

Long Memory and Volatility Models in Forecasting Exchange Rate of Nigerian Naira to United State Dollar

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Abstract

Financial time series such as stock prices, inflation rates, interest rates, and exchange rates are known to exhibit upward and downward trend and often possesses long memory and volatility behavior. These behaviors are crucial in the analysis, modeling and forecasting of time series data. Unfortunately, many analysts don't take into consideration the consequences of long memory and volatility while modeling financial time series data. Therefore, this paper intends to examine the effect of long memory and volatility in forecasting Exchange Rate of Nigerian Naira-United State Dollar. The data used for this study is obtained from Central Bank of Nigeria's website for the period of January 2002 to Feb 2020. Observations from time series and Autocorrelation function (ACF) plot shows that the data was not stationary. This was confirmed by the Augmented Dickey Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) unit root tests on the datasets. Fractional differencing was used to transform the series and ARFIMA(1,d,1) and ARFIMA(1,d,2) models were selected using AIC criterion. However, the residuals of these models were found to be serially correlated and heteroscedastic. These problems led to combining ARFIMA with GARCH model in order to adequately study long memory and volatility simultaneously. Therefore ARFIMA models were combined with GARCH(1,1) to form ARFIMA(1,d,1)-GARCH(1,1) and ARFIMA(1,d,2)-GARCH(1,1). The results of the forecast performance indicate that the best model is ARFIMA(1,d,2)-GARCH(1,1).

Keywords: Exchange Rate, Long memory, Volatility, ARFIMA and ARFIMA-GARCH.

INTRODUCTION

Financial time series such as stock prices, inflation rates, interest rates, and exchange rates exhibit long memory behavior. This makes long memory process to have increasing attention and important subject of both theoretical and empirical research. However, the well-known time series models such as Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) failed to capture this challenging property. This challenge motivated Granger (1980), Granger and Joyeux (1980), and Hosking

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(1981) to develop Fractional Integrated Autoregressive Moving Average (ARFIMA) model. This model takes fractional values in the interval $0 < d < 1$ and eliminate long memory components from time series data. ARFIMA model is widely used in the study of long memory processes but it is not suitable for series exhibiting high periods of volatility (Aliyu et al., 2023).

Many economic series show periods of stability followed by the periods of instability in volatility (Aliyu et al., 2023). To take care this, Autoregressive conditional heteroscedastic (ARCH) model was developed. A parsimonious generalization of the ARCH model is GARCH, which is proposed by Bollerslev (1986). Later on, Baillie *et al* (1995) considered time series data that exhibits both features of long memory and volatility and came up with ARFIMA-GARCH model, a hybrid of the two models. ARFIMA is the mean model that considers the mean behavior of a time series (long memory), and GARCH is a variance model that uses the residual series from the fitted ARFIMA to model the variance behavior (ARCH effect).

A recent contribution by Wiri and Tuaneh (2022) examined the monthly exchange rate of Naira per Dollar using the ARFIMA method. The presence of a long memory structure was also revealed in the series and ARFIMA (1, 0.0868, 1) was found to be appropriate on a monthly forecasting horizon. Musa et al., (2014) investigated the conditional volatility modeling of daily Dollar/Naira exchange rate using GARCH, Glosten Jagannathan Runkle GARCH (GJR-GARCH), Threshold GARCH (TGARCH) and Taylor Schwert GARCH (TS-GARCH) models.

Some researchers studied long memory behavior in the exchange rate without considering the volatility (see Wiri and Tuaneh (2022)) while some studied volatility without considering long memory (see Musa et al., (2014)). Therefore this research is aimed at hybridizing ARFIMA and GARCH models using Nigerian Naira to US Dollar in order to study long memory and volatility simultaneously and this procedure provides substantial improvements in terms of model fitting and forecasts (see Cheung and Chung (2009) and Fofana *et al.*, (2014)).

MATERIALS AND METHODS

Data

The data used for this study is a secondary data (monthly Exchange Rates of Nigerian Naira-United State Dollar from January, 2002 to February, 2020) obtained from Central Bank of Nigeria's website. The data was analyzed using R statistical software.

Stationary Test

In this study, we consider the Augmented Dickey Fuller Test (ADF) and Kwiatkowski-Philips-Schmidt-Shin (KPSS) for unit root test.

Augmented Dickey Fuller (ADF) Test

The ADF test was been proposed by Dickey and Fuller (1981) with the null hypothesis that the data generating process is non stationary. The ADF test is conducted by first considering autoregressive model of order one as follows:

$$y_t = c + \gamma_t + \phi_1 y_{t-1} + \varepsilon_t. \quad (1)$$

The ADF add lagged differences to these models so that

$$\begin{aligned} \Delta y_t &= c + \gamma_t + \phi_1 y_{t-1} + \delta_1 \Delta y_{t-1} \dots + \delta_p \Delta y_{t-p} + \varepsilon_t, \\ &= c + \gamma_t + \phi_1 y_{t-1} + \sum_{i=1}^p \delta_i \Delta y_{t-i} + \varepsilon_t, \end{aligned} \quad (2)$$

Where c is a constant, γ is the coefficient on a time trend, p the lag order of the autoregressive model and $\varepsilon_t \sim i.i.d(0, \sigma_\varepsilon^2)$.

Kwiatkowski-Philips-Schmidt-Shin(KPSS) Test

KPSS test was developed by Kwiatkowski *et al* (1992). The null hypothesis states that the Data Generating Process (DGP) is stationary. If the data is without a linear trend the test assumes the following DGP:

$$X_t = y_t + z_t \quad (3)$$

where $y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + v_t$, $v_t \sim i.i.d(0, \sigma_v^2)$ and z_t is a stationary process. The test statistic is given by:

$$KPSS = \frac{1}{T^2} \sum_{t=1}^T \frac{s_t^2}{\hat{\sigma}_\infty^2} \quad (4)$$

where $s_t = \sum_{j=1}^t \hat{w}_j$ with $\hat{w}_t = y_t - y$ and $\hat{\sigma}_\infty^2$ is an estimator of the long run variance of the process, z_t .

Long Memory Test

Before estimating the ARFIMA model, we must ensure that the data set exhibit long memory behavior. In this study, Hurst Exponent was considered for the long memory test.

Hurst Exponent

The Hurst exponent is a representation of a time series' long-memory. The word "Hurst exponent" comes from Harold Edwin Hurst, whose name is often synonymous with the letter H, and is used as the standard notation for coefficient. The long memory structure occurs when $0.5 < H < 1$. The Hurst exponent can be determined using the formula;

$$H = \frac{\log\left(\frac{R}{S}\right)}{\log(T)} \quad (5)$$

where T is the duration of the sample data and $\frac{R}{S}$ the corresponding value of the rescaled analysis.

Statistical Models and Diagnostic Tests

ARFIMA Model

The general form of an ARFIMA model of Granger and Joyeux (1980) and Hosking (1981) is given by:

$$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t, 0 < d < 1. \quad (6)$$

The term $(1-L)^d$ can be expanded using binomial series expansion as follows:

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k = 1 - dL + \frac{d(d-1)}{2!} L^2 \quad (7)$$

GARCH Model

The GARCH (r,s) model's variance equation is presented as

$$\sigma_t^2 = \omega + \sum_{i=1}^r \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2, \quad (8)$$

where σ_{t-j}^2 is the volatility at day $t-j$, $\omega > 0$, $\alpha_i \geq 0$ for $i = 1, \dots, r$, and $\beta_j \geq 0$ for $j = 1, \dots, s$, are parameters of the model to be estimated. Under the GARCH (r,s) model, the conditional variance of ε_t , σ_t^2 , depends on the squared innovations in the previous r periods, and the conditional variance in the previous s periods.

ARFIMA-GARCH Model

The ARFIMA-GARCH model is used to study long memory and volatility of a time series concurrently. The ARFIMA (p,d,q) -GARCH (r,s) model can be defined as follows:

$$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t \quad (9)$$

where $\varepsilon_t | \psi_{t-1} \sim N(0, \sigma_t^2)$ and $\psi_{t-1} = Y_1, Y_2, \dots, Y_{t-1}$.

Information Criterion

The Akaike Information Criterion (AIC) is a measure of the relative goodness of fit of a statistical model as well as the order of the model. Akaike (1974) suggests measuring the goodness of fit for some particular model by balancing the error of the fit against the number of parameters in the model. The formula for the AIC is:

$$AIC = 2k - 2 \ln(L), \quad (10)$$

where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model.

Diagnostic Tests

For a mean and hybrid modeling, it is important to test for serial correlation and presence of heteroscedasticity after models estimation. In this study, we consider a number of these tests.

Autoregressive Conditional Heteroscedastic-Lagrange Multiplier(ARCH-LM Test)

Engle (1982) proposed the ARCH-LM test to cater for issues of conditional heteroscedasticity in squared residuals under the null hypothesis that there is no heteroscedasticity in the model residuals. The test statistic is

$$Q = N(N + 2) \sum_{i=1}^N \frac{\rho_i}{(N-i)}, \quad (11)$$

The Q statistic has an asymptotic χ^2 distribution with n degrees of freedom provided that the squared residual is uncorrelated. N is the number of observation and ρ_i is the sample correlation coefficient between squared residuals $\hat{\varepsilon}_t^2$ and $\hat{\varepsilon}_{t-1}^2$. The null hypothesis of square residuals is that; $\{\hat{\varepsilon}_t^2\}$ are uncorrelated.

Portmanteau Test.

The Portmanteau test investigates the presence of autocorrelation in the residuals of a fitted model. Suppose the autocorrelation between ε_t and ε_{t-k} is $\rho_k = \text{Corr}(\varepsilon_t, \varepsilon_{t-k})$. Then, the null hypothesis states that all lags correlation are zero and is given as

$$H_0: \rho_1 = \rho_2 = \dots \rho_k = 0.$$

The test statistic, the modified Q statistic is given by

$$Q_1 = m(m + 2) \sum_{j=1}^K (m - j)^{-1} \hat{\rho}_{\hat{\varepsilon}}^2(j) \quad (12)$$

The Q_1 statistic approximately follows a χ^2 distribution with $K - p - q$ degrees of freedom.

Measures of Forecast Accuracy

Assessing the accuracy of forecasts obtained from time series model is important but a difficult task. To handle this vital stage of modeling, this section presents two measures of forecasting performance. These are Root Mean Square Error (RMSE) and Mean Absolute Error (MAE).

Root Mean Square Error

The RMSE is defined as the estimate of deviation of errors in forecasting. It is calculated by obtaining the square root of the difference among predicted and historical observations that are both squared and averaged over the sample. A smaller RMSE indicates a better model estimate.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (Y_t - \hat{Y}_t)^2}. \quad (13)$$

Mean Absolute Error

The MAE is regularly used for examining the suitability of time series model. The MAE is expressed as:

$$MAE = \frac{1}{n} \sum_{i=1}^n |Y_i - \hat{Y}_i|. \quad (14)$$

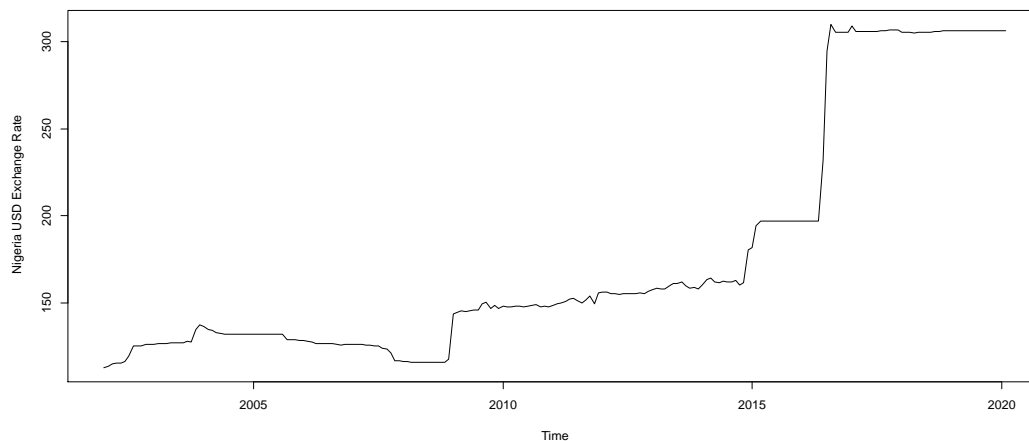
RESULTS AND DISCUSSION

This section provides summary statistics of the tests discussed in section two above.

Stationary and Long Memory Test

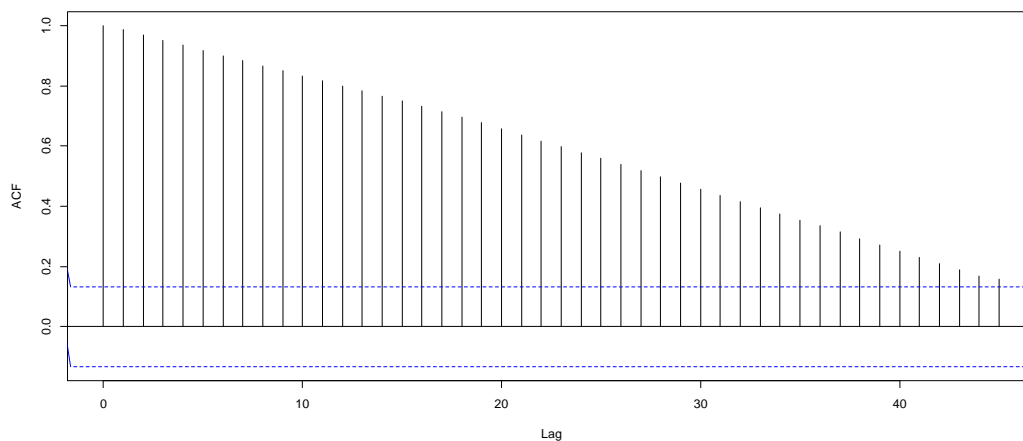
When modeling time series data, the first step is to check for stationarity and long memory behavior of the series.

Figure 1: Time Series Plot of Exchange Rate of the Monthly Nigerian Naira-United State Dollar



The time series plot of the Exchange Rate in Figure 1 exhibits trend indicating that the monthly exchange rate is changing over time.

Figure 2: The ACF of Exchange Rate



The ACF in Figure 2 shows a slow decay in the exchange rate, an indication of long memory and non-stationary.

Table 1: Showing ADF and KPSS test of Exchange Rate

Test	Statistic	P-Value	Decision	Remark
ADF	-1.6664	0.716	Fail to reject	Non stationary
KPSS	3.4033	0.01	Reject	Non stationary

The ADF and KPSS non-stationary test results as displayed in Table 1 show that the Exchange rate is non-stationary. The p-values of the ADF and KPSS test are greater and less than 0.05 respectively. The null hypothesis for the ADF and KPSS test are accepted and rejected respectively.

Table 2: Long Memory Test

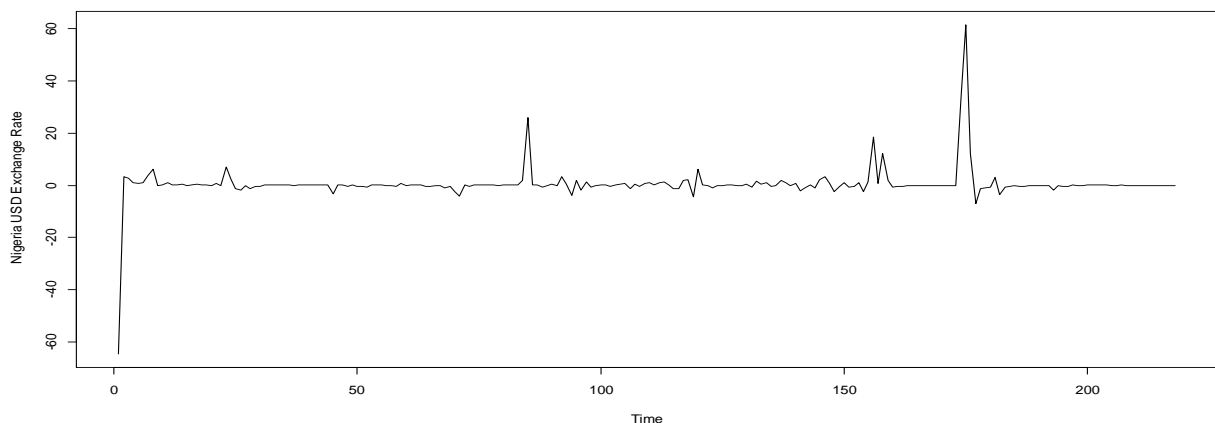
Test Type	Test Statistic
Hurst Exponent	0.83(0.000)

The Hurst exponent confirm the incidence of long memory in the Exchange rate series as shown in Figure 2 and in Table 2. The null hypothesis of no long meory is rejected because the p-value is less than the 0.05 significance level. The Hurst exponent estimator produces long memory value in the interval of $0 < d < 1$ suggesting that the ARFIMA model can be considered in modeling the exchange rate series.

Differencing

Since non-stationary time series exhibits trend characteristics as shown in Figure 1, it is important to transform the time series by applying transformation of fractional differencing since ARFIMA model is to be estimated. The transformation will help in stabilizing the mean and variance of the time series, and thus removing the long memory property.

Figure 3: Time Series Plot of the Fractional Differenced of Monthly Nigeria Naira-United State Dollar Exchange Rate



The plot in Figure 3 shows that the series is stationary after fractional differencing is carried out. In addition, the fractional differenced series indicate three periods of volatility in the exchange rate.

Table 3: Showing ADF and KPSS Test of Transformed Exchange Rate

Series	Test	Statistic	P-Value	Decision	Remark
First Differenced	ADF	-1.6664	0.001	Reject	stationary
First Differenced	KPSS	3.4033	0.261	Fail to reject	stationary
Frac. Differenced	ADF	-1.4281	0.002	Reject	stationary
Frac. Differenced	KPSS	5.0712	0.591	Fail to reject	stationary

The ADF and KPSS non-stationary test results as displayed in Table 3 show that the transformed Exchange rate is stationary. All the p-values of the ADF and KPSS test are less and greater than 0.05 respectively. The null hypothesis for the ADF and KPSS test are rejected and accepted respectively.

ARFIMA Model Identification

The parameters of the ARFIMA models were identified for the exchange rate series. The AIC of the identified ARFIMA models are displayed in Table 4. The values are compared and ARFIMA(1,d,1) and ARFIMA(1,d,2) are the best models with the minimum AIC.

Table 4: AIC Values for ARFIMA(p,d,q) Models

S/N	ARFIMA(p,d,q)	AIC
1	ARFIMA(1,d,0)	1361.632
2	ARFIMA(0,d,1)	1462.292
3	ARFIMA(1,d,1)	1320.425
4	ARFIMA(2,d,0)	1329.413
5	ARFIMA(0,d,2)	1396.070
6	ARFIMA(2,d,1)	1322.055
7	ARFIMA(1,d,2)	1321.955
8	ARFIMA(2,d,2)	1324.179

The Estimation of ARFIMA Models

The results of the estimated parameters of ARFIMA models for the studied series and their log-likelihood values are given in Table 5.

Table 5: Estimation of ARFIMA(p,d,q) with their Log-likelihood Values

ARFIMA(p,d,q)	ϕ_1	θ_1	θ_2	Log-Lik.
ARFIMA(1,d,1)	0.99(0.002)	-0.50(0.004)	-----	-656.20
ARFIMA(1,d,2)	0.99(0.001)	-0.58(0.010)	-0.10(0.003)	-656.00

Note: all standard error values are in parenthesis

All the parameters in the ARFIMA model are significant due to minimum standard error values.

The Diagnostic Tests of ARFIMA Models

The results of the serial correlation and heteroscedasticity analyses for ARFIMA models are shown in Table 6.

Table 6: Diagnostic Tests of the ARFIMA(p,d,q) Models.

ARFIMA(p,d,q)	Portmanteau Test	ARCH-LM Test
ARFIMA(1,d,1)	30.31(0.000)	56.62(0.000)
ARFIMA(1,d,2)	35.52(0.000)	69.43(0.000)

Portmanteau and ARCH-LM test at lag 40 and p-values are in parenthesis and null hypothesis is rejected at 5% levels.

The tests are based on the Ljung-Box test, and Engle heteroscedastic known as ARCH-LM test. The result shows the presence of serial correlation and heteroscedasticity in both models. This problem led to the estimation of hybrid ARFIMA-GARCH model in order to study long memory and volatility simultaneously.

The Estimation of ARFIMA-GARCH Models

The results of the estimated parameters of ARFIMA-GARCH models with their log-likelihood values are given in Table 7.

Table 7: The Estimation of ARFIMA(p,d,q)-GARCH(1,1)

Parameters	ARFIMA(1,d,1)-GARCH(1,1)	ARFIMA(1,d,2)-GARCH(1,1)
ϕ_1	0.99(0.002)	0.99(0.001)
θ_1	-0.50(0.004)	-0.58(0.010)
θ_2	-----	-0.10(0.003)
ω	10.9(0.001)	9.88(0.003)
α_1	0.87(0.002)	0.71(0.005)
β_1	0.11(0.001)	0.21(0.003)
Log-Lik.	-341.82	-461.52

Note: all standard error values are in parenthesis

All the parameters of the ARFIMA-GARCH model in Table 7 comes with smaller standard errors indicating adequacy of the model when compared with ARFIMA. Note that, log-likelihood values are larger compared to the log-likelihood values of ARFIMA models. This gives evidence of improvement in model fitting as a results of introducing GARCH to the ARFIMA models.

The Diagnostic Tests of Hybrid Models

The serial correlation and heteroscedasticity analyses of the ARFIMA-GARCH models are shown in Table 8.

Table 8: Diagnostic Tests of the ARFIMA(p,d,q)-GARCH(1,1) Models.

ARFIMA(p,d,q)-GARCH(1,1)	Portmanteau Test	ARCH-LM Test
ARFIMA(1,d,1)-GARCH(1,1)	47.92(0.8951)	66.81(0.7814)
ARFIMA(1,d,2)-GARCH(1,1)	56.76(0.9741)	73.63(0.8683)

Portmanteau and ARCH-LM test at lag 40 and p-values are in parenthesis and null hypothesis is accepted at 5% levels.

The diagnostic tests in Table 8 show better improvement in terms of large p-values compared to the corresponding estimates for ARFIMA. This development can also be attributed to the role of GARCH component in capturing volatility that the ARFIMA has failed to handle alone.

Forecasts Accuracy of ARFIMA-GARCH Models

In this study, RMSE and MAE are used to evaluate the forecast performance. Results given in Table 9 show that the ARFIMA (1,d,2)-GARCH(1,1) model produces a better forecast with minimum RMSE and MAE.

Table 9: Forecasts Accuracy Values of ARFIMA-GARCH Model

Candidate Models	RMSE	MAE
ARFIMA(1,d,1)-GARCH(1,1)	0.1131	0.1526
ARFIMA(1,d,2)-GARCH(1,1)	0.0214	0.0611

CONCLUSION

This paper considered long memory and volatility models in forecasting exchange rate of Nigerian Naira to United State Dollar. Monthly data for the period of January 2002 to February 2020 was used for this study. First and foremost, the stationarity of the exchange rate series was examined using ADF and KPSS tests which showed that the series is non-stationary. To make the exchange rate series stationary, fractional differencing was employed. Eight different ARFIMA models were estimated. Two best models, ARFIMA(1,d,1) and ARFIMA(1,d,2) were selected using AIC criterion. However, serial correlation and heteroscedasticity analyses using Portmanteau and ARCH-LM test indicated strong evidence of serial correlation and heteroscedasticity in the selected models residuals. These problem led to the estimation of the hybrid models, ARFIMA(1,d,1)-GARCH(1,1) and ARFIMA(1,d,2)-GARCH(1,1). Model estimation result reveals that; the hybrid models produced larger values of log-likelihood function and smaller values of standard errors than the ARFIMA models. These showed that the GARCH component has improved the models fitting of the exchange rate series and has captured volatility in the series adequately. Furthermore, the diagnostic tests of ARFIMA-GARCH indicated that; the model residuals are free from serial correlation and heteroscedasticity. Finally, the ARFIMA(1,d,1)-GARCH(1,1) is found to be the best model for forecasting Nigerian Naira to US Dollar with the lowest values of RMSE, MAE.

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