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Analysis of a Mathematical Model of the Population Dynamics of Drug Addiction Among Youths

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Abstract

Drug addiction amongst youths has been identified as one of the global menace that has affected negatively the development and productivity of young people which has come to stay from time pass to the present day and has resulted to various behavioural disorder such as criminality, psychiatric issues and death. In this paper, a 5 sub-class model is formulated to analyze the population dynamics of drug addiction among youths and the effect of addiction on their development and productivity towards contributing their quota to the human population. The model was developed using ordinary differential equations approach. It was determined that the basic reproduction number which is a vital tool for measuring the impact of drug addiction in human was obtained using the next generation operator method. Stability analysis was carried out and it was discovered that drug-free equilibrium exists and it was shown to be locally and globally asymptotically stable whenever effective basic reproduction number is less than one; and unstable whenever effective basic reproduction number is greater than one. It was discovered that drug abuse persisted among youths and are both locally and globally asymptotically stable whenever effective basic reproduction number is greater than one and unstable whenever effective basic reproduction number is less than one.

Keywords: Drug, Addiction, Model, Stability, Youth, Human Population, Substance

Introduction

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Drug misuses caused enormous danger to human health, and socio-economic development of individuals and nations according to WHO, (2018). In 2015, UNODC proved that out of one-fourth of a billion human that abuse drugs, about 29.5 million suffered substance/drug-related disorders (UNODC, 2018). Drug misuse caused about 183,000 deaths across the world in 2012 (UNODC, 2014). Worldwide, it was appraised that, in 2012, about 162 million and 324 million people aged between 15 and 64 had participated in an illegitimate drug use (UNODC 2014). Unlawful drug administration is said to be nonmedical use of varieties of drugs that are prohibited by local/international law (Degenhardt et al. 2004; GBD, 2005). Illegal use of harmful substances/drugs that results to worldwide threat to human population was carried out in the work of Donoghoe, (1996). Donoghoe, (1996) asserted that wrongful consumption of drug was responsible for the of the death of about 10,000 people around the world in 1990 of which about 62% of them are found in the developing countries. Apart from deaths resulting from illegal use of chemical/drugs, indecent administration of drug causes health challenges resulting to about 50% according to CMHA, (2005). In recent times, substance misuse has become a major issue to human health and societal development across the world making resources that should have been channeled to building infrastructural amenities to be used in construction of rehabilitation centres and purchase of resuscitating medicine for the victims of the habitual disorder. The present world is experiencing the devastating effects of drug abuse. It works as a hindrance to present civilization. It can also restrain us from accomplishing our goals or dreams of life. Generally, the use of illegal drugs or the use of prescription or over-the-counter medication in ways other than recommended. Hassan, (2013) in his work postulated that misuse of substances can induce sleep or result to untold happiness or insensitivity which has a negative effect on the psyche and physical wellbeing of people. Over dependence on substance use does not only cause a single problem but also a number of other factors (such as biological disorder, brain damage, societal problem and psychological behaviors). Hassan, (2013) suggested that treatment in rehabilitation centers may guarantee a better life for the patients who are habitual drug addicts. An addict can achieve a sustained recovery in his or her future life without drugs after getting treatment from rehabilitation centers. To control drug addiction, all stakeholders should increase public education among the citizens. There is need to introduce voluntary activities by government and non-governmental organizations to implement laws guiding the use of substances/drugs.

Bae, (2014) proposed a model that correlated repeated misuse of drug in the increase of infectious diseases around the world. On the other hand, a tobacco model has been established to obtain the periodic motion from tobacco addiction with complete recovery, relapsed rate and addiction stages (Bae, 2014). The use of mathematical models to explore the dynamics of drug use has been an interesting topic for a couple of researchers (Nyabadza et al. 2013; Samanta 2011; White and Comiskey, 2007). White and Comiskey, (2007) proposed a mathematical model to evaluate the role of treatment and relapse in the dynamics of heroin. Their work revealed among others that relapse of individuals who would have quit heroin use has a significant impact on the generation of new or secondary heroin users. Nyabadza *et al.*, (2013) constructed a mathematical model to examine the dynamics of crystal meth "Tik" abuse in the presence of drug-supply chains. Using the data from South Africa, their work suggests among others that programs aimed at encouraging light drug users to quit drug use can be more effective to control "Tik" Mathematical analysis has provided quantifiable insight into several areas. Numerous models have been published in the previous years studying various problems regarding practical life. The literature and development of mathematical epidemiology are as well documented as in (Bae, 2014; Biswas, 2014; Biswas *et al*. 2014; Biswas 2012; Biswas, 2011; Biswas *et al.* 2016; Muhammad *et al*. 2019; Khandy *et al.* 2017 and Orwa *et al*. 2019). However, the previous models did not consider control measure of

drug/substance abuse amongst youths which this work attempted to address since energetic young people form huge fraction of illicit substance users in human society.

Methodology

Model description and Formulation: We considered human population affected with drug abuse and subdivided them into the following mutually exclusive sub-classes; namely: Susceptible $S_h(t)$, Drug users $D_h(t)$, Drug Abuser $A_h(t)$, Disabled/retarded addicts $Q_h(t)$ and recovered $R_h(t)$. The total number of population $N(t)$ are thus given as:

$$N_h(t) = S_h(t) + D_h(t) + Q_h(t) + A_h(t) + R_h(t) \tag{1.1}$$

Susceptible human are recruited at rate Λ, susceptible human become drug users with force of infection

$$\lambda = \frac{\beta\left(\varepsilon D_h + A_h\right)}{N_h} \tag{1.2}$$

The model equations are:

$$\begin{aligned}
\dot{S}_h &= \Lambda - \lambda S_h - \mu S_h \\
\dot{D}_h &= \lambda S_h - \left(\alpha + \tau_D + \mu\right) D_h \\
\dot{A}_h &= \alpha D_h - \left(\tau_A + \mu + \sigma\right) A_h \\
\dot{Q}_h &= \sigma A_h - \left(\tau_Q + \mu + \delta\right) Q_h \\
\dot{R}_h &= \tau_D D_h + \tau_A A_h + \tau_Q Q_h - \mu R_h
\end{aligned} \tag{1.3}$$

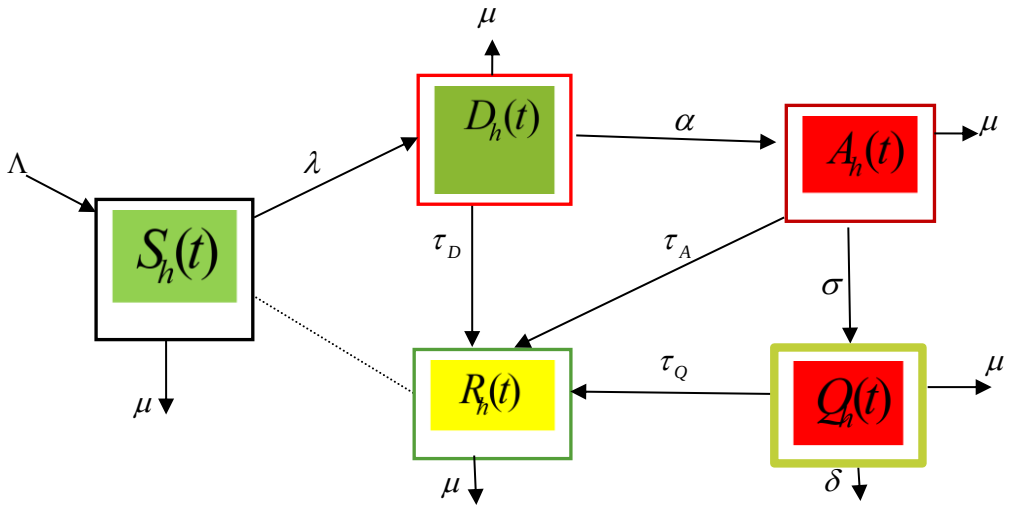

Figure 1.1: Flow diagram of drug addiction among youths

Table 1.1 State variable descriptions and values of drug abuse in young people

Variables	Description	Values (year/m)	Reference
$S_h(t)$	Susceptible human subclass	5000	Estimated
$D_h(t)$	Human drug user subclass	3000	Estimated
$A_h(t)$	Addicted human to drug abuse subclass	2500	Estimated
$Q_h(t)$	Human disabled /retarded drug addict subclass	500	Estimated
$R_h(t)$	Recovered human subclass from drug abuse	270	Estimated
Parameters	**Description**	**(Age between 12- 45)**	
Λ	Human recruitment rate into the population	1500	Mohammed *et al*. (2019)
μ	Natural death rate for human population	0.02	Mohammed *et al*. (2019)
α	Progression rate from $D_h(t)$ to $A_h(t)$	0.3	Mohammed *et al*. (2019)
σ	Progression rate from $A_h(t)$ to $D_h(t)$	0.001	Mohammed *et al*. (2019)
δ	Death rate due to addiction	0.05	Estimated
τ_Q	Rehabilitation rate	0.3	Orwa *et al*. (2019)
τ_A	Progression rate from $D_h(t)$ to $R_h(t)$	0.25	Orwa *et al*. (2019)
τ_D	Progression rate from $A_h(t)$ to $R_h(t)$	0.01	Mohammed *et al*. (2019)

Boundedness of solutions of the model

We consider the region $\Omega_2 = \left(S_h, \mathrm{D}_h, A_h, \mathrm{Q}_h, R_h \in \mathbb{R}_+^5 : \mathrm{N}_h \le \frac{\Lambda}{\mu} \right)$; We shall show that the set Ω_2 is positively invariant and an attractor of all positive solutions of the model (1.3).

Lemma 1: The set Ω_2 is positively invariant for the model (1.3).

Proof: The rate of change of the total population is given thus

$$\frac{dN_h}{dt} = \Lambda - \mu N_h - \delta Q_h \tag{1.4}$$

By standard comparison theorem (1.4) is reduced thus:

$$\frac{dN_h}{dt} \le \Lambda - \mu N_h \tag{1.5}$$

We solve (1.5) by the integrating factor method to obtain:

$$N_h e^{\mu t} \le \frac{\Lambda e^{\mu t}}{\mu} + N_h(0) - \frac{\Lambda}{\mu}$$

$$N_h = N_h(0)e^{-\mu t} + \frac{\Lambda}{\mu}\left(1 - e^{-\mu t}\right) \tag{1.6}$$

If $N_h \le \frac{\Lambda}{\mu}$ then $N_h(0) \le \frac{\Lambda}{\mu}$ such that Ω_2 is positively invariant set under the flow described in model (1.3). Hence, all solutions of the model (1.3) remains in the set Ω_2 and its boundary. Also, since solution paths cannot leave Ω_2, the solution remains non-negative for non-negative initial conditions. Solution exist for all time t. in this region/set, therefore model (1.3) is said to be well posed mathematically and epidemiologically.

Positivity of solution of the model (1.3).

Lemma 2: Let the initial data for the model (1) be $S_h(t)>0, D_h(t)>0, A_h(t)>0, Q_h(t)>0, R_h(t)>0.$

Then the solution $\left(S_h(t), D_h(t), A_h(t), Q_h(t), R_h(t)\right),$ with positive initial data will remain positive for all time $t>0$.

The Proof is shown below:

Let $t_1 = Sup\left\{t>0; S_h(t)>0, D_h(t)>0, A_h(t)>0, Q_h(t)>0, R_h(t)>0\right\}>0$

Model (1.3)

$$S_h^* = \Lambda - (\lambda+\mu)S_h \tag{1.7}$$

Using the method of integrating factor

I.F $exp\left\{(\mu t)+\int_0^t \lambda(\tau)dt\right\}$

Then solution to the above is to substitute the integrating factor to the above equation.

$$\frac{d}{dt}\left[S_h(t)\exp\left\{\mu t+\int_0^t \lambda(\tau)\,d\tau\right\}\right] = \Lambda\left[\exp\left\{\mu t+\int_0^t \lambda(\tau)\,d\tau\right\}\right] \tag{1.8}$$

Integrating from 0 to t_1

$$S(t_1)\exp\left\{\mu t_1+\int_0^t \lambda(\tau)\,d\tau\right\} - S_h(0) = \int_0^{t_1}\Lambda\left[\exp\left\{\mu y+\int_0^y \lambda(\tau)\,d\tau\right\}\right]dy \tag{1.9}$$

$$S_h(t_1) = S_h(0)\exp\left\{-\mu t_1-\int_0^t \lambda(\tau)\,d\tau\right\}+\left\{\exp\left\{-\mu t_1-\int_0^t \lambda(\tau)\,d\tau\right\}\right\}\times\int_0^{t_1}\Lambda\left[\exp\left\{\mu y+\int_0^y \lambda(\tau)\,d\tau\right\}\right]dy>0 \tag{1.10}$$

Similarly, it can be shown that all state variables of the model remain positive for all time, $t>0$ so that $D_h(t)>0$, $A_h(t))>0$, $Q_h(t)>0$, $R_h(t)>0$, for all time $t>0$.

Existence of drug-free equilibrium (DFE)

The model (1.3) will exhibit disease-free equilibrium point obtained by setting the right-hand sides (RHS) to zero and all the disease classes/compartments to zero. The result is shown as below:

$$\xi^* = \left(S_h^*, D_h^*, A_h^*, Q_h^*, R_h^*\right) = \left\{\frac{\Lambda}{\mu}, 0, 0, 0, 0\right\} \tag{1.11}$$

Local stability of drug-free equilibrium.

The stability of ξ^* is obtained by using the next generation operator method as used in Van Den Driessche and Watmough, (2002), so that the matrices F and V are computed as:

From model (1.3)

$$F = \begin{bmatrix} \frac{\beta ES}{N} & \frac{\beta S}{N} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} k_2 & 0 & 0 \\ -\alpha & k_3 & 0 \\ 0 & -\sigma & k_4 \end{bmatrix}$$

Where, $k_2 = (\alpha + \tau_D + \mu)$ $k_3 = (\tau_A + \mu + \sigma)$ and $k_3 = (\tau_Q + \mu + \delta)$

Therefore, the spectral radius is given by

$$\rho(FV^{-1}) = \frac{\beta(\varepsilon k_3 + \alpha)}{k_2 k_3} \tag{1.12}$$

Whenever $\rho = 1$, $FV^{-1} = \frac{\beta(\varepsilon k_3 + \alpha)}{k_2 k_3}$

Where R_0 is the effective basic reproduction number of young people addicted to drug abuse.

$$R_0 = \frac{\beta(\varepsilon k_3 + \alpha)}{k_2 k_3} \tag{1.13}$$

Existence of drug persistence equilibrium point (DPEP)

Denoting the drug persistence equilibrium point (DPEP) of model (1) by $\xi^{**} = (S_h^{**}, \mathrm{D}_h^{**}, \mathrm{A}_h^{**}, \mathrm{Q}_h^{**}, R_h^{**})$ and solving model (1.3) in terms of force of infection at equilibrium state gives:

$$\left.\begin{aligned} S_h^{**} &= \frac{\Lambda}{k_1},\ D^{**} = \frac{\Lambda\lambda}{k_1 k_2},\ A_h^{**} = \frac{\alpha\Lambda\lambda}{k_1 k_2 k_3},\ Q_h^{***} = \frac{\sigma\alpha\Lambda\lambda}{k_1 k_2 k_3 k_4}, \\ R^{**} &= \frac{\Lambda\lambda\left[\tau_D k_3 k_4 + \tau_A \alpha k_4 + \tau_Q \sigma\alpha\right]}{\mu k_1 k_2 k_3 k_4}, \end{aligned}\right\} \tag{1.13}$$

Recall that

$$\lambda^{**} = \frac{\beta\lambda(\Lambda\varepsilon k_3 + \alpha\Lambda)}{N_h k_1 k_2 k_3} \tag{1.14}$$

Substituting for the values of N^{**} in the above,

$$N^{**} = \frac{\Lambda}{k_1} + \frac{\Lambda\lambda}{k_1 k_2} + \frac{\alpha\Lambda\lambda}{k_1 k_2 k_3} + \frac{\sigma\alpha \wedge \lambda}{k_1 k_2 k_3 k_4} + \frac{\Lambda\lambda\left[\tau_D k_3 k_4 + \tau_A \alpha k_4 + \tau_Q \sigma\alpha\right]}{\mu k_1 k_2 k_3 k_4} \tag{1.15}$$

Simplifying gives

$$N^{**} = \frac{m_1 + m_2 \lambda^{**}}{m_3} \tag{1.16}$$

Where $m_1 = \Lambda\mu k_2 k_3 k_4$

$$m_2 = \left[\Lambda k_3 k_4 \mu + \alpha\Lambda\mu k_4 + \sigma\alpha\Lambda\mu + \Lambda\left(\tau_D k_4 + \tau_A \alpha k_4 + \tau_Q \sigma\alpha\right)\right]$$
$$m_3 = \mu k_1 k_2 k_3 k_4$$

$$\lambda^{**} = \frac{\beta m_3\left(\Lambda\varepsilon k_3 + \alpha\Lambda\right)\lambda^{**}}{m_1 k_1 k_2 k_3 + m_2 k_1 k_2 k_3 \lambda^{**}} \tag{1.17}$$

Solving gives

$$\left(m_1 k_1 k_2 k_3\right)\lambda^{**} + \left(m_2 k_1 k_2 k_3\right)\lambda^{**2} - \left(\beta m_3\left(\varepsilon k_3 + \alpha\right)\Lambda\right)\lambda^{**} = 0 \tag{1.18}$$

Simplifying (1.18) we obtain

$$A_1\lambda^{**2} + A_2\lambda^{**} = 0 \tag{1.19}$$

$$\lambda^{**}(A_1\lambda^{**} + A_2) = 0 \tag{1.20}$$

Either $\lambda^{**} = 0$ or $A_1\lambda^{**} + A_2 = 0$ (1.21)

But

$$A_1 = k_1 k_2 k_3\left[k_3 k_4 \mu + \alpha\mu k_4 + \sigma\alpha\mu + (\tau_D k_4 + \tau_A \alpha k_4 + \tau_Q \sigma\alpha k_4)\right]\Lambda$$
$$A_2 = k_1 k_2 k_3 \times \mu\, k_2 k_3 k_4 \Lambda - \beta\Lambda(\varepsilon k_3 + \alpha) \times k_1 k_2 k_3 k_4 \mu$$

Simplifying we obtain

$$A_2 = k_2 k_3\left(1 - \frac{\beta(\varepsilon k_3 + \alpha)}{k_2 k_3}\right)$$

$$A_2 = k_2 k_3\left(1 - R_0\right) \tag{1.22}$$

$A_2 < 0$ if $R_0 > 1$

So the system of model (1.3) exhibits a unique (stable) drug persistence equilibrium point whenever $R_0 > 1$ if $\lambda > 0$ for $R_0 > 1$

Local stability of drug persistence equilibrium point

We analyze local stability of DPE by setting up the Jacobian of the model (1.3) expressed in terms of force of infection as:

$$\mathrm{J}\left(\varepsilon_1^{**}\right) = \begin{bmatrix} -(\lambda+\mu) & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 \\ 0 & 0 & 0 & 0 & -\mu \end{bmatrix}$$

The upper triangular matrix of the $\mathrm{J}\left(\varepsilon_1^{**}\right)$ is obtained as

$$J\left(\varepsilon_1^{**}\right)=\begin{bmatrix} -\lambda-\mu & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 \\ 0 & 0 & 0 & 0 & -\mu \end{bmatrix}$$

The eigenvalues are,

$\lambda_1=-\left(\lambda^{**}+\mu\right),\ \lambda_2-k_2<0,\ \lambda_3=-k_3<0,\ \lambda_4=-k_4<0$

$\lambda_5=-\mu<0$.

From (1.21)

$$\lambda^{**}=-\frac{A_2}{A_1}=\frac{k_2k_3\left(R_0-1\right)}{k_1k_2k_3\left(k_3k_4\mu+\alpha\mu k_4+\sigma\alpha\mu+(\tau_D k_4+\tau_A\alpha k_4+\tau_Q\sigma\alpha k_4)\right)\Lambda}>0 \qquad (1.23)$$

If $R_0>0$

Then,

$\lambda_1=-\left(\lambda^{**}+\mu\right)<0$ if $R_0>1$

We conclude that the drug persistence equilibrium point (DPE) for drug additions among youth is locally asymptotically stable (LAS) when $R_0>1$.

Global asymptotic stability of drug-free equilibrium (DFE) of model (1.3)

Lemma: The drug-free equilibrium (DFE) of the model (1.3) is globally asymptotically stable (GAS) in region Ω whenever $R_O<1$

Proof:

To prove global asymptotic stability of DFE according to Augusto *et al.* (2017) we let $X=\left(S_h,R_h\right)$ and $Z=\left(D_h,A_h,Q_h\right)$

(a) $\frac{dX}{dt}=F\left(X,0\right)$ (1.24)

(b) $\frac{dZ}{dt}=G\left(X,Z\right)$ (1.25)

Where $D_h=A_h=Q_h=0$ with $F\left(X,0\right)$ being the RHS of $\left(S_h,R_h\right)$ and $G\left(X,0\right)$ the RHS of $\left(D_h,A_h,Q_h\right)$

Reduce system $\frac{dF}{dt}=F\left(X,0\right)$ is given as

$$\begin{aligned} S_h^*&=\Lambda-\mu S_h \\ R_h^*&=-\mu R_h \end{aligned} \qquad (1.26)$$

Therefore, solving (1.26) thus;

$S_h^*-\mu S_h=\Lambda$

$$\int S_h^* e^{\mu t} = \int \Lambda e^{\mu t}$$

$$\frac{dt}{dt}\left[S_h^* e^{\mu t}\right]_0^t = \frac{\Lambda}{\mu}\left[e^{\mu t}\right]_0^t$$

$$S_h^* e^{\mu t} - S_h^*(0) = \frac{\Lambda}{\mu}\left[e^{\mu t} - 1\right]$$

Divide both sides by $e^{\mu t}$

$$S_h^*(t) = S_h^*(0)e^{-\mu t} + \frac{\Lambda}{\mu}\left[1 - e^{\mu t}\right] \tag{1.27}$$

Taking the limit

$$\underset{t\to\infty}{Lim} S_h^*(t) = \frac{\Lambda}{\mu}$$

$$X^* = \left(S_h^*, \mathrm{D}_h^*, \mathrm{A}_h^*, Q_h^*, \mathrm{R}_h^*\right) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right) \tag{1.28}$$

The asymptotic dynamics are independent of initial conditions in region Ω. Hence, the solutions of the model (1.3) converges globally in region Ω. According to previous study by Augusto *et al*. (2017), it is essential to prove that $G(X, Z)$ satisfies the two stated conditions as given below

$$\begin{aligned} &i. \quad G(X,0) = 0 \ \ and \\ &ii. \quad G(X,Z) = D_Z G(X^*,0)Z - G(X,Z), G(X,Z) \geq 0 \end{aligned} \tag{1.29}$$

Where;

$$\left(X^*,0\right) = \left(\frac{\Lambda}{\mu},0,0,0,0\right)$$

$D_Z G(X^*,0)$ is the Jacobian of $G(X,Z)$ taken with respect to the infected classes and evaluated at $(X^*,0)$

$$D_Z G\left(X^*,0\right) = \begin{bmatrix} \frac{\beta\varepsilon_1 S_h^*}{N^*} - k_1 & \frac{\beta S_h^*}{N^*} & 0 \\ \alpha & -k_2 & 0 \\ 0 & \sigma & -k_3 \end{bmatrix} \tag{1.30}$$

$$\left(\frac{\beta\varepsilon_1 S_h^* - k_1}{N_h^*}\right)D_h + \frac{\beta S_h^* A_h}{N_h^*} - \frac{\beta\left(\varepsilon_1 D_h + A_h\right)S}{N_h} + k_1 D_h$$

$$\frac{\beta\varepsilon_1 S_h^* D_h}{N_h^*} - \frac{\beta\varepsilon_1 D_h S_h}{N_h} + \frac{\beta S_h^* A_h}{N_h^*} - \frac{\beta S_h A_h}{N_h} = \frac{\beta\varepsilon_1 S_h^*}{N_h^*}\left(1 - \frac{N_h^*}{S_h^*}\frac{S_h}{N_h}\right)D_h + \frac{\beta S_h^*}{N_h^*}\left(1 - \frac{N_h^*}{S_h^*}\frac{S_h}{N_h}\right)A_h$$

$$G(X,Z)=\begin{bmatrix}\left(\frac{\beta\varepsilon_1 S_h^* D_h}{N_h^*}+\frac{\beta S_h^* A_h}{N_h^*}\right)\left(1-\frac{N_h^*}{S_h^*}\frac{S_h}{N_h}\right)\\0\\0\end{bmatrix} \quad (1.31)$$

Since we have $S_h^*=\frac{\Lambda}{\mu}$, in $D_h, S_h \le S_h^*$ thus $N_h \le N_h^*$. Whenever the young people population is at equilibrium we have $\left(1-\frac{N^*}{S^*}\frac{S}{N}\right)>0.$ thus $\hat{G}(X,Z)\ge 0$

Hence, the DFE is globally asymptotically stable as shown in Castillo-Chavez and Songs, (2004).

Global stability of drug persistence equilibrium point (DPEP)

Theorem: The drug persistence equilibrium of the model (1.3) is globally asymptotically stable whenever $R_0>1$.

Proof: Let $R_0>1$, then the existence of the drug persistence equilibrium point is guaranteed. We shall show this by setting up a common quadratic Lyapunov function as thus:

$$F(x_{1,}.....x_n)=\sum_{i-1}^{n}\left(x_i...x_n^*\right)^2 \quad (1.32)$$

Therefore;

$$\begin{aligned}&\Rightarrow \Lambda-\lambda S_h-\mu S_h+\alpha D_h-\tau_A A_h-\mu A_h-\sigma A_h+\sigma A_h-\tau_Q Q_h-\mu Q_h-\delta Q_h\\&\quad+\tau_D Q_h+\tau_A A_h+\tau_Q Q_h-\mu R_h+\lambda S_h-\alpha D_h-\tau_D D_h-\mu D_h\\&\Rightarrow \Lambda-\mu S_h-\mu A_h-\mu Q_h-\mu D_h-\mu R_h-\delta Q_h\\&\Lambda-\mu\left(S_h+D_h+A_h+Q_h+R_h\right)-\delta Q_h\\&\Lambda=\mu\left(S_h^*+D_h^*+A_h^*+Q_h^*+R_h^*\right)-\delta Q_h^* \quad (1.33)\end{aligned}$$

We shall form a Lyapunov function with the following state variables

$$F=(S_h,D_h,A_h,\mathrm{Q}_h,R_h)=\frac{1}{2}\left[\left(S_h-S_h^*\right)+\left(D_h-D_h^*\right)+\left(A_h-A_h^*\right)+\left(Q_h-Q_h^*\right)+\left(R_h-R_h^*\right)\right]^2 \quad (1.34)$$

Now differentiating the above equations.

$$\frac{dF}{dt}=$$

$$\left[\left(S_h-S_h^*\right)+\left(D_h-D_h^*\right)+\left(A_h-A_h^*\right)+\left(Q_h-Q_h^*\right)+\left(R_h-R_h^*\right)\right]\times\frac{d}{dt}\left(S_h+D_h+A_h+Q_h+R_h\right) \quad (1.35)$$

It implies that;

$$\frac{dF}{dt}=\left[\left(S_h-S_h^*\right)+\left(D_h-D_h^*\right)+\left(A_h-A_h^*\right)+\left(Q_h-Q_h^*\right)+\left(R_h-R_h^*\right)\right]$$
$$\times\mu\left(S_h^*+D_h^*+A_h^*+Q_h^*+R_h^*\right)-\delta Q_h^*$$
$$-\mu\left(S_h+D_h+A_h+Q_h+R_h\right)-\delta Q_h$$

$$\frac{dF}{dt}=-\left[\left(S_h-S_h^*\right)+\left(D_h-D_h^*\right)+\left(A_h-A_h^*\right)+\left(Q_h-Q_h^*\right)+\left(R_h-R_h^*\right)\right]\times$$
$$\left[\mu\left(S_h-S_h^*\right)+\mu\left(D_h-D_h^*\right)+\mu\left(A_h-A_h^*\right)+\mu\left(Q_h-Q_h^*\right)+\mu\left(R_h-R_h^*\right)+\delta\left(Q_h-Q_h^*\right)\right] \quad (1.36)$$

This shows that $\frac{dF}{dt}$ is negative and $\frac{dF}{dt}=0$ if and only if $S_h=S_h^*, D_h=D_h^*, A_h=A_h^*, Q_h=Q_h^*, R_h=R_h^*$

In addition, every solution of model (1.3) with the initial data approaches ξ^{**} at $t\to\infty$

Therefore, the largest compact invariant set in $\left\{\left(S_h,D_h,A_h,Q_h,R_h\right)\in\Omega:\frac{dF}{dt}\le 0\right\}$ is a singleton set therefore, it follows from LaSalle's invariant principles that the drug persistence equilibrium ξ^{**} is globally asymptotically stable in Ω whenever $R_0>1$.

Numerical simulation

Effects of Treatment of young people from drug abuse

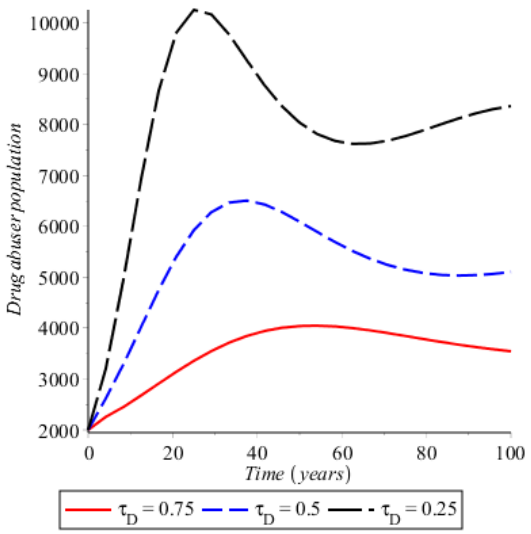

Fig. a

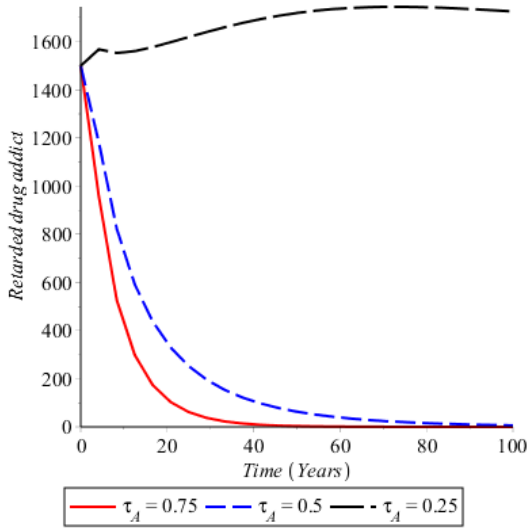

Fig. b

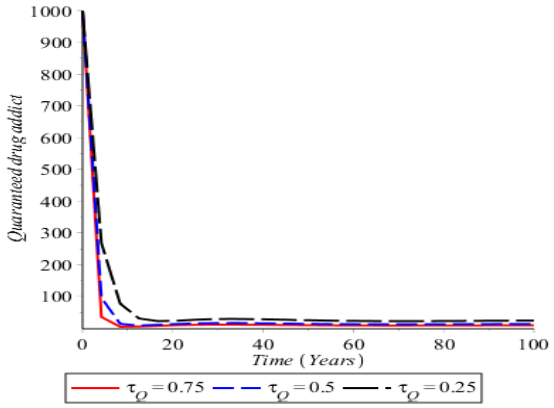

Fig. c

The above simulations in fig. a, b & c, shows the impact of drug abuse among youth/young people at various stages and their corresponding control measures proposed. From the graphs, it was discovered that at different drug abuse level, the adopted control measure indicate reduction in illicit drug use by young people. In this paper, we discovered that the effective control measure in drug abuse amongst youths is quarantine/rehabilitation before administrating any control (treatment or counselling/education).

Effect of recovery rate of young people from drug missuses

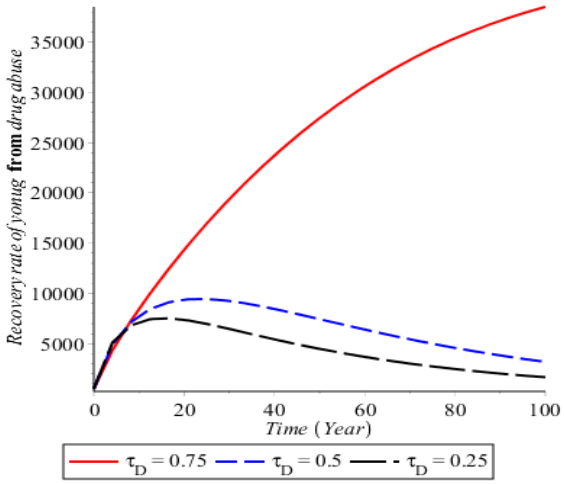

Fig. d

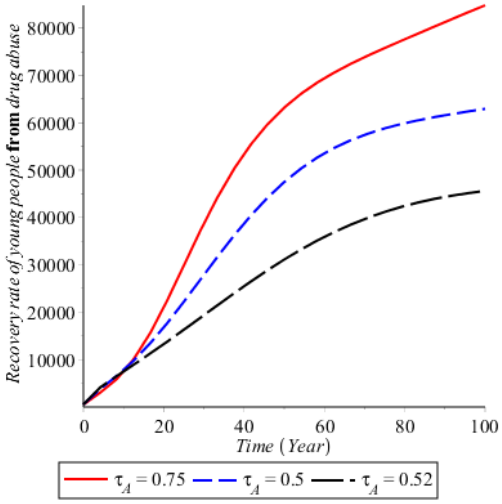

Fig. e

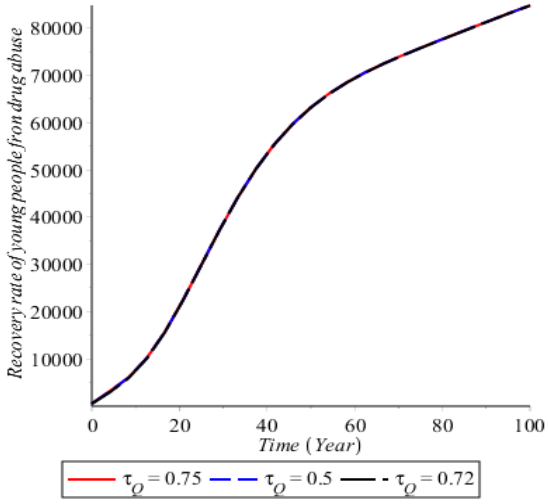

Fig. f

Figures (d, e & f) shows the dynamism of recovery rate of young people from drug abuse due to the treatment administered at different stages of abuse. From fig. d, it was discovered that administering treatment at high rate to youths at early stage of abuse facilitates quick recovery. Fig. e shows that the rate of recovery from drug abuse among youth when they had reached addiction stage is more difficult compared to that of the early drug abuse stage. Also, fig. f has shown that it is nearly impossible to decipher that all youthful exuberant are due to drug missuses.

Discussion

Meticulously, drug addiction can be controlled among youth whenever the initial size of young people population involved in drug abuse is small enough such that the control reproduction number is brought below unity. This is possible by involving all stakeholders in intensifying public awareness campaign on the health danger of drug use in the human population. Also, proper rehabilitation and treatment of all young persons who are addicted to drug should be carried out as a matter of urgency as shown from the simulation analysis above.

Conclusion

In this work, a 5 compartmental model was formulated using ordinary differential equations to study the effect of drug miss/addiction among youths. It was discovered that the model exhibits drug-free equilibrium point, which was shown to be locally and globally asymptotically stable when the basic reproductive number is less than unity. This means that drug abuse can be eradicated from human population with effective control measures put in place. Also, the analysis shows that the model exhibit drug abuse persistence equilibrium point, which was further proved to be locally and globally asymptotically stable if the control reproductive number is greater than unity. This means drug addiction will persist in human population if control measures are not properly enforced by all the stakeholders.

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