

# Forecasting the Exchange Rate of Nigerian Naira to United State' Dollar using Arima-Garch Model

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## Abstract

*In order to model and forecast exchange rates in both developed and emerging countries, majority of time series analysts have employed various technical and fundamental approaches, the forecast outcome differs depending on the approach chosen or implemented. In this view, this study is about hybridization of Autoregressive Integrated Moving Average (ARIMA) with Generalized Autoregressive Conditional Heteroscedastic (GARCH) model in forecasting exchange rate using monthly data of the Nigerian Naira against the U.S. Dollar for the period of January 2002 to February 2020. The stationarity of the exchange rate series is examined using unit root test of Augmented Dickey Fuller (ADF) test and Kwiatkowski-Philips-Schmidt-Shin (KPSS) which showed that the series is non stationary. To make the exchange rate series stationary, the data was transformed by first differencing and appropriate ARIMA models were obtained using Box-Jenkins method. ARIMA (0,1,1) and ARIMA(0,1,2) models were selected using AIC criteria and the residuals of these models were found to be serially correlated and heteroscedastic; hence the need for the hybridization of ARIMA with GARCH model. Therefore ARIMA models were hybridized with GARCH(1,1) to form ARIMA(0,1,1)-GARCH(1,1) and ARIMA(0,1,2)-GARCH(1,1). The results of forecast performance indicates that the best model is ARIMA(0,1,1)-GARCH(1,1) which has the lowest Root Means Square Error (RMSE) and Mean Absolute Error (MAE).*

**Keywords:** Exchange Rate, Trend, Volatility, ARIMA and ARIMA-GARCH.

## INTRODUCTION

Modeling and forecasting of macroeconomic variables have been a challenge to economists and policymakers. Though various models of estimating and forecasting economic variables have been developed, however it requires a lot of effort to identify a suitable model for a given series. This is due to the fact that; time series data behaves differently and exhibit different characteristics over time. For example, when time series data is non-stationary, Autoregressive Integrated Moving Average (ARIMA) model can be a good choice for trend analysis. This model is capable of eliminating patterns because it allows the series to be differenced (Box and Jenkins, 1976). If there is presence of volatility in the series, variance models such as, Generalized Autoregressive Conditional Heteroscedasticity

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(GARCH) can be employed to handle the volatility components(Lawal, Umaru and Yar', 2013).

The ARIMA-GARCH model is a combination of two models: ARIMA and GARCH. ARIMA is a model that considers the mean behavior of a time series (trend components), and GARCH is a variance model that uses the residual series from the fitted ARIMA to model the variance behavior (ARCH effect)(Yaziz *et al.*, 2013).

Nwankwo (2014) applied the ARIMA model to the annual exchange rate (Naira to Dollar) within the periods 1982-2011. Different ARIMA models were fitted and ARIMA (1,1,0) model was found to be appropriate to study the data because it had the lowest AIC. Similarly, the diagnostic analysis identified the selected model as the best fit to the data. Musa *et al.*,(2014) investigated the volatility modeling of daily Dollar/Naira exchange rate using GARCH, Glosten Jagannathan Runkle GARCH (GJR-GARCH), Threshold GARCH (TGARCH) and Taylor Schwert GARCH (TS-GARCH models by using daily data from June 2000 to July 2011. It was found that the GJR-GARCH and TGARCH models showed the existence of statistically significant asymmetric effect.The forecasting ability was subsequently assessed using the symmetric lost functions which are the Mean Absolute Error (MAE), Root Mean Absolute Error (RMAE), Mean Absolute Percentage Error (MAPE) and their inequality coefficients. The results showed that TGARCH model provide the most accurate forecasts. This model captured all the necessary stylized facts such as persistent, volatility clustering and asymmetric effects.

In order to model and forecast exchange rates in both developed and emerging countries, majority of time series analysts have employed various technical and fundamental approaches, the forecast outcome differs depending on the approach chosen or implemented (Onasanya and Adeniji, 2013). Some researchers applied ARIMA to study trend without considering volatility component (see Nwankwo (2014)) and some applied GARCH to study volatility without considering trend component (see Musa *et al.*,(2014)). In these view, this study is aim at hybridizing ARIMA with GARCH model in forecasting exchange rate using monthly data of the Nigerian Naira against the U.S. in order to study trend and volatility simultaneously because combining these models improves forecasting accuracy and are effective in overcoming the limitations of the model components (Dritsaki, 2018).

## METHODOLOGY

### Data Collection and Description

The data for this study is a secondary data obtained from Central Bank of Nigeria website. There are 218 data points from January 2002 to February 2020. The data was analyzed using R statistical software.

### Stationarity Test

In this study, we used Augmented Dickey Fuller Test (ADF) and Kwiatkowski-Philips-Schmidt-Shin (KPSS) Test for unit root test.

### Augmented Dickey Fuller (ADF) Test

The ADF test has been proposed by Dickey and Fuller (1981) with the null hypothesis that the data generating process is non stationary. The ADF test is carried out by first considering autoregressive model of order one as follows:

$$y_t = c + \gamma_t + \phi_1 y_{t-1} + \varepsilon_t. \quad (1)$$

The ADF add lag differences to these models so that

$$\Delta y_t = c + \gamma_t + \phi_1 y_{t-1} + \delta_1 \Delta y_{t-1} \dots + \delta_p \Delta y_{t-p} + \varepsilon_t,$$

$$= c + \gamma_t + \phi_1 y_{t-1} + \sum_{i=1}^p \delta_i \Delta y_{t-i} + \varepsilon_t, \tag{2}$$

where  $c$  is a constant,  $\gamma$  is the coefficient on a time trend,  $p$  is the lag order of the autoregressive model and  $\varepsilon_t \sim i.i.d(0, \sigma_\varepsilon^2)$ .

**Kwiatkowski-Philips-Schmidt-Shin(KPSS)Test**

This test was proposed by Kwiatkowski *et. al.*(1992). The null hypothesis is stated as: the data generating process is stationary. If the data is without a linear trend, the test assumes the following data generating process:

$$X_t = y_t + z_t \tag{3}$$

where  $y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + v_t$ ,  $v_t \sim i.i.d(0, \sigma_v^2)$  and  $z_t$  is a stationary process. The test statistic is given by:

$$KPSS = \frac{1}{T^2} \sum_{t=1}^T \frac{s_t^2}{\hat{\sigma}_\infty^2} \tag{4}$$

where  $s_t = \sum_{j=1}^t \hat{w}_j$  with  $\hat{w}_t = y_t - y$  and  $\hat{\sigma}_\infty^2$  is an estimator of the long run variance of the process  $z_t$ .

**Autoregressive Integrated Moving Average (ARIMA) Model**

The non-seasonal ARIMA model is basically denoted as ARIMA(p,d,q) model, where the parameters p,d,q are the AR, differenced and MA terms greater than or equal to zero respectively.

The general form of the ARIMA model using the backward shift operator is:

$$\phi(B)(1 - B)^d y_t = \theta(B)\varepsilon_t \tag{5}$$

**GARCH Model**

The GARCH (r,s) model's variance equation can be written as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^r \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^s \beta_j \sigma_{t-j}^2, \tag{6}$$

where  $\sigma_{t-j}^2$  is the volatility at day  $t - j$ ,  $\omega > 0$ ,  $\alpha_i \geq 0$  for  $i = 1, \dots, r$ , and  $\beta_j \geq 0$  for  $j = 1, \dots, s$ , are the model's parameters be estimated according to the GARCH (r,s) model,  $\varepsilon_t$ ,  $\sigma_t^2$ , conditional variance is dependent on the squared innovations in the previous  $r$  periods, as well as the conditional variance over the prior  $s$  periods. To establish a satisfactory volatility model fit for financial time series, the GARCH models are sufficient.

**THE ARIMA-GARCH Model**

The ARIMA-GARCH model is used to study trend and volatility of a time series concurrently (Dritsaki, 2018). The ARIMA (p,d,q)-GARCH (r,s) model can be defined as follows:

$$\phi(L)(1 - L)^d Y_t = \theta(L)\varepsilon_t, \tag{7}$$

$$\varepsilon_t | \psi_{t-1} \sim N(0, \sigma_t^2), \tag{8}$$

where  $\sigma_t^2$  can be represented as (6) and  $\psi_{t-1} = Y_1, Y_2, \dots, Y_{t-1}$ .

### Model selection criterion

The Akaike Information Criterion (AIC) is a measure of the relative goodness of fit of a statistical model as well as the order of the model. The formula for the AIC is:

$$AIC = 2k - 2 \ln(L), \quad (9)$$

where  $k$  is the number of parameters in the statistical model, and  $L$  is the maximized value of the likelihood function for the estimated model.

### Diagnostic tests

For a mean and hybrid modeling, it is important to test for serial correlation and presence of heteroscedasticity after model's estimation. In this study, we consider a number of these tests.

### Autoregressive Conditional Heteroscedastic-Lagrange Multiplier (ARCH -LM) Test

Engle (1982) proposed the ARCH-LM test to cater for issues of conditional heteroscedasticity in squared residuals under the null hypothesis that there is no heteroscedasticity in the model residuals and the test statistic is

$$Q = N(N + 2) \sum_{i=1}^N \frac{\rho_i}{(N-i)}, \quad (10)$$

where the statistic,  $Q$  has an asymptotic  $\chi^2$  distribution with  $n$  degrees of freedom if the squared residuals is uncorrelated.  $N$  is the number of observation and  $\rho_i$  is the sample correlation coefficient between squared residuals  $\hat{\varepsilon}_t^2$  and  $\hat{\varepsilon}_{t-1}^2$ . The null hypothesis of squared residuals is that;  $\{\hat{\varepsilon}_t^2\}$  are uncorrelated.

### Portmanteau Test

The Portmanteau test investigates the presence of autocorrelation in the residuals of a fitted model. Suppose the autocorrelation between  $\varepsilon_t$  and  $\varepsilon_{t-k}$  is  $\rho_k = \text{Corr}(\varepsilon_t, \varepsilon_{t-k})$ . Then, the null hypothesis states that all lags correlation are zero,  $H_0: \rho_1 = \rho_2 = \dots = \rho_k = 0$ . The test statistic is the modified  $Q$  statistic and is given by:

$$Q_1 = m(m + 2) \sum_{j=1}^K (m - j)^{-1} \hat{\rho}_{\hat{\varepsilon}}^2(j). \quad (11)$$

The  $Q_1$  statistic approximately follows a  $\chi^2$  distribution with  $K - p - q$  degrees of freedom.

### Measures of Forecast Accuracy

Assessing the accuracy of forecasts obtained from time series model is an important but a difficult task. To handle this vital stage of modeling, this section presents two measures of forecasting performance. These are Root Mean Square Error (RMSE) and Mean Absolute Error (MAE).

### Root Mean Square Error

The RMSE is defined as the estimate of deviation of errors in forecasting. It is calculated by obtaining the square root of the squared difference between the observed and predicted values. A smaller RMSE indicates a better model estimate.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (Y_t - \hat{Y}_t)^2}. \quad (12)$$

### Mean Absolute Error

The MAE is regularly used for examining the suitability of time series model. The MAE is express by:

$$MAE = \frac{1}{n} \sum_{i=1}^n |Y_i - \hat{Y}_i|. \quad (13)$$

## RESULTS AND DISCUSSION

### Stationarity Test

When modeling time series data, the first step is to check if the data under study is stationary.

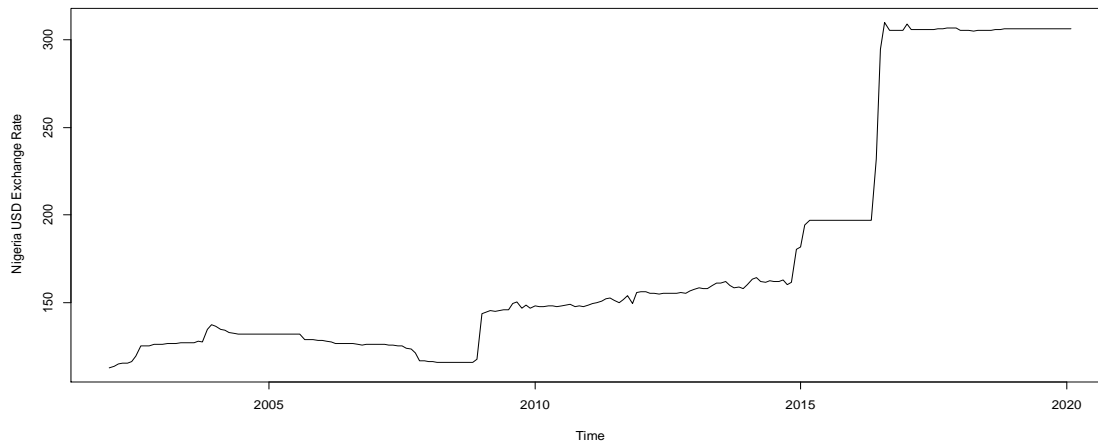


Figure 1: Time Series plot of Exchange Rate

The time series plot of the Exchange Rate in Figure 1 exhibits trend indicating that the monthly exchange rate is changing over time.

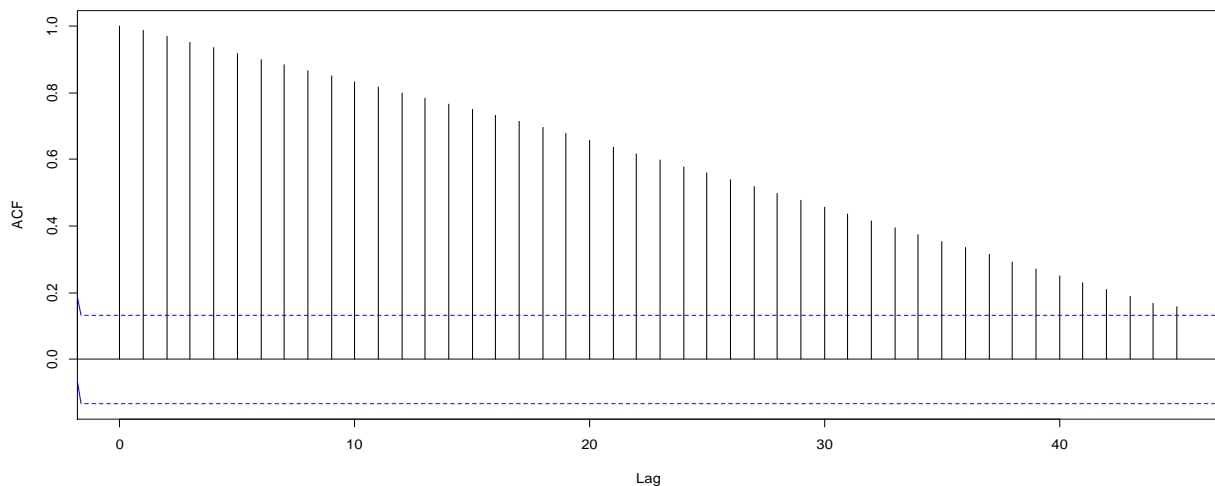


Figure 2: The ACF plot of Exchange Rate

The ACF in Figure 2 shows a slow decay in the Exchange rate which is an evidence of non-stationary series.

Table 1: ADF and KPSS test of Exchange Rate

Test	Statistic	P-Value	Decision	Remark
ADF	-1.6664	0.716	Fail to reject	Nonstationary
KPSS	3.4033	0.01	Reject	Nonstationary

The ADF and KPSS non-stationary test results as displayed in Table 1 shows that the Exchange Rate is non-stationary. The p-values of the ADF and KPSS test are greater and

less than 0.05 respectively. The null hypothesis for the ADF and KPSS test are accepted and rejected respectively. The non-stationary test indicates that the ARIMA is the appropriate model to be used for studying the Exchange Rate series.

**Differencing**

Since non-stationary time series exhibits trend characteristics as shown in Figure 1 and 2 as well as Table 1, it is important to transform the time series by first differencing. The transformation will help in stabilizing the mean and variance of the time series, and thus removing the trend.

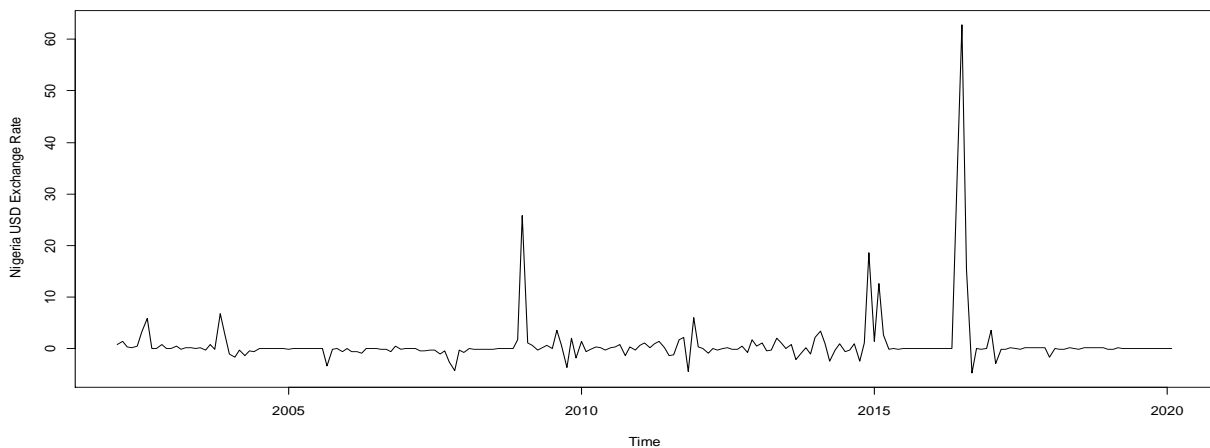


Figure 3: Time Series Plot of the First Differenced Exchange Rate

The plot in Figure 3 shows that the series is stationary after first differencing was carried out. In addition, the first differenced series indicate periods of high volatility in the MNGNUSDER in late 2009, early 2015 and 2016. It is also called the evidence of volatility clustering (Ding, 2011). The volatility clustering may be due to shock caused by Nigerian government policies of naira devaluation or otherwise.

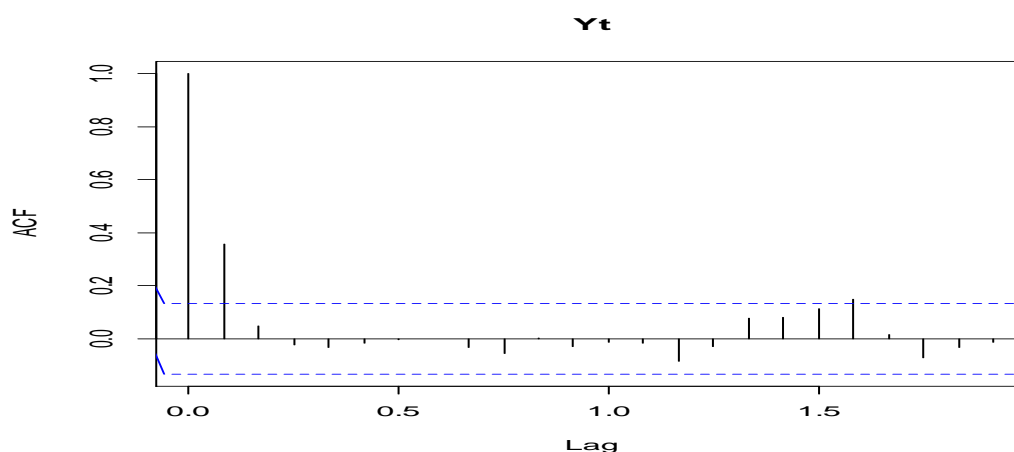


Figure 4: The ACF for First Differenced Exchange Rate

Figure 4: ACF indicates that the MNGNUSDER is stationary after the first differencing as most of the correlation values are within the confidence limit.

Table 2: ADF and KPSS test of Transformed Exchange Rate

Series	Test	Statistic	P-Value	Decision	Remark
First Differenced	ADF	-1.6664	0.001	Reject	Stationary
First Differenced	KPSS	3.4033	0.261	Fail to Reject	Stationary

The ADF and KPSS non-stationary test results as displayed in Table 2 show that the transformed Exchange Rate is stationary. All The p-values of the ADF and KPSS test are less and greater than 0.05 respectively. The null hypothesis for the ADF and KPSS test are rejected and accepted respectively.

### ARIMA Modeling

Now that the trend and variability in the Exchange Rate are stabilized, the ARIMA modeling is carried out in this section following the Box and Jenkins modeling approach. The approach involves the model identification, estimation and diagnostic analysis.

### ARIMA Model Identification

The AIC criterion of the ARIMA models are displayed in Table 3.

Table 3:AIC Values for ARIMA(*p,d,q*) Models

S/N	ARIMA( <i>p,d,q</i> )	AIC
1	ARIMA(1,1,0)	1317.78
2	<b>ARIMA(0,1,1)</b>	<b>1310.38</b>
3	ARIMA(1,1,1)	1310.39
4	ARIMA(2,1,0)	1311.12
5	<b>ARIMA(0,1,2)</b>	<b>1310.27</b>
6	ARIMA(2,1,1)	1312.26
7	ARIMA(1,1,2)	1312.26
8	ARIMA(2,1,2)	1314.26

The AIC of the identified ARIMA models are displayed in Table 3. The values are compared and ARIMA(0,1,1) and ARIMA(0,1,2) are the models with the minimum AIC. This suggests the two models are a better fit for Exchange Rate.

### Parameter Estimation of ARIMA Models

The parameters of the ARIMA were estimated using the Exchange Rate series. The results of the estimated parameters of ARIMA model for the studied series and their log-likelihood value are given in Table 4.

Table 4: Estimation of ARIMA (*p,d,q*) with their Log-likelihood Values

ARIMA( <i>p,d,q</i> )	$\theta_1$	$\theta_2$	Log-Lik.
ARIMA(0,1,1)	0.52(0.052)	-----	-689.19
ARIMA(0,1,2)	0.57(0.068)	0.10(0.067)	-672.14

Note: all standard error values are in parenthesis except first.

All the parameters in the ARIMA model are significant due to minimum standard error values.

**The Diagnostic Tests of ARIMA Models**

The results of the serial correlation and heteroscedascity analyses for ARIMA are shown in Table 5.

Table 5: Diagnostic Tests of the ARIMA (p,d,q) Models.

ARIMA(p,d,q)	Portmanteau Test	ARCH-LM Test
ARIMA(0,1,1)	26.43(0.000)	52.71(0.000)
ARIMA(0,1,2)	33.76(0.000)	63.84(0.000)

Portmanteau and ARCH-LM test at lag 40 and p-values are in parenthesis and null hypothesis is rejected at 5% levels.

The tests are based on the Ljung-Box test, and Engle heteroscedastic known as ARCH-LM test. Results show presence of serial correlation and heteroscedasticity in the residuals of ARIMA models because the P-values are less than the 0.05 significance level. In view of this, hybridizing ARIMA with GARCH to form ARIMA- GARCH model has to be considered in order to cater for the heteroscedasticity in the residuals of the fitted ARIMA models.

**The Estimation of ARIMA-GARCH**

The results of the estimated parameters of ARIMA-GARCH models with their log-likelihood values are given in Table 6.

Table 6: The Estimation of ARIMA (p,d,q)-GARCH (1,1)

Parameters	ARIMA(0,1,1)-GARCH(1,1)	ARIMA(0,1,2)-GARCH(1,1)
$\theta_1$	0.52(0.052)	0.57(0.068)
$\theta_2$	-----	0.10(0.067)
$\omega$	10.49(10.781)	10.46(10.531)
$\alpha_1$	0.92(0.1.092)	0.87(3.599)
$\beta_1$	0.00(0.266)	0.00(0.913)
Log-Lik.	-593.6715	-592.6995

Note: all standard error values are in parenthesis except first.

**The Diagnostic Tests of ARIMA (p,d,q)-GARCH (1,1)**

The serial correlation and heteroscedascity analyses of the ARIMA-GARCH models are shown in Table 7.

Table 7: Diagnostic Tests of the ARIMA(p,d,q)-GARCH(1,1) Models.

ARIMA(p,d,q)-GARCH	Portmanteau Test	ARCH-LM Test
ARIMA(0,1,1)-GARCH(1,1)	32.67(0.214)	61.28(0.307)
ARIMA(0,1,2)-GARCH(1,1)	48.95(0.311)	69.19(0.389)

Portmanteau and ARCH-LM test at lag 40 and p-values are in parenthesis and null hypothesis is rejected at 5% levels.



The diagnostic tests show better improvement in terms of large p-values compared to the corresponding estimates for ARIMA models. This development can also be attributed to the role of GARCH component in capturing variability that the ARIMA has failed to handle alone.

**Forecasts Accuracy of ARIMA-GARCH**

RMSE and MAE are used to evaluate the forecast performance. Results are given in Table 8 show that the ARIMA(0,1,1)-GARCH(1,1) model produces a better forecast with minimum RMSE and MAE.

Table 8: Forecasts Accuracy Values of ARIMA-GARCH and ARFIMA-GARCH Model

Candidate Models	RMSE	MAE
ARIMA(0,1,1)-GARCH(1,1)	0.4831	0.7914
ARIMA(0,1,2)-GARCH(1,1)	0.8832	2.0821

Finally, the ARIMA(0,1,1)-GARCH(1,1) is found to be the best model with the lowest values of RMSE, MAE. This model adequately captured volatility in the exchange rate.

**CONCLUSION**

This paper focuses on hybridizing ARIMA and GARCH model in forecasting exchange rate using time series methodology. Monthly data of exchange rate of Nigerian Naira to United State Dollar for the period ranging from January 2002 to February 2020 are used for this purpose. First of all, the stationarity of the exchange rate series is examined using unit root test such as ADF and KPSS tests which showed the series as non stationary. Hence, to make the exchange rate series stationary, the exchange rates are transformed using first differencing. Eight different ARIMA models were estimated using the Box-Jenkins approach. The most appropriate obtained models among different models using AIC are ARIMA(0,1,1) and ARIMA(0,1,2). However, the serial correlation and heteroscedascity analyses indicated strong evidence of serial correlation and heteroscedasticity in ARIMA model's residuals due to small p-values of Portmanteau and ARCH-LM test statistic. Therefore, these problem lead to the estimation of hybrid models, ARIMA(0,1,1)-GARCH(1,1) and ARIMA(0,1,2)-GARCH(1,1). From the fitting of these hybrid models it was found that the ARIMA-GARCH models produced larger log-likelihood values than the ARIMA models. These showed that the GARCH component has improved the models fitting of the Exchange Rate series and has captured adequately the volatility in the series. In addition, the diagnostic tests ARIMA-GARCH indicated larger p-values for Portmanteau and ARCH-LM test statistic values than the ARIMA. Again, the increase in the p-value could be attributed to the role of the GARCH model in eliminating the volatility effect.

Finally, the ARIMA(0,1,1)-GARCH(1,1) is found to be the best model with the lowest values of RMSE, MAE. This model adequately captured volatility in the exchange rate.

Further research, should consider hybridizing ARFIMA with GARCH model in forecasting exchange rate.

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