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A Novel Hybrid ARFURIMA-APARCH Model for Modeling Interminable Long Memory and Asymmetric Effect in Time Series

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Abstract
Abstract: *In this paper, we introduce a new hybrid model namely Autoregressive Fractional Unit Root Integrated Moving Average-Asymmetric Power Autoregressive Conditional Heteroscedasticity (ARFURIMA-APARCH) model. The Nigeria daily COVID19 records and Bitcoin to EURO exchange rate that exhibit a type of Long Memory (LM) called Interminable LM (ILM), volatility and asymmetric (leverage) effect were used to show the applications of the proposed ARFURIMA-APARCH model. The existing Autoregressive Fractional Integrated Moving Average-Asymmetric Power Autoregressive Conditional Heteroscedasticity (ARFIMA-APARCH) model were estimated and compared with the ARFURIMA-APARCH model. Results showed that the new hybrid model is better based on goodness-of-fit, serial correlation tests and forecast measures of accuracy. As a conclusion, our study showed that the ARFURIMA-APARCH model performed better compared to the ARFIMA-APARCH hybrid model. Therefore, the ARFURIMA-APARCH model is a better option for modeling ILM, volatility and leverage effect of health and financial data. Future study should focus on the application of the developed hybrid ARFURIMA-APARCH model using some major economic indicators, for example, Gross Domestic Product (GDP), currency exchange rate, stock price index, interest rate and other financial data.*

Keywords: Interminable, Autocorrelations, ARFURIMA-APARCH model, Forecasting and Leverage effect.

INTRODUCTION

Long memory is defined as the long-range dependence between observations collected over time. According to Granger &Joyeux (1980), Hosking (1981) and Musa & Jibrin (2021), data is said to have long memory characteristics when its plot exhibit a slow decay Autocorrelation Function (ACF). The Interminable Long Memory (ILM) is a type of long memory and can be considered a component of variation observed in large and non-stationary time series. According to Rahman & Jibrin (2019), ILM are time series that exhibits a strong long-range dependence, have slow decaying ACF, strong, positive and perfect autocorrelation, fractional

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difference values in the interval of $1 < d < 2$ and they are generated by Fractional Unit Root Integrated (FURI)process.

Furthermore, persistence also called long memory and volatility clustering in return is obtained using statistical volatility models. In the last three decades, many volatility models with long memory features were introduced and some include Bollerslev (1986) and Ding *et al.,* (1993). Baillie *et al,* (1996) introduced fractional differencing on the traditional Generalized Autoregressive Conditional Heteroscedastic (GARCH) model which is named the Fractionally Integrated GARCH (FIGARCH) model. However, to effectively study some stylized facts such as long memory, volatility clustering and leverage effect in financial returns, Ding *et al.,* (1993) introduced the Asymmetric Power Autoregressive Conditional Heteroscedastic (APARCH) model. The power returns, absolute returns and estimate of ACF of power returns are used to study various market indexes and other types of data. Bollerslev (1986) introduced the GARCH model to study the volatility in time series.

The present study assumed that many time series are long memory both in the original series and log-transformed series called the returns. Also, there is substantial correlation in absolute returns than squared returns, a stylized fact of high-frequency financial returns called long memory. This is similar to the observations of Cont (2005), Belkhouja *et al.* (2008) and Ding (2011). Similarly, it has been shown that residuals of non-stationary long memory mean models like Autoregressive Fractional Integrated Moving Average(ARFIMA), Autoregressive Tempered Fractional Integrated Moving Average(ARTFIMA), Autoregressive Fractional Unit Root Integrated Moving Average(ARFURIMA) and other mean models, regularly show the presence of variability (see Zhou & He, 2009; Jibrin *et al.,* 2015; Duppati *et al.,* 2016). The magnitude of the variability may possibly affect the reliability of forecasts from mean models. Likewise using the long memory variance models alone like the FIGARCH and APARCH model may equally cause absurd predictions. Therefore joining the mean model with the FIGARCH or APARCH to develop a hybrid model for studying mean and variability concurrently will produce better forecast.

To this effect, Baillie *et al.,* (1996) introduced a hybrid equation called the ARFIMA-GARCH model to study the long memory and variance in the inflation of the united state of America concurrently. Similarly, Ambach &Ambach (2018) introduced the periodic hybrid time series model called ARFIMA-P-GARCH. Jibrin (2019) introduced the ARFURIMA-GARCH model while Kabala (2020) introduced the ARTFIMA-GARCH model. All the listed hybrid models are introduced to study long memory only in the mean but not in volatility. In addition, none of the earlier hybrid models are used to study long memory both in mean and variance combined with leverage effect. On this basis, the current study intends to introduce a hybrid model, namely: the ARFURIMA-APARCH model.

The use of hybrid models to analyze the interminable long memory in mean and long memory volatilities of time series will ensure adequate elimination of noise signals that can affect modeling techniques. This current study wants to introduce the hybrid model, the ARFURIMA-APARCH hybrid model with fractional difference values in the interval $1 < d < 2$ to apply for the study of the ILM in mean and long memory in variance combined with leverage effect in time series respectively. Nonetheless, these models shall be developed to improve the precision of fitting and diagnostic testing, as well as to aid in improving the forecast accuracy measures. Other strengths of the hybrid ARFURIMA-APARCH model includes low information criteria, large log-likelihood values and minimum accuracy measures whose indicates forecasting power.

METHODOLOGY

Many financial and economic indices are found to be nonstationary and ILM with fractional differencing value, $1 < d < 2$ (see Phillips and Shimotsu, 2005; Boutahar, 2007). Consequently, Jibrin (2019) introduced the ARFURIMA model to study time series with these properties and characteristics. Given a time series$\{\boldsymbol{X_t}\}, \boldsymbol{t} = \boldsymbol{1}, \dots, \boldsymbol{T}$,assumed to be ILM, volatility and asymmetry, the ARFURIMA model of order p, d and q denoted by ARFURIMA(p,d,q) can be defined as:

$$\phi(L)\{(1-L)(1-d^*(1+L))\}X_t = \theta(L)\varepsilon_t \quad (1)$$

It is clear that the ARFURIMA model lack requirements of studying time series with volatility, ILM, trend and leverage effect concurrently.

The Proposed ARFURIMA-APARCH Model

Many time series exhibits trend, volatility, a long memory and leverage effect (see Gajda *et al.*, 2018; Lyócsa *et al.,* 2019; Boubaker *et al.,* 2020; Lembang *et al.,* 2021; Chinhamu *et al.,* 2022). The current study assumed that the model in (1) failed to account for the volatility, ILM, trend and leverage effect concurrently that are present and dwelled in the time series$\{X_t\}, t = 1, \dots, T$;have residuals$\{\varepsilon_t\}, t \geq 0$, that are serially correlated and heteroscedastic;and cannot account for the high degree of relationship that exists in absolute returns of a time series as observed for similar mean modelsby Safadi & Pereira (2010) and Rahman &Jibrin (2018). Consequently, the current study wants to introduce the hybrid ARFURIMA-APARCH model. The ARFURIMA(*p,d,q*)-APARCH(*s,v*) is for the study of the ILM, long memory in volatility and leverage effect in time series. Following Engle (1982; 2001), ε_t in eq.(1) is considered to be a stochastic process defined as:

$$\varepsilon_t = u_t\sigma_t, \quad (2)$$

where $E(u_t) = 0$, $Var(u_t) = 1$ and σ_t is positive and changes with respect to time, t. This implies that the process, $\{u_t\}$, is assumed to be serially uncorrelated expressed as:

$$u_t \sim iid(0,1) \quad (3)$$

Ding *et al.,* (1993) introduced the APARCH(*s,v*) model to study σ_t as:

$$\sigma_t^\delta = \omega + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta, \quad (4)$$

Now, to develop the hybrid ARFURIMA-APARCH model, eq.(4) can be expressed as:

$$\sigma_t = \left[\omega + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta\right]^{\frac{1}{\delta}}. \quad (5)$$

Substituting eq.(5) in (2), the following is obtained

$$\varepsilon_t = u_t\left[\omega + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta\right]^{\frac{1}{\delta}}. \quad (6)$$

Let $\varepsilon_t = u_t\left[\omega + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta\right]^{\frac{1}{\delta}}$ in eq. (1) so that the ARFURIMA(*p,d,q*)-APARCH(*s,v*) can be represented as:

$$\phi(L)\{(1-L)(1-d^*(1+L))\}X_t = \theta(L)\left[\omega + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta\right]^{\frac{1}{\delta}}u_t. \quad (7)$$

where X_t is dependent variable,$u_t$is the error terms that are independent and identically distributed random variables and $0 < d^* < 1$such that $d^* = d - 1$, $1 < d < 2$, both d and d^* are fractional differencing values. The $\phi(L) = 1 - \phi_1 L - \cdots \phi_p L^p$, $\theta(L) = 1 - \theta_1 L - \cdots \theta_q L^q \alpha(L) = \alpha_1 L^1 + \cdots, \alpha_r L^r$ and $\beta(L) = \beta_1 L^1 + \cdots, \beta_v L^v$ are called characteristics polynomial and all their roots are expected to lie in the unit root circle while L is the lag operator. The ω,

$\phi_1, \dots, \phi_p$, $\theta_1, \dots, \theta_q$, $\alpha_1, \dots, \alpha_r$, $\beta_1, \dots, \beta_v$, δ, d and γ are parameters of the model and must be estimated from the sample data. The γ and δ are the leverage and power term respectively.

Since the $\{(1-L)(1-d^*(1+L))\}$ in eq. (7) is the filter that is used to eliminate the non-stationary, ILM and unwanted noise signals in $X_t$so that the components of the ARFURIMA-APARCH are estimated using the transformed series. Therefore, let $\{(1-L)(1-d^*(1+L))\}$=1 so that eq. (7) become

$$\phi(L)X_t = \theta(L)\left[\omega + \alpha_{\mathbf{1}}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta\right]^{\frac{1}{\delta}}u_t. \quad (8)$$

The ARFURIMA(p,q)-APARCH(s,v) is

$$X_t = \left(\phi(L)\right)^{-1}\theta(L)\left[\omega + \alpha_{\mathbf{1}}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta\right]^{\frac{1}{\delta}}u_t. \quad (9)$$

In addition, when $p = q = s = v = 1$, the ARFURIMA(*1,1*)-APARCH(*1,1*) can be shown be

$$X_t = (\phi_1)^{-1}\theta_{\mathbf{1}}\left[\omega + \alpha_{\mathbf{1}}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta_{\mathbf{1}}\sigma_t^\delta\right]^{\frac{1}{\delta}}u_t. \quad (10)$$

The ARFIMA-APARCH models

The ARFIMA(*p,d,q*)-APARCH(*s,v*)model can be defined as follows:

$$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t \quad (11)$$

$$\varepsilon_t = Z_t\sigma_t,\ Z_t \sim N(0, \sigma_t) \quad (12)$$

where σ_t^2 can be represented as (4).

Maximum Likelihood Estimation Method for Hybrid Models

Here, the procedures for maximum likelihood estimation of the proposed hybrid model is discussed. Given $X = (x_1, \dots, x_t)'$, to obtain the estimates of the ARFURIMA-APARCH parameters, the time series X_t, is filtered by the fractional filter in (13)

$$Y_t = \left\{\{(1-L)(1-d^*(1+L))\}X_t\right\}. \quad (13)$$

Following Baillie*et al.* (1996) and Kang &Yoon (2013), we assume Y_t in (13) to be normal. The ARFURIMA(*p,d,q*)-APARCH(*s,v*) model parameters are estimated by using the non-linear optimization procedures to maximize the logarithm of the normal likelihood function given in (14).

$$\ln\{L(d, \phi, \theta, \omega, \alpha, \beta, \delta)\} = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln|\Sigma| - \frac{1}{2}Y'\Sigma^{-1}Y, \quad (14)$$

where n is the number of observations, Σ is the (n x n)covariance matrix of Y depending on $d, \phi, \theta, \omega, \alpha, \beta and \delta$. Also, $|\Sigma|$ is the determinant of Σ.

RESULTS AND DISCUSSION

This section presents the application of the ARFIMA, ARFURIMA, ARFIMA-APARCH and the proposed ARFURIMA-APARCH models by using Nigeria daily COVID19 records and Bitcoin to EURO exchange rate(BITCER)).

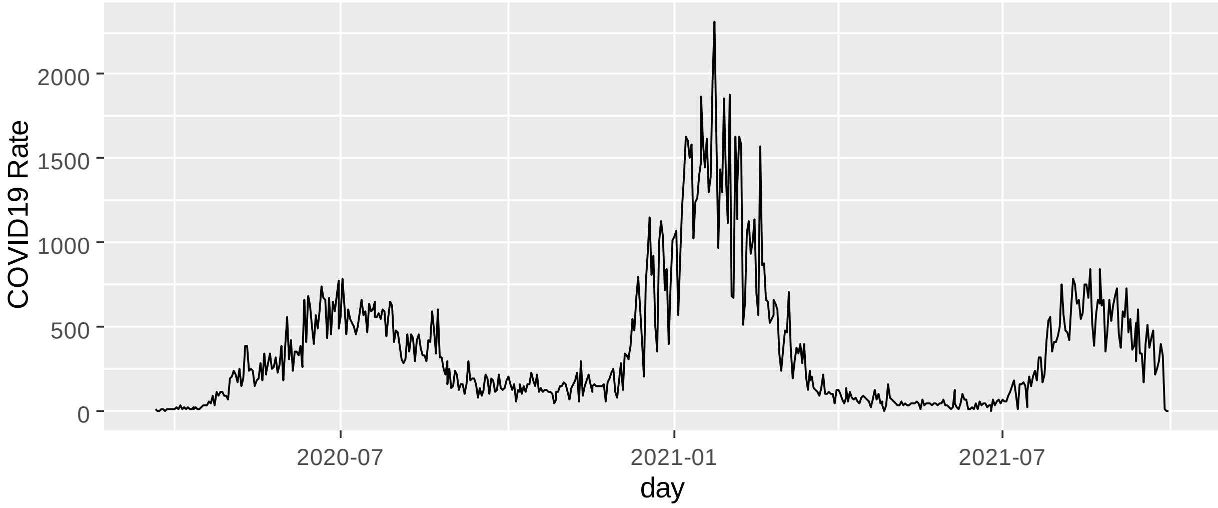

Figure1: ThePlot of Daily Time Series of Nigerian COVID19 Rate (20-03-2020)-(30-09-2021).

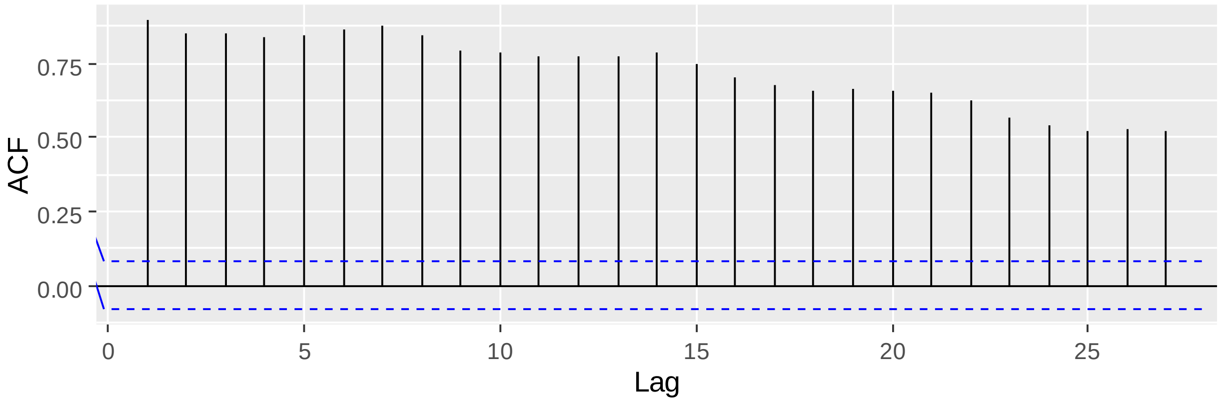

Figure 2: The Plot of ACF of Nigerian COVID19 Rate(20-03-2020)-(30-09-2021).

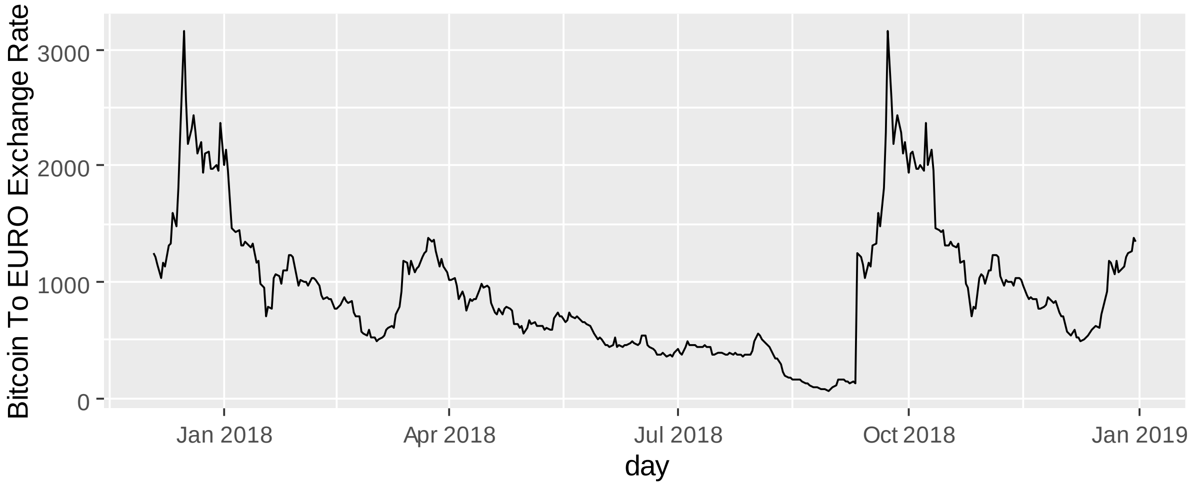

Figure 3: The Plot of Daily Time Series of Bitcoin to EURO Exchange Rate.

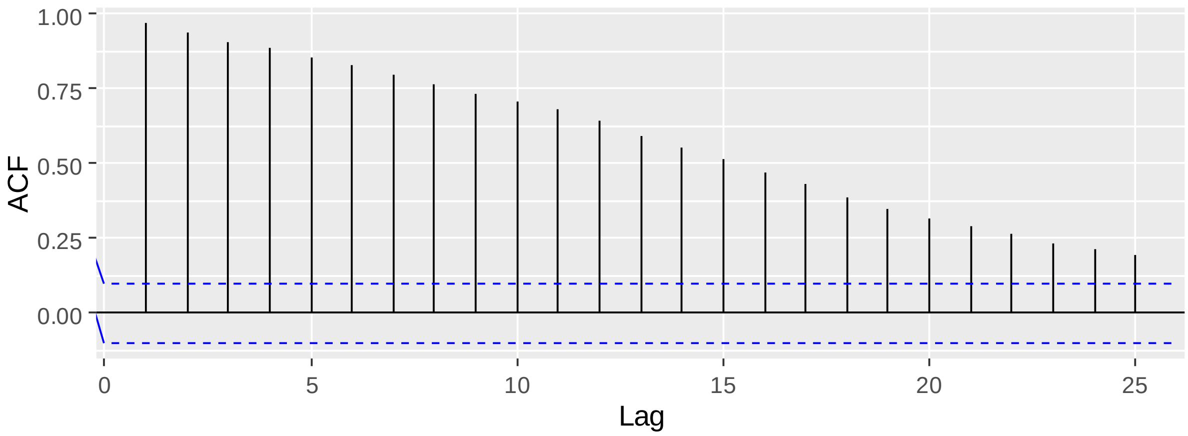

Figure 4: The Plot of ACF of Bitcoin to EURO Exchange Rate.

The time series plot of the COVID19 and BITCER is shown in Fig. 1 and 3 respectively. Both the time series graph exhibits deterministic trend behavior. The ACF of the COVID19 and BITCER are shown in Fig. 2 and 4 respectively. Both the ACF indicates a very slow decay and strong autocorrelations which is evidence of the ILM process. Similarly, time series data that show this type of behavior can produce fractional difference values in the intervals, $1 < d < 2$ (see Jibrin *et al.*,2020). Therefore, on average, both the COVID19 and BITCER are not stationary. Furthermore, the fractional difference value in the COVID19 and BITCER is further estimated and is displayed in Table 1. The two series, COVID19 and BITCER produced fractional difference values in the intervals, $1 < d < 2$ indicating that they are generated by ILM and FURI process.

Table1:Geweke and Porter-Hudak Long Memory Parameter Estimation

Data	Time Series	Volatility
COVID19	1.2842	0.5873
BITCER	1.4661	0.3013

These fractional difference estimates confirmed the ILM attributes as shown in the ACF Fig. 2 and 4. In addition, the fractional unit root differencing values obtained indicate that the fractional unit root differencing approach would be an appropriate transformation method to apply in eliminating the unwanted ILM and trend in both the COVID19 and BITCER. Also, the ARFURIMA is the appropriate model to consider in modeling the COVID19 and BITCER.

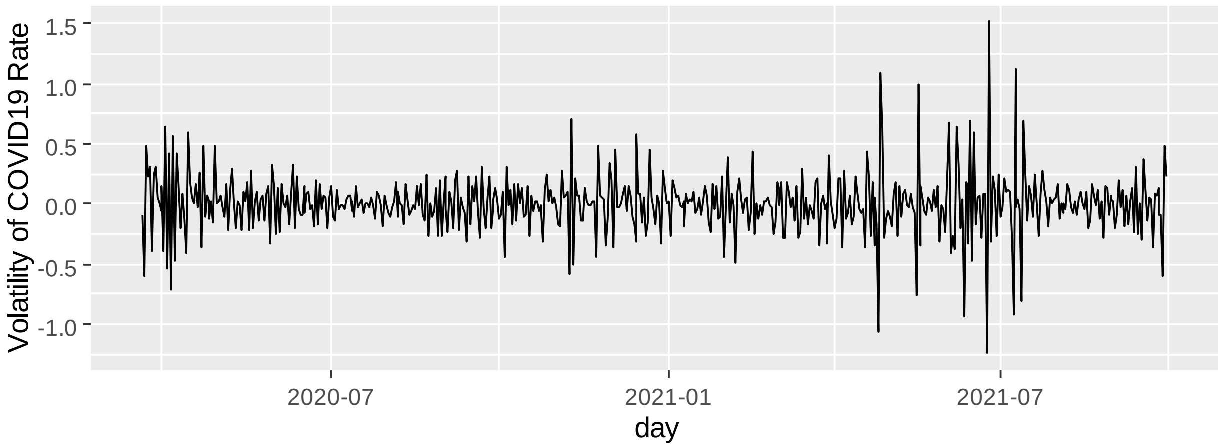

Figure 5: The Plot of Daily Volatility of the Nigerian COVID19 Rate.

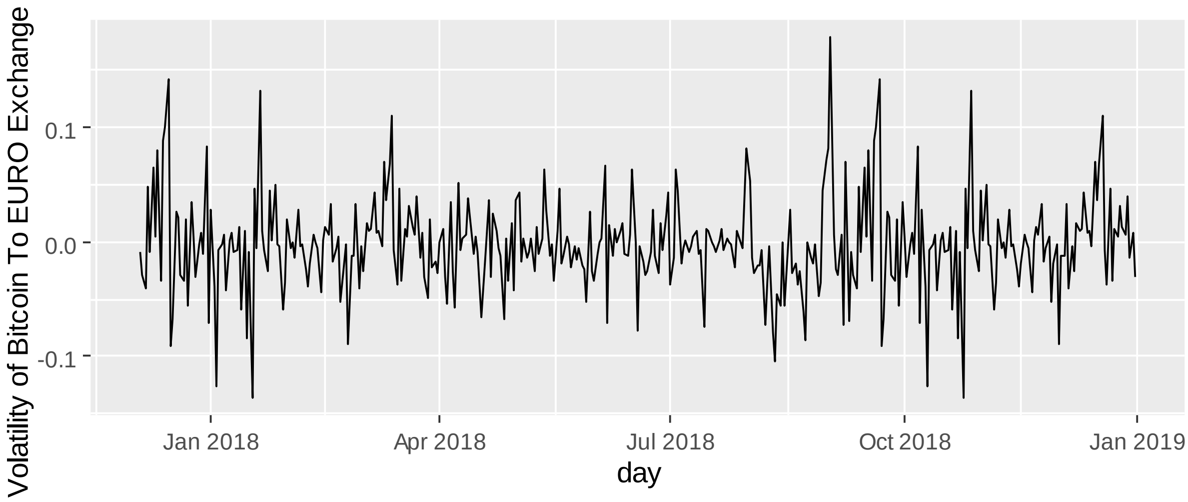

Figure 6: The Plot of Daily Volatility of the Bitcoin to EURO Exchange Rate.

ARFIMA-APARCH and ARFURIMA-APARCH Modeling

Before estimating the hybrid models that involved the asymmetric model APARCH, it is important to investigate the presence of asymmetric effect in the Nigeria daily COVID19 and Bitcoin to EURO exchange rate. This is similar to the position of Brooks (2008).

Table 2: Asymmetric Test for COVID19 and BITCER

Variables	Sgin Bias	Negative Bias	Positive Bias	Joint Effect
COVID19	1.031 (0.303)	0.514 (0.608)	**7.487** **(0.000)**	**68.962** **(0.000)**
BITCER	0.519 (0.604)	1.099 (0.273)	**3.089** **(0.002)**	**14.710** **(0.002)**

As shown in Table 2, both series show evidence of asymmetric effect also known as leverage effect. This is because the p-values for the positive sign bias and joint effect are less than 0.05 significance levels. These results indicate the importance of using the ARFIMA-APARCH and the new developed ARFURIMA-APARCH models to concurrently study the ILM, long memory in volatility and asymmetric effect in the COVID19 and bitcoin to EURO exchange rate. Many hybrid models were estimated and throughout the model fitting, the order p, qwere set to be $1-3$ while order of set to $s=1, v=1$. The Akaike Information Criteria (AIC) values are compared and ARFIMA(1,d,1)-APARCH(1,1) and ARFIMA(0,d,1)-

APARCH(1,1) are the models with the minimum AIC for COVID19 and BITCER respectively. Similarly, the ARFURIMA(0,d,2)-APARCH(1,1) and ARFURIMA(2,d,1)-APARCH(1,1) are the models with the minimum AIC for COVID19 and BITCER respectively. These suggest that the selected hybrid models can be considered for studying the COVID19 and BITCER.

The Estimation and Diagnostic Tests of ARFIMA-APARCH and ARFURIMAAPARCH Models

The results of the parameters estimation and serial correlation analysis of both ARFIMA-APARCH and ARFURIMA-APARCH models with their AIC and log-likelihood values are given in Tables 3 and 4 respectively.

Table 3: The Estimation of Hybrid Candidates Model Using COVID19 Series

Parameters	ARFIMA(1,d,1)-APARCH(1,1)	ARFURIMA(0,d,2)-APARCH(1,1)
d	0.49(0.0.6125)	1.28(0.1656)
$\boldsymbol{\phi}_1$	0.99(0.0306)	---------
$\boldsymbol{\theta}_1$	-0.63(0.5289)	-0.99(0.0010)
$\boldsymbol{\theta}_2$	---------	0.21(0.0005)
ω	0.01(0.0281)	0.03(0.0002)
$\boldsymbol{\alpha}_1$	0.08(0.0155)	0.09(0.0003)
$\boldsymbol{\beta}_1$	0.91(0.0026)	0.90(0.0010)
$\boldsymbol{\gamma}_1$	-0.04(1.0142)	**-0.83(0.0009)**
δ	1.49(0.0685)	**1.67(0.0004)**
Skew	1.14(0.2368)	1.18(0.0011)
Shape	9.70(53.5422)	6.73(0.0026)
AIC	11.607	**10.923**
Log-Likelihood	-3222.59	**-3032.17**
Portmanteau Test	27.395[0.0731]	**52.224[0.8962]**
ARCH-LM Test	6.1042[0.0344]	**9.8739[0.4516]**

Note: standard errors in parenthesis and p-values are in square brackets

Table 4: The Estimation of Hybrid Candidates Model Using BITCER Series

Parameters	ARFIMA(0,d,1)-APARCH(1,1)	ARFURIMA(2,d,1)-APARCH(1,1)
d	0.49(0.0423)	1.47(0.0002)
$\boldsymbol{\phi}_1$	------------	0.07(0.0003)
$\boldsymbol{\phi}_2$	------------	0.09(0.0003)
$\boldsymbol{\theta}_1$	0.99(0.0306)	-0.93(0.0010)
ω	286.10(1725.7915)	1.82(0.0013)
$\boldsymbol{\alpha}_1$	0.32(0.1037)	0.05(0.0002)
$\boldsymbol{\beta}_1$	0.59(0.0567)	0.92(0.0010)
$\boldsymbol{\gamma}_1$	-0.05(0.1112)	-0.99(0.0010)
δ	1.57(1.3277)	1.82(0.0013)
Skew	2.61(1.3821)	1.16(0.0011)
Shape	60.00(10.0388)	3.41(0.0018)
AIC	13.112	**9.556**
Log-Likelihood	-1833.167	-1331.5775
Portmanteau Test	520.50[0.0000]	14.16[0.1659]
ARCH-LM Test	75.043[0.000]	15.43[0.1170]

Note: standard errors in parenthesis, p-values are in square brackets

The parameters of the ARFURIMA-APARCH models come with smaller standard errors than parameters of the ARFIMA-APARCH models indicating the adequacy of the proposed hybrid

ARFURIMA-APARCH models. The AIC values for the new developed hybrid model is smaller. Also, the log-likelihood values for ARFURIMA-APARCH are larger than the log-likelihood values of ARFIMA-APARCH models indicating the goodness-of-fit of the proposed hybrid models to both COVID19 and BITCER. These results show evidence of improvement in model fitting as a result of introducing the APARCH model to both the ARFIMA and ARFURIMA models. The ARFURIMA(0,2)-APARCH(1,1) is

$$X_t = [u_t - \theta_{\mathbf{1}} u_{t-1} - \theta_{\mathbf{2}} u_{t-2}]\left[\omega + \alpha_{\mathbf{1}}(|\varepsilon_t| - \gamma\varepsilon_t)^{\delta} + \beta_{\mathbf{1}}\sigma_t^{\delta}\right]^{\frac{1}{\delta}} \quad (15)$$

and when is fitted to the Nigeria daily COVID19 records is

$$X_t = [u_t + 0.99u_{t-1} - 0.21u_{t-2}][0.03 + 0.09(|\varepsilon_t| + 0.83\varepsilon_t)^{1.67} + 0.9\sigma_t^{1.67}]^{\frac{1}{1.67}}. \quad (16)$$

Similarly, the ARFURIMA(2,1)-APARCH(1,1)

$$X_t = (\phi_1 X_{t-1} - \phi_2 X_{t-2})^{-1}(u_t - \theta_{\mathbf{1}} u_{t-1})\left[\omega + \alpha_{\mathbf{1}}(|\varepsilon_t| - \gamma\varepsilon_t)^{\delta} + \beta_{\mathbf{1}}\sigma_t^{\delta}\right]^{\frac{1}{\delta}} \quad (17)$$

and when fitted to the daily Bitcoin to EURO exchange rate becomes

$$X_t = (0.07X_{t-1} - 0.09X_{t-2})^{-1}[u_t + 0.93u_{t-1}][1.82 + 0.05(|\varepsilon_t| + 0.99\varepsilon_t)^{1.82} + 0.92\sigma_t^{1.82}]^{\frac{1}{1.82}} \quad (18)$$

3.3 The Foreacst Accuracy Measures of Hybrid Models

The Mean Error (MSE), Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) are used in the literature to evaluate forecasts performance (see Hyndman & Athanasopolous, 2013; Papailias &Dias, 2017). The current applied the same methods.

Table 5: Forecast Accuracy Measures for Hybrid Models

	ARFIMA-APARCH			ARFURIMA-APARCH		
Variables	MSE	RMSE	MAE	MSE	RMSE	MAE
COVID19	23214.47	152.36	88.63	0.9548	0.9771	0.7388
BITCOIN	104616.20	323.44	221.04	0.9877	0.9938	0.6959

The forecast accuracy measures results of COVID19 and Bitcoin to EURO exchange rate using the hybrid ARFIMA-APARCH and ARFURIMA-APARCH models are in Table 5. Compared to the hybrid ARFIMA-APARCH model, the ARFURIMA-APARCH model produces a better forecast performance with minimum accuracy measures.

CONCLUSION

This paper presented a new modelling formulation for the ILM series with fractional differencing values in the interval of$1 < d < 2$. A new hybrid ARFURIMA-APARCH model was developed that captures ILM, long memory in variance components and leverage effects. The models' parameters were estimated using the Maximum Likelihood (ML) estimator. Model calibration was achieved using COIVD19 and Bitcoin to EURO exchange rate datasets. In addition, the proposed hybrid model was compared with existing ARFIMA-APARCH competing methods. The results from the comparative analysis using AIC, log-likelihood, portmanteau test, ARCH-LM test and forecast accuracy measures revealed that the hybrid ARFURIMA-APARCH model is the best. Also, the ARFURIMA-APARCH model adequately captured the asymmetric effect and power terms based on the time series used. The diagnostic confirms that the residuals from the ARFURIMA-APARCH model can be regarded as serially uncorrelated and homoscedastic.

This study contributes to the literature on long memory time series modeling. Since most of the health, financial and economic data are non-stationary, if the series show evidence of ILM characteristics, they are predictable. As work in modelling the conditional volatility of stock prices has found that the stock volatility responds asymmetrically to positive or negative shocks, the current study shows that COVID19 data volatility tends to rise higher as positive shocks give rise to higher volatility than negative shocks. In view of this, it can be concluded that there is an asymmetry in the conditional volatility of Nigeria's daily COVID19 data. In this study, it can be stated that an novel, efficient, and sufficient hybrid model was developed for eliminating unwanted noise signals and variability in large time series. Specifically, the proposed hybrid ARFURIMA-APARCH model can be used to fit the data with ILM, volatility and asymmetric effect characteristics. In addition, this hybrid model contributes to the study of the ILM and volatilities in health and financial time series. This study can be a reference to other researchers to dwell more on the ILM modeling with volatility simultaneously. Future works may focus on the application of the developed hybrid ARFURIMA-APARCH model using major economic indicators, for example, Gross Domestic Product (GDP), currency exchange rate, stock price index, interest rate and other financial data.

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