_n = \max(D_n + c_n\ , 0)$ (18)

kth Sub-problem $(k = (n-1),\ (n-2), \cdots, 3, 2, 1)$

This is the general case and the subproblem is stated as follows:

$$\left.\begin{array}{l} Min\ \ w=(H-h)D_k+hE_k \\ s.t. \\ \quad D_k \ge E_{k+1} - c_k' \\ \quad D_k \ge D_{k+1} \\ \quad D_k \ge 0 \\ \quad E_k \ge D_k + c_k \\ \quad E_k \ge E_{k+1} \\ \quad E_k \ge 0 \end{array}\right\} \qquad (19)$$

The optimal values of D_k *and* E_k are obtained as follows:

$\left.\begin{array}{l} D_k \ge E_{k+1} - c_k' \\ D_k \ge D_{k+1} \\ D_k \ge 0 \end{array}\right\}$ solution set is $D_k \ge \max(E_{k+1} - c_k'\ ,\ D_{k+1}\ , 0)$

i.e. $D_k = \max\left(E_{k+1} - c'_k\ ,\ D_{k+1}, 0\right)$, $k = (n-1),\ (n-2), \cdots, 3, 2, 1$

$$\left.\begin{array}{l} E_k \geq D_k + c_k \\ E_k \geq E_{k+1} \\ E_k \geq 0 \end{array}\right\} \text{ solution set is } E_k \geq \max\left(D_{k+1} - c_k\ ,\ E_{k+1}, 0\right)$$

i.e. $E_k = \max\left(D_k + c_k\ ,\ E_{k+1}, 0\right)$, $k = (n-1),\ (n-2), \cdots, 3, 2, 1$

$$\left.\begin{array}{l} D_k = \max\left(E_{k+1} - c'_k\ ,\ D_{k+1}, 0\right), \\ E_k = \max\left(D_k + c_k\ ,\ E_{k+1}, 0\right) \end{array}\right\} k = (n-1),\ (n-2), \cdots, 3, 2, 1 \qquad (20)$$

In each sub-problem the D_k are obtained before the E_k. The last of these are which are partial sums of c_k *and* c'_k. D_1 *and* E_1 are then substituted into equation (6) to yield the minimum value of the objective function of the dual DP problem which by Duality Theorem of symmetric duals is equal to the maximum value of the objective function of the primal DP problem for manpower planning.

NUMERICAL ILLUSTRATION

The data in Table 1 show (a) the average monthly salary (c_j) of junior staff on wastage for the year 2001 to 2012 and (b) the average monthly salary (c'_j) of recruited junior staff for the year 2001 to 2012.

Table 1: Average monthly salary for **junior staff** on wastage and recruitment

Year	2001 (1)	2002 (2)	2003 (3)	2004 (4)	2005 (5)	2006 (6)	2007 (7)	2008 (8)	2009 (9)	2010 (10)	2011 (11)	2012 (12)
c_j	33286	32045	35770	35918	36637	37552	38437	39126	33065	32281	38084	40124
c'_j	30148	32281	33665	34305	37545	34305	37894	36157	32981	30467	37688	36645

We considered average monthly salary for a period of up to 12 years so that our results can give good estimates of staff wastage (x_j) and recruitment (y_j) when initial number of junior staff (h) and future capacity staff strength (H) are known. In 2012, A newly established university XYZ had 162 junior staff. Based on the present salary trend, determine the optimal annual number of staff on wastage and recruitment that will maximize total accruable revenue to the institution in the next 12 years (i.e. by the year 2024) when the junior staff strength is planned to be 393.

Solution

We have 12 periods in this example, $n = 12$ and we proceed to evaluate the D_k *and* E_k $(k = 12, 11, 10, \cdots, 2, 1)$ using system (20)

$$\left.\begin{array}{l} D_k = \max\left(E_{k+1} - c'_k,\ D_{k+1}, 0\right) \\ E_k = \max\left(D_k + c_k,\ E_{k+1},\ 0\right) \end{array}\right\}, \left(k = 12, 11, 10, \cdots, 2, 1\right) \qquad (20)$$

$$D_{12} = \max\left(-c'_{12}, 0\right) = 0$$
$$E_{12} = \max\left(D_{12} + c_{12}, 0\right) = D_{12} + c_{12} = c_{12}$$
$$D_{11} = \max\left(E_{12} - c'_{11}, D_{12},\ 0\right) = E_{12} - c'_{11} = c_{12} - c'_{11}$$
$$E_{11} = \max\left(D_{11} + c_{11}, E_{12}, 0\right) = D_{11} + c_{11} = c_{12} - c'_{11} + c_{11}$$

$D_{10} = \max(E_{11} - c'_{10}, D_{11}, 0) = E_{11} - c'_{10} = c_{12} - c'_{11} + c_{11} - c'_{10}$
$E_{10} = \max(D_{10} + c_{10}, E_{11}, 0) = D_{10} + c_{10} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{10}$
$D_{9} = \max(E_{10} - c'_{9}, D_{10}, 0) = D_{10} = c_{12} - c'_{11} + c_{11} - c'_{10}$
$E_{9} = \max(D_{9} + c_{9}, E_{10}, 0) = D_{9} + c_{9} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{9}$
$D_{8} = \max(E_{9} - c'_{8}, D_{9}, 0) = D_{9} = c_{12} - c'_{11} + c_{11} - c'_{10}$
$E_{8} = \max(D_{8} + c_{8}, E_{9}, 0) = D_{8} + c_{8} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8}$
$D_{7} = \max(E_{8} - c'_{7}, D_{8}, 0) = E_{8} - c'_{7} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7}$
$E_{7} = \max(D_{7} + c_{7}, E_{8}, 0) = D_{7} + c_{7} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7}$
$D_{6} = \max(E_{7} - c'_{6}, D_{7}, 0) = E_{7} - c'_{6} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6}$
$E_{6} = \max(D_{6} + c_{6}, E_{7}, 0) = D_{6} + c_{6} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6}$
$D_{5} = \max(E_{6} - c'_{5}, D_{6}, 0) = E_{6} - c'_{5} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{5}$
$E_{5} = \max(D_{5} + c_{5}, E_{6}, 0) = E_{6} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6}$
$D_{4} = \max(E_{5} - c'_{4}, D_{5}, 0) = E_{5} - c'_{4} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4}$
$E_{4} = \max(D_{4} + c_{4}, E_{5}, 0) = D_{4} + c_{4} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4} + c_{4}$
$D_{3} = \max(E_{4} - c'_{3}, D_{4}, 0) = E_{4} - c'_{3} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4} + c_{4} - c'_{3}$
$E_{3} = \max(D_{3} + c_{3}, E_{4}, 0) = D_{3} + c_{3} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4} + c_{4} - c'_{3} + c_{3}$
$D_{2} = \max(E_{3} - c'_{2}, D_{3}, 0) = E_{3} - c'_{2} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4} + c_{4} - c'_{3} + c_{3} - c'_{2}$
$E_{2} = \max(D_{2} + c_{2}, E_{3}, 0) = E_{3} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4} + c_{4} - c'_{3} + c_{3}$
$D_{1} = \max(E_{2} - c'_{1}, D_{2}, 0) = E_{2} - c'_{1} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4} + c_{4} - c'_{3} + c_{3} - c'_{1}$
$E_{1} = \max(D_{1} + c_{1}, E_{2}, 0) = D_{1} + c_{1} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{8} - c'_{7} + c_{7} - c'_{6} + c_{6} - c'_{4} + c_{4} - c'_{3} + c_{3} - c'_{1} + c_{1}$
$D_{1} = (c_{3} + c_{4} + c_{6} + c_{7} + c_{8} + c_{11} + c_{12}) - (c'_{1} + c'_{3} + c'_{4} + c'_{6} + c'_{7} + c'_{10} + c'_{11})$
$= 265011 - 238472$
$= \mathbf{26,539}$
$E_{1} = (c_{1} + c_{3} + c_{4} + c_{6} + c_{7} + c_{8} + c_{11} + c_{12}) - (c'_{1} + c'_{3} + c'_{4} + c'_{6} + c'_{7} + c'_{10} + c'_{11})$
$= 298297 - 238472$
$= \mathbf{59825}$

The dual objective function value is given as:

$(H - h)D_{1} + hE_{1} = 231(26539) + 162(59825)$
$= 6130509 + 9691650$
= ₦15,822,159 which is the recruitment/wastage policy **cost** of the manpower planning problem obtained from the DP model.

To obtain the solution to the primal DP problem we proceed as follows:

$$(H - h)D_{1} + hE_{1} = \sum_{j=1}^{12} (c_{j}x_{j} - c'_{j}y_{j}) \tag{21}$$

LHS $= 231(c_{3} + c_{4} + c_{6} + c_{7} + c_{8} + c_{11} + c_{12}) - 231(c'_{1} + c'_{3} + c'_{4} + c'_{6} + c'_{7} + c'_{10} + c'_{11})$
$+162(c_{1} + c_{3} + c_{4} + c_{6} + c_{7} + c_{8} + c_{11} + c_{12}) - 162(c'_{1} + c'_{3} + c'_{4} + c'_{6} + c'_{7} + c'_{10} + c'_{11})$
$= 162c_{1} + 0c_{2} + 393c_{3} + 393c_{4} + 0c_{5} + 393c_{6} + 393c_{7} + 393c_{8} + 0c_{9} + 0c_{10} + 393c_{11} + 393c_{12}$

•

$- 393c'_{1} - 0c'_{2} - 393c'_{3} - 393c'_{4} - 0c'_{5} - 393c'_{6} - 393c'_{7} - 0c'_{8} - 0c'_{9} - 393c'_{10} - 393c'_{11} - 0c'_{12}$

- R.H.S

$= x_1c_1 + x_2c_2 + x_3c_3 + x_4c_4 + x_5c_5 + x_6c_6 + x_7c_7 + x_8c_8 + x_9c_9 + x_{10}c_{10} + x_{11}c_{11} + x_{12}c_{12}$

- $- y_1c'_1 - y_2c'_2 - y_3c'_3 - y_4c'_4 - y_5c'_5 - y_6c'_6 - y_7c'_7 - y_8c'_8 - y_9c'_9 - y_{10}c'_{10} - y_{11}c'_{11} - y_{12}c'_{12}$
- $\Rightarrow \; x_1 = 162,\; x_2 = 0,\; x_3 = 393,\; x_4 = 393,\; x_5 = 0,\; x_6 = 393,\; x_7 = 393,\; x_8 = 393,$
- $x_9 = 0,\; x_{10} = 0,\; x_{11} = 393,\; x_{12} = 393,$
- $y_1(i.e.x_{13}) = 393,\; y_2(i.e.x_{14}) = 0,\; y_3(i.e.x_{15}) = 393,\; y_4(i.e.x_{16}) = 393,\; y_5(i.e.x_{17}) = 0,$
- $y_6(i.e.x_{18}) = 393,\; y_7(i.e.x_{19}) = 393,\; y_8(i.e.x_{20}) = 0,\; y_9(i.e.x_{21}) = 0,\; y_{10}(i.e.x_{22}) = 393,$
- $y_{11}(i.e.x_{23}) = 393,\; y_{12}(i.e.x_{24}) = 0$
- The primal objective function value is:
  - $162(33286) + 393(35770 + 35918 + 37552 + 38437 + 39126 + 38084 + 40124)$
- $- 393(30148 + 33665 + 34305 + 34305 + 37894 + 30467 + 37688)$
  - $= 5392332 + 393(265011) - 393(238472)$
  - = ₦15,822,159
- The dual objective function value $\left(i.e.(H - .h)D_1 + hE_1\right)$ and that of the primal $(i.e.\sum_{j=1}^{12}(c_jx_j - c'_jy_j))$ are equal in this solution. This is in agreement with the Duality Theorem for symmetric duals.

DISCUSSION OF RESULTS

The existing DP models in LP form in RaO (1990) and Nirmala and Jeeva (2010) which are based on only recruitment cost are only merely stated in a manner that cannot be solved by simplex method or its variants. Hence the DP models were not solved. But the proposed DP model in LP form can be solved by both LP and DP methods using either computer implementation or manual. The existing DP models used only recruitment factor to the exclusion of wastage factor in the mathematical formulations. In real life, both recruitment and wastage take place from time to time. Our proposed DP model incorporates both recruitment and wastage costs in line with what exists in practical situations. The optimal solution to the junior staff problem from the DP model algorithm reveals that there should be no staff wastage in periods 2, 5, 9 and 10, and no recruitment in periods 2, 5, 8, 9 and 12 if the total accruable revenue from human resources to the organization is to be maximized. The dual objective function value $\left(i.e.(H - .h)D_1 + hE_1\right)$ and that of the primal $(i.e.\sum_{j=1}^{12}(c_jx_j - c'_jy_j))$ are equal in this solution. This is in agreement with the Duality Theorem for symmetric duals.

CONCLUSION:

In this paper, we have developed a backward recursive DP model based on recruitment and wastage factors. The model algorithm has been applied to solve practical manpower problems using data from XYZ University. It is observed that based on present record of periodic staff salaries, initial and final manpower needs, we can from our two-factor DP model determine periodic optimal recruitment and wastage schedules for a given time horizon. This encourages business organization to plan ahead.

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