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Optimal Allocation of Buses to Routes Using Linear Programming as a Tool in a Transport Service Authority: A Case Study of Kano State Transport Authority, Nigeria

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Abstract
In this paper, new profit maximization for Kano State transport authority resulting from optimal allocation of buses to inter-state routes is considered taking into consideration all the constraints associated. The problem was modeled using linear programming and the TORA (a software for solving linear programming problems) was used to obtained the solution to the modeled problem. The maximum objective value of ₦2,203,900.00 was obtained daily after 16 iterations and this a better result when compared to the current traditional or intuitive schedule by the authority that yielded ₦2,036,000.00 daily. This recommended schedule will yield additional ₦167,900.00 daily and over ₦5,000,000.00 monthly when implemented.

Keywords: KSTA, TORA, Linear Programming, Transport Service, Sensitivity Analysis, Objective Value.

INTRODUCTION
Linear programming (LP) is an optimal decision making tool where the objective is a linear function and the constraints on the decision problems are also linear equalities and/or inequalities. It is the most commonly applied form of constrained optimization. The cardinal elements of any constrained optimization are decision variables, objective function, constraints and variable bounds. In LP, all the mathematical expressions for the objective function and constraints are linear. One might imagine that the restriction to linear models severely limits the ability to model real-world problems; but this is not so. An amazing range of problems can be modeled using LP from airline scheduling to least cost petroleum

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processing and distribution (Charnes, et al., 1953; Kulkarni et.al 2015; Nabasirye, et.al 2011) Chinnech, 2000; Ramsey, 2012).

According to Hiller et al. (1995), linear programming is a generalization of linear algebra use in modeling so many real life problems ranging from scheduling airline routes to shipping oil from refineries to cities for the purpose of finding solutions capable of meeting daily requirements. Hiller argued that the reason for the great versatility of linear programming is due to the ease at which constraints can be incorporated into the linear programming model.

Akpan and Iwok, (2016) reported that, linear programming plays an important role in improving management decision, despite that, it is still regarded as new science but it has proven to be capable of solving problems such as production planning, allocation of resources, inventory control and advertisement.

A linear programming problem must have a linear relationship between variables and constraints, the model must also have an objective function, structural constraint and a non-negativity constraint. Thus, the general form of a linear programming model with n decision variables and n constraints are given as follows:

Optimize (Max or Min) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subject to $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1$ (maximization)

$a_{21}x_2 + a_{22}x_2 + \ldots + a_{2n}x_n \ge b_2$ (minimization)

$a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n = b_3$ (equality)

$a_{1m}x_1 + a_{2m}x_2 + \ldots + a_{mn}x_n [\le \text{ or } \ge]\ b_m$

$x_i \ge 0, \forall i = 1,2,3,\ldots,n$ (Non- negativity restriction)

Linear programming helps in dealing with the problem of allocation of limited resources among different competitive activities in the most optimal manner. It is concerned with determining the optimal allocation of scarce resources to meet certain activities. However, Linear programming is applicable only to problem where the constraints and the objective function are linear. Reducing problems to a set of linear equation is usually very difficult (Nyor et al. 2014).

Brief History of Kano State Transport Authority (KSTA)

Kano State Transport Authority was established by the then military Governor, Col. Idris Garba in the year 1988. And it serves as one of the major means of transportation to various states in the country from Kano State.

KSTA is located at Naibawa along Police Station Street and is the terminus one of the KSTA while terminus two is at kofar Nasarawa. The head office is located at Sharada Phase II, beside vehicle inspection officers (VIOs) office, opposite of Ministry of Works.

KSTA has the following organizational chart:

Figure 1: KSTA Organization Chart

The staff strength of KSTA stands at about more than 100, under the Chief Executive/Managing Director Bashir Nasiru Aliko Koki.
According to KSTA rules, passengers are to note the following adherences:

1. The Authority does not accept liability for loss of goods. Passengers are therefore advised to take good care of their goods/properties while waiting to board our vehicles and while on transit.
2. Preaching is strictly prohibited in our vehicle.
3. The habit of smoking in our vehicle is strictly prohibited.
4. You can only enter our vehicle when you have paid correct money and obtained a ticket for the journey.
5. Ticket can only be issued to prospective commuters when they maintain a single queue.
6. Tickets should be in passengers' possessions until the end of the journey.
7. Heavy loads, bags and boxes are paid for and tickets obtained before they are loaded on the vehicle.
8. Intentional damage to our vehicle seats, glasses etc. will not be accepted.
9. Female passengers are seated at the back of the vehicle while male passengers are seated at the front.

Kano State Transport Authority (KSTA) is a state-owned transport service that operates in Kano, Kano State, Nigeria, and commuting inter-state routes service.
Therefore, this study covers the inter-state route service of the KSTA. These routes are as follows:

Kano – Yola, Kano – Taraba, Kano – Bida, Kano – Minna, Kano – Sokoto, Kano – Zamfara, Kano – Bauchi, Kano–Gombe, Kano – Benue, Kano – Lafia, Kano – Guru, Kano – Abuja, Kano – Katsina, Kano – Borno, and Kano – Jos.
The objective of the study was to apply linear programming model to optimally allocate the available buses of the Transport Authority to the service routes.
Application of linear programing model to routinely allocate the available buses of the transport Authority is the cardinal objective of this research. The simulated results obtained is compared to the current manual method of allocating the resources to observe the best method of allocation for optimum profit, since the profit maximization and service delivery is the target of every organization. Abubakar et al. (2020) carried out a research on ATS Multi-Concept Worldwide Ltd in Katsina State where recommendations were made to enhance the profit accrued to the company. The work by Nyor et al. (2014), Mula *et al.* (2005) and

Abdullahi et al. (2021) motivated us to carried this research. KSTA operates 15 inter State route services and has more than 250 fifteen seater HIACE buses among which not more than 50 are used for the routing services in the authority. According to research and interview with the KSTA officials weekly, the Buses incur costs in four ways: fuel consumption, percentage parking levy, routine service and maintenance (repair).

Sensitivity Analysis

Sensitivity analysis is a technique used for determining how the independent variable values will impact a specific dependent variable under a given assumption(s). It helps the researcher, reader and policy makers to know how sensitive a model is to changes in the value of parameters of the model and to change in the structure of the model (Dahiya et al. 2019; Mula et al.. *2005;* Lakhtaria 2012). Meanwhile, Goyal et al.. (2017) stated that sensitivity analysis dictates how the uncertainty in the output, or sensitivity is an output of a mathematical model or a complex system that can be assigned to different sources of uncertainty in its inputs or to change in an input while keeping the other inputs constant.

RESEARCH METHODOLOGY

The data below were collected on 23 January 2020 from KSTA headquarters which is located at Naibawa opposite Federal Road Safety Corps (FRSC) office. Data on Bus services were obtained from the interview with the KSTA scheduling officer, Mal. Usman I. Usman while data on services and maintenance were obtained from the interview with KSTA engineer in person of Mal. Rabiu Usman. Table 1: Data on Bus Route

S/N	Route	Freq. of Trips Per week	Fuel consumption in Liters	Percentage Parking levy (%)	No. of Buses Per Route Per Day	Max. No. of Trips per Bus Per Day	No. of Hours Per Half Trip	Transport Fare Per Person (₦)
1	Kano-Yola	4	145	10	4	1	9	3000
2	Kano-Taraba	4	172	10	3	1	11	4000
3	Kano-Bida	4	103	10	1	1	10	3000
4	Kano-Minna	4	90	10	4	1	8	2500
5	Kano-Sokoto	4	83	10	5	1	7	2000
6	Kano-Zamfara	4	48	10	3	1	4	1200
7	Kano-Bauchi	4	48	10	3	1	4	1200
8	Kano-Gombe	4	83	10	4	1	5	1800
9	Kano-Benue	4	145	10	2	1	9	3000
10	Kano- Lafia	4	124	10	1	1	6	2500
11	Kano-Guru	4	55	10	1	1	5	1200
12	Kano-abuja	4	83	10	3	1	6	1800
13	Kano-Katsina	4	48	10	5	2	2	500
14	Kano-Borno	4	103	10	5	1	6	2500
15	Kano-Jos	4	48	10	1	1	4	1500
TOTAL			1378	150	45	16	96	31700

Source: (KSTA, 2019)

From Table 1: Route: this column shows all the (15) inter states routes that KSTA buses ply. Frequency of Trips per week: This is the number of times a bus plies a particular route in a week. Note that trips mean (to and fro) the destination. For example, Kano-Yola trip means the bus has gone from Kano-Yola and back. A bus that has gone from Kano-Yola only has made half trip.

Table 2: Data on Bus Services

S/N	Required Service Items	Amount (₦)
1	Oil Filter	500
2	5-liter Engine oil	4800
3	Oil Treatment	600
	TOTAL	5900

Source: (KSTA, 2019)
Note: Buses in KSTA are serviced four times in a month, which is after 7 days.

Table 3: Data on Bus Repair /Maintenance

S/N	Repair/Maintenance Items	Cost (₦)	Duration it lasts
1	Tyre	108000	4 months
2	Front bearing	14000	3 months
3	Break disk	6000	6 months
4	Break pad	1500	2 months
5	Break lining	4000	1 months
6	Car battery	17000	6 months
7	Shocks filling	1500	2 months
8	Sparking plugs	3000	3 months
9	Fuel pump	3000	2 months
10	Release bearing	3000	6 month
	Total	161000	

Source: (KSTA, 2019)

MODEL CONSTRUCTION

In this section, the model developed based on the information provided by the officials of KSTA.

Table 4: Daily Cost of Bus Services

S/N	Required Service Items	Amount per week (₦)	Amount per month (₦)	Daily Services Cost Per bus (₦)
1	Oil Filter	500	2000	$2000 \div 30 = 66.7$
2	5-Litre Engine Oil	4800	19200	$19200 \div 30 = 640$
3	Oil Treatment	600	2400	$2400 \div 30 = 80$
	Total	5900	23600	787

Table 5: Daily Cost of Bus Repair /Maintenance

S/N	Repair/Maintenance items	Cost(₦)	Duration it last	Duration in Days	Cost Per Day(₦)
1	Tyre	108000	4 months	$4 \times 30 = 120$	900
2	Front Bearing	14000	3 months	$3 \times 30 = 90$	156
3	Break Disk	6000	6 months	$6 \times 30 = 180$	33
4	Break Pad	1500	2 months	$2 \times 7 = 14$	107
5	Break Lining	4000	1 months	$1 \times 30 = 30$	133
6	Car Battery	17000	6 months	$6 \times 30 = 180$	94
7	Shocks filling	1500	2 months	$2 \times 30 = 60$	25
8	Sparking plug	3000	3 months	$3 \times 30 = 90$	33
9	Fuel pump	3000	2 months	$2 \times 30 = 60$	50
10	Release Bearing	3000	6 months	$6 \times 3 = 180$	17
	TOTAL	161000			1548

Table 6. Daily Contribution per Route

S/N	Fuel consumption per Bus (Liters)	Cost of Fuel consumption per Bus (₦)	Percentage parking levy per Bus (%)	Monetary parking levy (₦)	Number of Buses per Route per Day	Max No. of Trips per Bus per Day	Equivalent No. of Buses Per Route Per Day	No. Of Hours Per Half Trip	Transport Fare Per Person	Daily cost of a Bus Services	Daily Cost of a Bus Repair/Maintenance	Return Per Bus Per Trip (N)	Daily Total Expenditure Per Bus (N)	Daily Contribution Per Bus (N)
1	145	21000	10	4500	4	1	4	9	3000	787	1548	90000	27835	62165
2	172	25000	10	6000	3	1	3	11	4000	787	1548	120000	33335	86665
3	90	13000	10	3750	4	1	4	8	2500	787	1548	75000	19085	55915
4	103	15000	10	4500	1	1	1	10	3000	787	1548	90000	21835	68165
5	83	12000	10	3000	5	1	5	7	2000	787	1548	60000	17335	42665
6	48	7000	10	1800	3	1	3	4	1200	787	1548	36000	11135	24865
7	48	7000	10	1800	3	1	3	4	1200	787	1548	36000	11135	24865
8	83	12000	10	2700	4	1	4	5	1800	787	1548	54000	17035	36965
9	145	21000	10	4500	2	1	2	9	3000	787	1548	90000	27835	62165
10	124	18000	10	3750	1	1	1	6	2500	787	1548	75000	24085	50915
11	55	8000	10	1800	1	1	1	5	1200	787	1548	36000	11135	24865
12	83	12000	10	2700	3	1	3	6	1800	787	1548	54000	17035	36965
13	48	7000	10	750	5	2	10	2	500	787	1548	15000	10085	4915
14	103	15000	10	3750	5	1	5	6	2500	787	1548	75000	21085	53915
15	48	7000	10	2250	1	1	1	4	1500	787	1548	45000	11585	33415
TOTAL:	1378	200000	150	47550	45	16	50	96	31700	11805	23220	951000	281575	669425

Table 7. Daily Contribution per Route

S/N	Route	Monetary perking levy per route (₦)	Equivalent No. of Buses per route per day	Fuel Consumption Per Route (₦)	Transport fare per person (₦)	Daily cost of bus services per route (₦)	Daily Cost of Repair/Maintenance (₦)	Return per Route (₦)	Daily total expenditure per route (₦)	Daily Contribution per Route (₦)
1	Kano-Yola	18000	4	84000	3000	3148	6192	360000	111340	248660
2	Kano-Taraba	18000	3	75000	4000	2361	4644	360000	100005	259995
3	Kano-Bida	4500	1	15000	3000	787	1548	90000	21835	68165
4	Kano-Minna	15000	4	52000	2500	3148	6192	300000	76340	223660
5	Kano-Sokoto	15000	5	60000	2000	3935	7740	300000	86675	213325
6	Kano-Zamfara	5400	3	21000	1200	2361	4644	108000	33405	74595
7	Kano-Bauchi	5400	3	21000	1200	2361	4644	108000	33405	74595
8	Kano-Gombe	10800	4	48000	1800	3148	6192	216000	68140	147860
9	Kano-Benue	9000	2	42000	3000	1574	3096	180000	55670	124330
10	Kano-Nasarawa	3750	1	18000	2500	787	1548	75000	24085	50915
11	Kano-Guru	1800	1	8000	1200	787	1548	36000	12135	23865
12	Kano-Abuja	8100	3	36000	1800	2361	4644	162000	51105	110895
13	Kano-Katsina	7500	10	70000	500	7870	15480	150000	37850	112150
14	Kano-Borno	18750	5	75000	2500	3935	7740	375000	105425	269575
15	Kano-Jos	2250	1	7000	1500	787	1548	45000	11585	33415
	TOTAL	14,3250	50	632,000	31,700	39,350	77,400	2,865,000	829,000	2,036,000

KSTA Problem Formulation

The problem formulation is based on the information under the problem situation and Table 6. The problem is formulated under the assumption that all the 50 buses that commute the inter states route are working daily.

Based on the interview with KSTA, availability of passengers and the number of other transport services plying the same routes, determine the number of buses that KSTA can assign to these routes. The following shows the possible number of buses that can be scheduled to the routes:

Table 8: List of the routes and possible number of buses allocated

Inter – State Routes	Possible No. of Buses	Inter – State Routes	Possible No. of Buses
Kano – Yola	4	Kano – Benue	2-3
Kano – Taraba	3-4	Kano – Lafia	1
Kano – Bida	1	Kano – Guru	1
Kano – Minna	3-5	Kano – Abuja	2-3
Kano – Sokoto	5	Kano – Katsina	10
Kano – Zamfara	3-4	Kano – Borno	5
Kano – Bauchi	3-4	Kano – Jos	1
Kano – Gombe	4		

It should be strongly noted that our allocation must not exceed 50 buses available for the inter-state route services.

Below is our problem formulation:

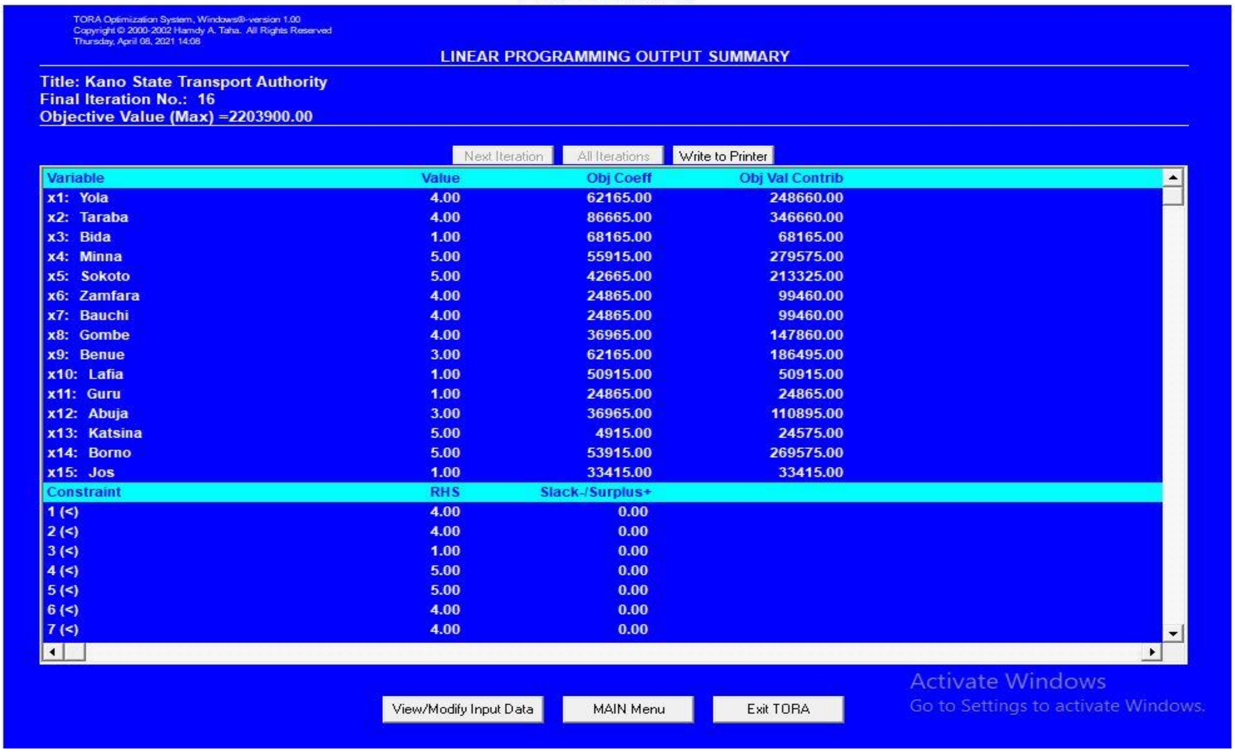
$$62165x_1 + 86665x_2 + 55915x_3 + 68165x_4 + 42665x_5 + 24865x_6 + 24865x_7 + 36965x_8 + 62165x_9 + 50915x_{10} + 24865x_{11} + 36965x_{12} + 4915x_{13} + 53915x_{14} + 33415x_{15}$$

$$x_1 \le 4$$
$$x_2 \le 4$$
$$x_3 \le 1$$
$$x_4 \le 5$$
$$x_5 \le 5$$
$$x_6 \le 4$$
$$x_7 \le 4$$
$$x_8 \le 4$$
$$x_9 \le 3$$
$$x_{10} \le 1$$
$$x_{11} \le 1$$
$$x_{12} \le 3$$
$$x_{13} \le 10$$
$$x_{14} \le 5$$
$$x_{15} \le 1$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \le 50$$

NUMERICAL RESULTS AND DISCUSSION

In this section, result of the Problem formulated will be analyzed and discussed. The formulated problem was solved using TORA – computer software used in solving Linear Programming problems. The software was developed by Taha (2002).

LINEAR PROGRAMMING


TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Thursday, April 08, 2021 14:08

LINEAR PROGRAMMING OUTPUT SUMMARY

Title: Kano State Transport Authority
Final Iteration No.: 16
Objective Value (Max) =2203900.00

Variable	Value	Obj Coeff	Obj Val Contrib
x1: Yola	4.00	62165.00	248660.00
x2: Taraba	4.00	86665.00	346660.00
x3: Bida	1.00	68165.00	68165.00
x4: Minna	5.00	55915.00	279575.00
x5: Sokoto	5.00	42665.00	213325.00
x6: Zamfara	4.00	24865.00	99460.00
x7: Bauchi	4.00	24865.00	99460.00
x8: Gombe	4.00	36965.00	147860.00
x9: Benue	3.00	62165.00	186495.00
x10: Lafia	1.00	50915.00	50915.00
x11: Guru	1.00	24865.00	24865.00
x12: Abuja	3.00	36965.00	110895.00
x13: Katsina	5.00	4915.00	24575.00
x14: Borno	5.00	53915.00	269575.00
x15: Jos	1.00	33415.00	33415.00

Constraint	RHS	Slack-/Surplus+
1 (<)	4.00	0.00
2 (<)	4.00	0.00
3 (<)	1.00	0.00
4 (<)	5.00	0.00
5 (<)	5.00	0.00
6 (<)	4.00	0.00
7 (<)	4.00	0.00

Figure 2: KSTA summary output of the solution of formulated problem

Constraint	RHS	Slack-/Surplus+
1 (<)	4.00	0.00
2 (<)	4.00	0.00
3 (<)	1.00	0.00
4 (<)	5.00	0.00
5 (<)	5.00	0.00
6 (<)	4.00	0.00
7 (<)	4.00	0.00
8 (<)	4.00	0.00
9 (<)	3.00	0.00
10 (<)	1.00	0.00
11 (<)	1.00	0.00
12 (<)	3.00	0.00
13 (<)	10.00	5.00-
14 (<)	5.00	0.00
15 (<)	1.00	0.00
16 (<)	50.00	0.00

Figure 3: KSTA summary output of the solution of the formulated problem.

Figures 2 and 3 show the summary output of the linear programming formulated. The variables , , represent Yola, Taraba, Bida, Minna, Sokoto, Zamfara, Bauchi, Gombe, Benue, Lafia, Guru, Abuja, Katsina, Borno and Jos respectively. The optimal solution of the problem under consideration was reached after 16th iterations with objective value of ₦2,203,900 as it can be seen on the TORA window screen. Each objective value contribution is obtained by multiplying number of final allocation of buses (value) and objective value contribution.

Sensitivity Analysis

Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1: Yola	62165.00	4915.00	infinity	0.00
x2: Taraba	86665.00	4915.00	infinity	0.00
x3: Bida	68165.00	4915.00	infinity	0.00
x4: Minna	55915.00	4915.00	infinity	0.00
x5: Sokoto	42665.00	4915.00	infinity	0.00
x6: Zamfara	24865.00	4915.00	infinity	0.00
x7: Bauchi	24865.00	4915.00	infinity	0.00
x8: Gombe	36965.00	4915.00	infinity	0.00
x9: Benue	62165.00	4915.00	infinity	0.00
x10: Lafia	50915.00	4915.00	infinity	0.00
x11: Guru	24865.00	4915.00	infinity	0.00
x12: Abuja	36965.00	4915.00	infinity	0.00
x13: Katsina	4915.00	0.00	24865.00	0.00
x14: Borno	53915.00	4915.00	infinity	0.00
x15: Jos	33415.00	4915.00	infinity	0.00
Constraint	**Current RHS**	**Min RHS**	**Max RHS**	**Dual Price**
1 (<)	4.00	0.00	9.00	57250.00
2 (<)	4.00	0.00	9.00	81750.00
3 (<)	1.00	0.00	6.00	63250.00
4 (<)	5.00	0.00	10.00	51000.00
5 (<)	5.00	0.00	10.00	37750.00
6 (<)	4.00	0.00	9.00	19950.00

Figure 4: Sensitivity Analysis Report

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<)	4.00	0.00	9.00	57250.00
2 (<)	4.00	0.00	9.00	81750.00
3 (<)	1.00	0.00	6.00	63250.00
4 (<)	5.00	0.00	10.00	51000.00
5 (<)	5.00	0.00	10.00	37750.00
6 (<)	4.00	0.00	9.00	19950.00
7 (<)	4.00	0.00	9.00	19950.00
8 (<)	4.00	0.00	9.00	32050.00
9 (<)	3.00	0.00	8.00	57250.00
10 (<)	1.00	0.00	6.00	46000.00
11 (<)	1.00	0.00	6.00	19950.00
12 (<)	3.00	0.00	8.00	32050.00
13 (<)	10.00	5.00	infinity	0.00
14 (<)	5.00	0.00	10.00	49000.00
15 (<)	1.00	0.00	6.00	28500.00
16 (<)	50.00	45.00	55.00	4915.00

Figure 5: Sensitivity Analysis Report

Observing from the Figures 4 and 5, the sensitivity analysis report. Sensitivity analysis helps the researcher, reader and policy makers to know how sensitive a model is to changes in the value of parameters of the model and to change in the structure of the model. From the above figures, more buses allocation can be assigned to these routes in questions except for Kano-Katsina route for more profit maximization. The table show that based on the availability, buses can increase for Kano-Yola from current recommended 4 to up to maximum 9, Kano-Taraba 4 to up to maximum 9, Kano- Bida 1 to up to maximum 6, Kano-Minna 5 to up to maximum 10, Kano-Sokoto 5 to up to maximum 10, Kano- Zamfara 4 to up to maximum 9, Kano-Bauchi 4 to up to maximum 9, Kano-Gombe 4 to up to maximum 9 , Kano-Benue 3 to up to maximum 8, Kano-Lafia 1 to up to maximum 6, Kano-Guru 1 to up to maximum 6, Kano-Abuja 3 to up to maximum 8, Kano- Borno 5 to up to maximum 10 and Kano-Jos 1 to up to maximum 6 to make more profit for the State. The corresponding dual prices are obtained in the sensitivity analysis table.

Table 9: Current KSTA versus Recommendation schedules

S/No.	Route	KSTA Current Schedule	Recommended Schedule	S/No.	Route	KSTA Current Schedule	Recommended Schedule
1	Kano – Yola	4	4	9	Kano – Benue	2	3
2	Kano – Taraba	3	4	10	Kano – Lafia	1	1
3	Kano – Bida	1	1	11	Kano – Guru	1	1
4	Kano – Minna	4	5	12	Kano – Abuja	3	3
5	Kano – Sokoto	5	5	13	Kano – Katsina	10	5
6	Kano – Zamfara	3	4	14	Kano – Borno	5	5
7	Kano – Bauchi	3	4	15	Kano – Jos	1	1
8	Kano – Gombe	4	4				

Table 9 shows the current KSTA bus schedule from Kano to various routes outlined in the table as against the recommended schedule obtained for maximum profit after finding solution to the modeled problem.

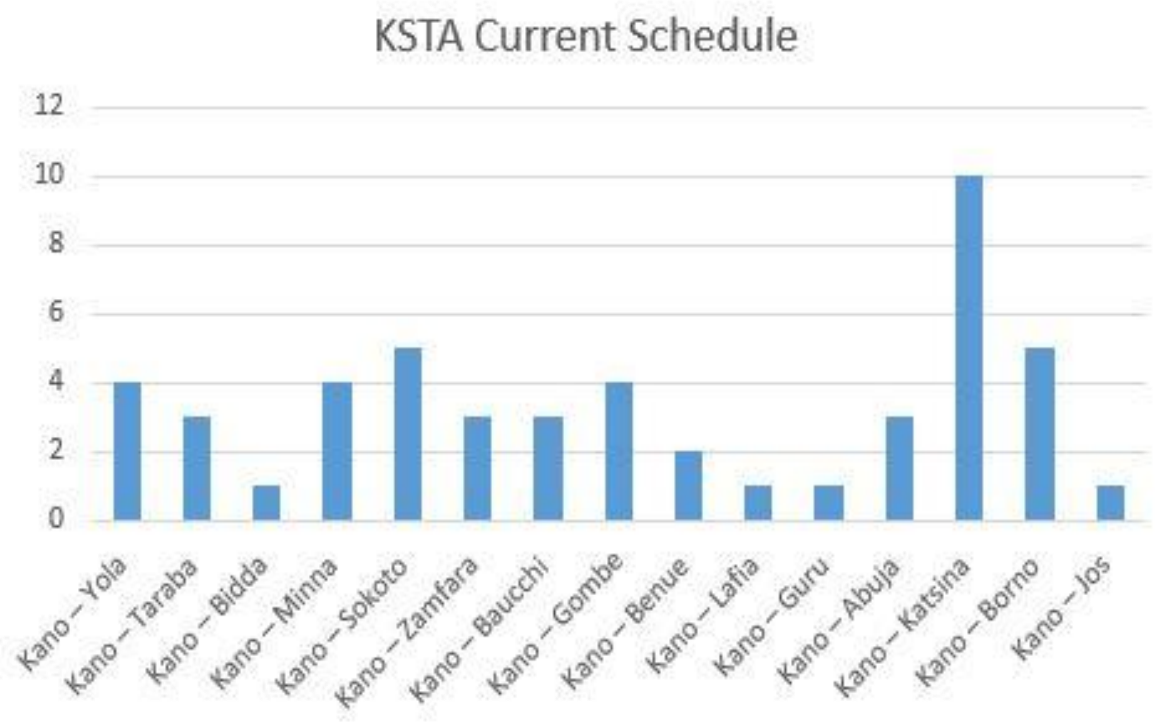


Figure 5: Current Schedule of KSTA

Figure 6: Recommended Schedule of KSTA

Figures 5 and 6 show the traditional or intuitive way of scheduling and recommend scheduling results obtained from the solution of the formulated model respectively. The recommended chart suggests that some allocation of buses should be either increased, decreased or be left as it was to achieve maximum profit possible.

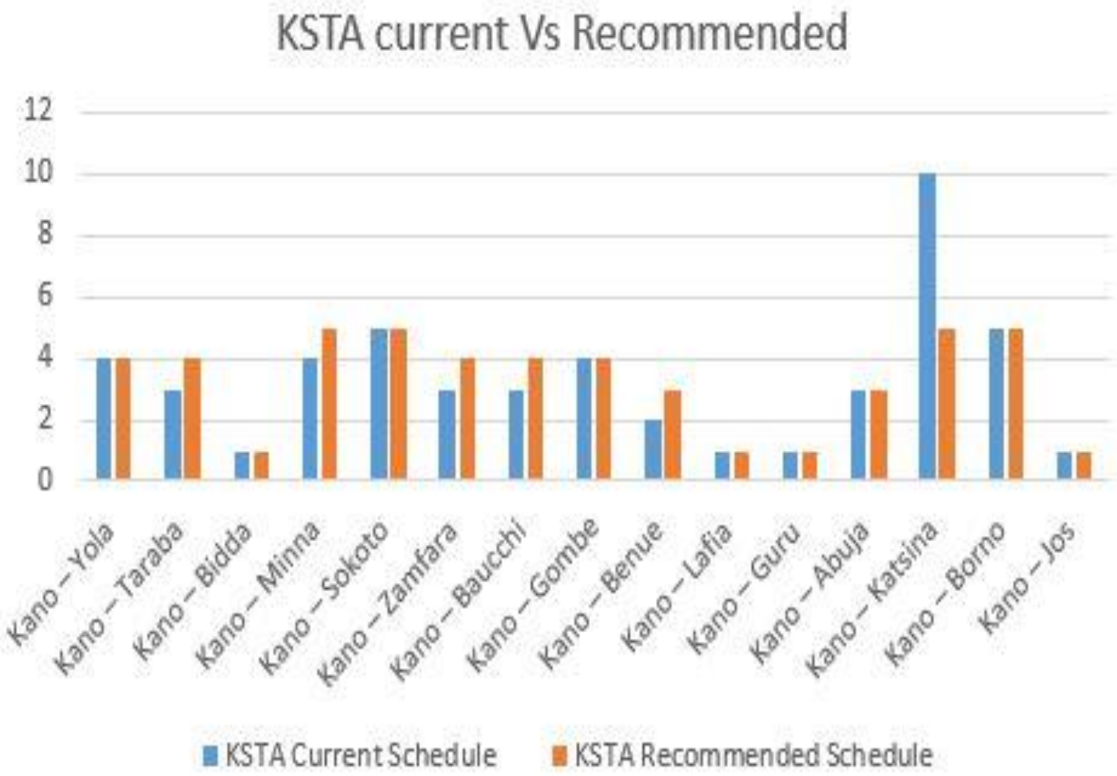


Figure 7: Current versus Recommended Schedule for KSTA

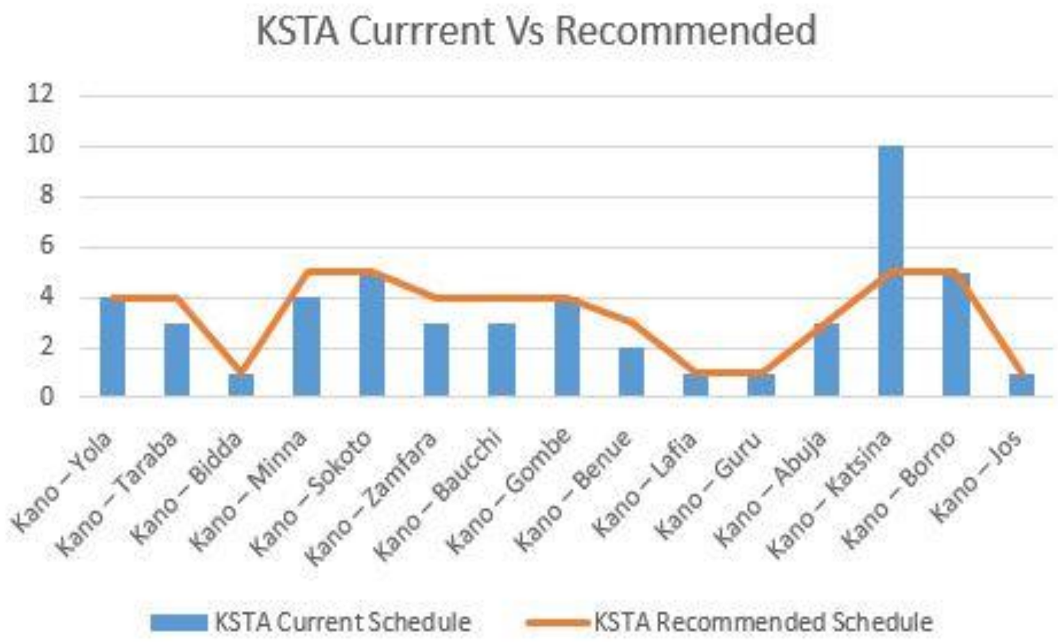


Figure 8: Current versus Recommended Schedule for KSTA.

Figures 7 and 8 is the combination of Figures 5 and 6 on the same chart for clearer understanding of recommendation. From the two charts above, it is observed that number of buses allocated to Yola, Sokoto, Gombe, Lafia, Guru, Abuja, Borno and Jos routes from Kano should remain as they were in the current schedule. However, allocation of buses to some routes should increase for instance, Taraba, Minna, Zamfara, Bauchi, and Benue to maximize more profit. Surprisingly, from the solution obtained it suggested that allocation of buses to Katsina be reduced from current 10 buses to 5 buses. This can be justified by minimal profit recorded from that route considering daily expenditure and contribution on the route.

CONCLUSION

The current KSTA schedule as seen in Table 7 yielded a daily contribution of ₦ 2,036,000. Meanwhile, our recommended schedule will yield ₦2,203,900 when implemented. Also, considering the sensitivity analysis report more profit can be recorded with more availability of the buses to the routes if the recommendation is followed strictly. The current solution of the model shows that KSTA could getting additional ₦ 167,900 daily and over ₦5,000,000 monthly and this is by no means a small amount accrued to a State from a transportation unit when implemented and it would help Kano state to boost internally generated revenue (IGR).

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