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Linear Programming Application in optimal allocation of Buses to Inter and Intra State Routes from Katsina State Transport Authority Service, Nigeria

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Abstract
In this paper, we study Linear Programming application in optimal allocation of Buses to inter and intra state routes from Katsina State transport authority services to maximize profit and boost the state's internally generated revenue. The problem was formulated from the data collected at their headquarters and by taking into consideration all the constraints. The TORA (a software for finding solution to linear programming problems) was used to obtained the solution to the model which yielded the maximum objective value of ₦729, 578 daily after 16 iterations. This gives better result when compared to the current initiative schedule by the authority that yields ₦584, 652 per day. By the recommendation schedule, the State will be getting additional ₦144,926.00 daily which translates to 24.8% daily profit increase and over ₦4,000,000.00 monthly when implemented.

Keywords: KTSTA, TORA, Linear Programming, Transport Services, Sensitivity analysis, Objectives Value.

INTRODUCTION
Linear Programming (LP), also called Linear Optimization is a method to achieve the best outcome (such as maximum profit or minimum cost) in the mathematical model whose requirements are represented by linear relationships (Lucey, 2002). Linear programming is a special case of mathematical programming (also known as mathematical optimization) (Kanu *et al,* 2014).

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Its feasible region is a convex polytope, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective is a real valued affine (linear) function defined on this polyhedron. A linear programming algorithm finds a point in the polytope

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where this function has the smallest (or largest) value if such a point exists (Ng *et al,* 2018, Salimifard *et al*, 2012).

Salimifard *et al*, (2012) stated that Linear programming contributed in attaining the optimum use of productive resources. It shows how a decision-maker can employ the productive factors effectively by selection and distribution of these resources. Linear programming approach brings quality life because of good decisions made. Linear programming approaches made possible and practical solutions since there could be other constraints operating outside the problem that must be taken into consideration. Production of many units does not translate to all been sold. Hence, necessary modification of its mathematical solution is required for the sake of convenience to the decision-maker (Bertsimas & Tsitsiklis, 1997, Fang & Puthenpura 1993).

However, according to Lieberman (2001) Linear programming major application is to problems where the constraints and the objective functions are linear. Factors such as uncertainty, weather conditions etc. are not taken into consideration. There is restriction to linear objective function (Hiller *et al*, 1995).

Linear programs are problems that can be expressed in canonical forms as

Optimize (Max or Min) $Z = c_1x_1 + c_2x_2 + ... + c_nx_n$

Subject to $a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$ (for maximization)

$a_{21}x_2 + a_{22}x_2 + ... + a_{2n}x_n \geq b_2$ (for minimization)

$a_{31}x_1 + a_{32}x_2 + ... + a_{3n}x_n = b_3$ (for equality)

… … …

$a_{1m}x_1 + a_{2m}x_2 + ... + a_{mn}x_n [\leq \text{ Or } \geq] b_m$

$x_i \geq 0, \forall i = 1,2,3,...,n$ (Non negativity restriction)

Brief History of Katsina State Transport Authority (KTSTA)

Katsina State Transport Authority is a governmental and non-profitable Organization created to provide an affordable, safe, and comfortable transportation to the people of Katsina State and Nigeria at large. KTSTA like other sister transport Organization uses different category of buses in carrying out its operations. It was inherited from Kaduna State Transport Authority when the state was created in 1987 by the Military Administration of General Ibrahim Babangida Badamasi. And it serves as one of major means of transportation within the country. KTSTA render the following services: Inter State services; Intra State services; International Route (Katsina-Senegal (Qualaha)); and on Request (Hire): It requires no specific route. KSTA is located at Tashar Batsari road Katsina.

Katsina State transport authority (KTSTA) operates 52-seater Marcopolo 1721, 30-seater Fuso Rosa, 18-seater Toyota Hiace, and 30-seater Toyota Coaster. According to research and interview with the KTSTA officials, the Buses incur costs in four ways: fuel consumption, percentage parking levy, routine service and maintenance (repair). Furthermore, this study is limited to the first two services – inter and intra state services due to interest. These routes are as follows:

Inter States Routes Possible No of Buses:

Katsina-Abuja, Katsina-Kaduna, Katsina-Kano, Katsina-Sokoto, Katsina-Bauchi, Katsina-Gombe,

Katsina-Minna, Katsina-Jos, and Katsina-Zaria

Intra States Routes Possible No of Buses:
Katsina-Daura, Katsina-Funtua, Katsina-Malumfashi, Katsina-Dutsin-Ma, Katsina-Mani, Katsina-Kankia, Katsina-Batsari, Katsina-Jibia, and Katsina-Ingawa

The main objective of the study was to apply linear programming model to optimally allocate the available buses of the Transport Authority to the service routes. The simulation results will be compare to the existing traditional method of allocation to see which one gives optimal result for profit maximization. Abubakar et.al (2020) conducted a research on Yoghurt Production by ATS Multi-Concept Worldwide Ltd in Katsina State, Nigeria where they were able to identified and recommend to the company on the quantity of production of some categories of yoghurt to be produce for profit maximization. The work carried out by Nyor *et al* (2014) inspired us to conduct this research

Sensitivity Analysis

Sensitivity analysis is a technique used for determining how the independent variable values will impact a specific dependent variable under a given assumption(s). It helps the researcher, reader and policy makers to know how sensitive a model is to changes in the value of parameters of the model and to change in the structure of the model (Monika and Ashish, 2019). Meanwhile, Goyal *et al* (2017) stated that sensitivity analysis dictate how the uncertainty in the output, or sensitivity in as output of a mathematical model or a complex system, can be assigned to different sources of uncertainty in its inputs or to change in an input while keeping the other inputs constant.

RESEARCH METHODOLOGY

In this section, data were collected and transform into linear programming problem taking into consideration all the constraints. Below data were collected in October, 2020 from KTSTA headquarters which is located at Tashar Batsari Road, Katsina.

Table 1: Data on Bus Routes

S/N	Route	Frequency of trip per week	Fuel Consumption (Ltrs) (PMS)	Percentage Parking Levy	No. of Buses Per route per Day	Max No. Of trips per Bus Per Day	No. Of hours per half Trip	Transport Fare per Person (₦)
1	KATSINA-ABUJA	DAILY	150	10	2	1	7	2,500
2	KATSINA-KADUNA	"	90	"	3	1	4	1,600
3	KATSINA-KANO	"	50	"	3	2	3	700
4	KATSINA-SOKOTO	"	150	"	1	1	6	3,000
5	KATSINA-BAUCHI	"	150	"	1	1	6	2,500
6	KATSINA-GOMBE	"	185	"	1	1	8	3,500
7	KATSINA-MINNA	"	170	"	1	1	7	3,500
8	KATSINA-JOS	"	150	"	2	1	7	2,500
9	KATSINA-ZARIA	"	70	"	3	1	3	1,200
10	KATSINA-DAURA	"	25	"	4	3	0.5	250
11	KATSINA-FUNTUA	"	55	"	2	2	1	600
12	KATSINA-MALUMFASHI	"	80	"	4	5	1	500
13	KATSINA-DUTSINMA	"	20	"	2	2	0.5	150
14	KATSINA-MANI	"	20	"	5	3	0.5	120
15	KATSINA-KANKIA	"	15	"	4	3	0.5	150
16	KATSINA-BATSARI	"	15	"	2	4	0.5	100
17	KATSINA-JIBIA	"	15	"	3	4	0.5	100
18	KATSINA-INGAWA	"	25	"	2	3	0.5	160
	TOTAL		1,435	180	45	39	56.5	23,130

Table 2: Data on Bus Service

S/N	Required Service Items	Amount (₦)
1	Oil filter	500
2	5-litre Engine oil	4200
3	Oil treatment	600
	Total	5300

Note: Buses in KTSTA are serviced twice in a month, which is after 15 days

Table 3: Data on bus repair/maintenance

S/N	Repair/maintenance items	Cost (₦)	Duration it lasts
1	Tyre	112,000	5 months
2	Front bearing	14,000	6 months
3	Break disk	6,000	1 year
4	Break pad	2,000	1 month
5	Break lining	4,000	1 month
6	Car battery	17,000	1 year
7	Shocks filling	1,500	3 months
8	Sparkling plugs	3,000	1 year
9	Fuel pump	3,500	1 year
10	Release bearing	3,000	1 year
	Total	166,000	

MODEL CONSTRUCTION

In this section, data obtained at KTSTA headquarter were used to formulate the model and subsequently solve the model

Table 4: Daily cost of Bus service

S/N	Required Service Items	Amount Per 15 Days (₦)	Amount Per Month (₦)	Daily Service cost per bus (₦)
1	Oil filter	500	1000	1000÷30 = 33
2	5-litre Engine oil	4200	8400	8400÷30 = 280
3	Oil treatment	600	1200	1200÷30 = 40
	Total	5300	10,600	353

Note: Buses in KTSTA are serviced twice in a month. This means that a bus in KTSTA consumes ₦10,600 in a month. Dividing service amount per month by 30 gives us ₦353 as the cost of servicing a bus in a day

Table 5: Daily cost of repair/maintenance

S/N	Repair/maintenance items	Cost (₦)	Duration it lasts	Duration in days	Cost per day (₦)
1	Tyre	112,000	5 months	5x30 = 150	747
2	Front bearing	14,000	6 months	6x30 = 180	78
3	Break disk	6,000	1 year	12x30 = 360	17
4	Break pad	2,000	1 month	30x1 = 30	67
5	Break lining	4,000	1 month	1x30 = 30	133
6	Car battery	17,000	1 year	12x30 = 360	47
7	Shocks filling	1,500	3 months	3x30 = 90	17
8	Sparkling plugs	3,000	1 year	12x30 = 360	08
9	Fuel pump	3,500	1 year	12x30 = 360	10
10	Release bearing	3,000	1 year	12x360 = 360	08
	Total	166,000			1,132

Table 6: Daily contribution per bus

Fuel consumption per bus (ltrs)	Fuel contribution per bus (₦)	Percentage parking levy per bus (%)	Monetary parking levy (₦)	No. of buses per route per day	Max. no. of buses per route per day	Equivalent no. of buses per route per day	No. of hours per half trip	Transport fare per person	Daily cost of a bus service	Daily cost of a bus repair/ maintenance	Return per bus per trip (₦)	Daily total expenditure per bus (₦)	Daily contribution per bus (₦)
150	24,750	10	3,250	2	1	2	7	2,500	353	1,132	65,000	29,480	35,515
90	14,850	10	2,080	3	1	3	4	1,600	353	1,132	41,600	18,415	23,185
50	8,250	10	910	3	2	6	3	700	353	1,132	18,200	10,645	7,555
150	24,750	10	3,000	1	1	1	6	3,000	353	1,132	60,000	29,235	30,765
150	24,750	10	3,250	1	1	1	6	2,500	353	1,132	65,000	26,485	35,515
185	30,525	10	4,550	1	1	1	8	3,500	353	1,132	91,000	36,560	54,440
170	28,050	10	4,550	1	1	1	7	3,500	353	1,132	91,000	34,085	56,915
150	24,750	10	3,250	2	1	2	7	2,500	353	1,132	65,000	29,485	33,512
70	11,550	10	1,560	3	1	3	3	1,200	353	1,132	31,200	14,595	16,605
25	4,125	10	325	4	3	12	0.5	250	353	1,132	9,000	5,935	3,051
55	9,165	10	780	2	2	4	1	600	353	1,132	21,600	11,430	10,170
80	13,200	10	650	4	5	20	1	500	353	1,132	18,000	15,335	2,665
20	3,300	10	195	2	2	4	0.5	150	353	1,132	5,400	4,980	420
15	2,475	10	165	5	3	15	0.5	120	353	1,132	4,320	4,116	204
20	3,300	10	195	4	3	12	0.5	150	353	1,132	5,400	4,980	420
15	2,475	10	130	2	4	8	0.5	100	353	1,132	3,600	4,090	-490
15	2,475	10	130	3	4	12	0.5	100	353	1,132	3,600	4,090	-490
25	4,125	10	208	2	3	6	0.5	160	353	1,132	5,760	5,818	-58
1,435	236,865	180	29,169	45	39	113	56.5	23,130	6,354	20,376	604,680	289,764	311,899

Note:

- 'Daily cost of service per bus' and 'daily cost of repair/maintenance per bus' are obtained in tables 4 and 5 respectively.
- 'Equivalent number of buses per route' is obtained by multiplying 'Number of buses per route' by 'number of trips per day'. Remember, a trip means to and fro Katsina.
- 'Return per bus per trip' is obtained by multiplying 'transport fare' per passenger by 2

- 'Total expenditure per bus' is obtained by adding 'fuel consumption', Monetary parking levy', 'daily service per bus' and 'daily repair/maintenance per bus'.

Table 7: Daily contribution per route

S/N	Routes (Katsina to-)	Monetary parking levy per route	Equivalent no. of buses per route per day	Fuel consumption per route	Transport fare per person	Daily cost of bus service per route	Daily cost of bus repair/maintenance per route	Return per route	Daily total expenditure per route	Daily contribution per route
1	Abuja	6,500	2	49,500	2,500	706	2,264	130,000	58,970	71,030
2	Kaduna	6,240	3	44,550	1,600	1,059	3,396	124,800	55,245	69,555
3	Kano	5,460	6	49,500	700	2,118	6,792	109,200	63,870	45,330
4	Sokoto	3,000	1	24,750	3,000	353	1,132	60,000	29,235	30,765
5	Bauchi	3,250	1	54,750	2,500	353	1,132	65,000	59,485	5,515
6	Gombe	4,550	1	30,525	3,500	353	1,132	91,000	36,560	54,440
7	Minna	4,550	1	28,050	3,500	353	1,132	91,000	34,085	56,915
8	Jos	6,500	2	49,500	2,500	706	2,264	130,000	58,970	71,030
9	Zaria	4,680	3	34,650	1,200	1,059	3,396	93,600	43,785	49,815
10	Daura	3,900	12	49,500	250	4,236	13,584	108,000	71,220	36,780
11	Funtua	3,120	4	36,660	600	1,412	4,528	86,400	45,720	40,680
12	Malumfashi	13,000	20	264,000	500	7,060	22,640	360,000	306,700	53,300
13	Dutsin-Ma	780	4	13,200	150	1,412	4,528	21,600	19,920	1,680
14	Mani	2,475	15	37,125	120	7,060	16,980	64,800	61,875	2,925
15	Kankia	2,340	12	39,600	150	1,412	13,584	64,800	59,760	5,040
16	Batsari	1,040	8	19,800	100	5,295	9,059	28,800	32,720	-3920
17	Jibia	1,560	12	29,700	100	4,236	13,584	43,200	49,080	-5,880
18	Ingawa	1,248	6	24,750	160	2,118	6,792	34,560	34,908	-348
	Total	74,193	113	880,110	23,130	39,889	127,916	1,706,760	1,122,108	584,652

KTSTA Problem Formulation

The Problem is Formulated using the information on Table 6. The problem is formulated under the assumption that all the 113 buses that commute intra and inter states routes were working daily optimally. Based on the interview with KTSTA, availability of passengers and the number of other transport services plying the same routes, among other factors, determine the number of buses that KTSTA can assign to these routes. The following shows the possible number of buses that can be scheduled to routes:

Table 8: Possible number of buses

Inter- State Routes	Possible number of Buses	Intra-State Routes	Possible number of buses
Katsina-Abuja	2-3	Katsina-Daura	12
Katsina-Kaduna	3-4	Katsina-Funtua	4
Katsina-Kano	6	Katsina-Malumfashi	20
Katsina-Sokoto	1	Katsina-Dutsin-Ma	4
Katsina-Bauchi	1	Katsina-Mani	15
Katsina-Gombe	1	Katsina-Kankia	12
Katsina-Minna	1	Katsina-Batsari	8
Katsina-Jos	2-3	Katsina-Jibia	12

Katsina-Zaria	3-4	Katsina-Ingawa	6

Noted that the allocation must not exceed 113 buses available for intra and inter states services. Below is formulated problem

$$35515x_1 + 23185x_2 + 7555x_3 + 30765x_4 + 35515x_5 + 544409x_6 + 56915x_7 + 33512x_8 + 16605x_9 + 3051x_{10} + 10170x_{11} + 2665x_{12} + 420x_{13} + 204x_{14} + 420x_{15} - 490x_{16} - 490x_{17} - 58x_{18}$$

$$x_1 \leq 4$$

$$x_2 \leq 4$$

$$x_3 \leq 6$$

$$x_4 \leq 1$$

$$x_5 \leq 1$$

$$x_6 \leq 1$$

$$x_7 \leq 1$$

$$x_8 \leq 3$$

$$x_9 \leq 4$$

$$x_{10} \leq 12$$

$$x_{11} \leq 4$$

$$x_{12} \leq 20$$

$$x_{13} \leq 4$$

$$x_{14} \leq 15$$

$$x_{15} \leq 12$$

$$x_{16} \leq 8$$

$$x_{17} \leq 12$$

$$x_{18} \leq 6$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} \leq 113$$

NUMERICAL RESULTS AND DISCUSSION

In this section, result of the problem formulated will be analyze and discuss. The formulated problem was solved using TORA – computer software used in solving Linear Programming problems. The software was developed by Hamdy Taha (2002 edition). The software was run HP Laptop with processor 1.3GHz, 4.00 GB RAM and 64-bit operating system.

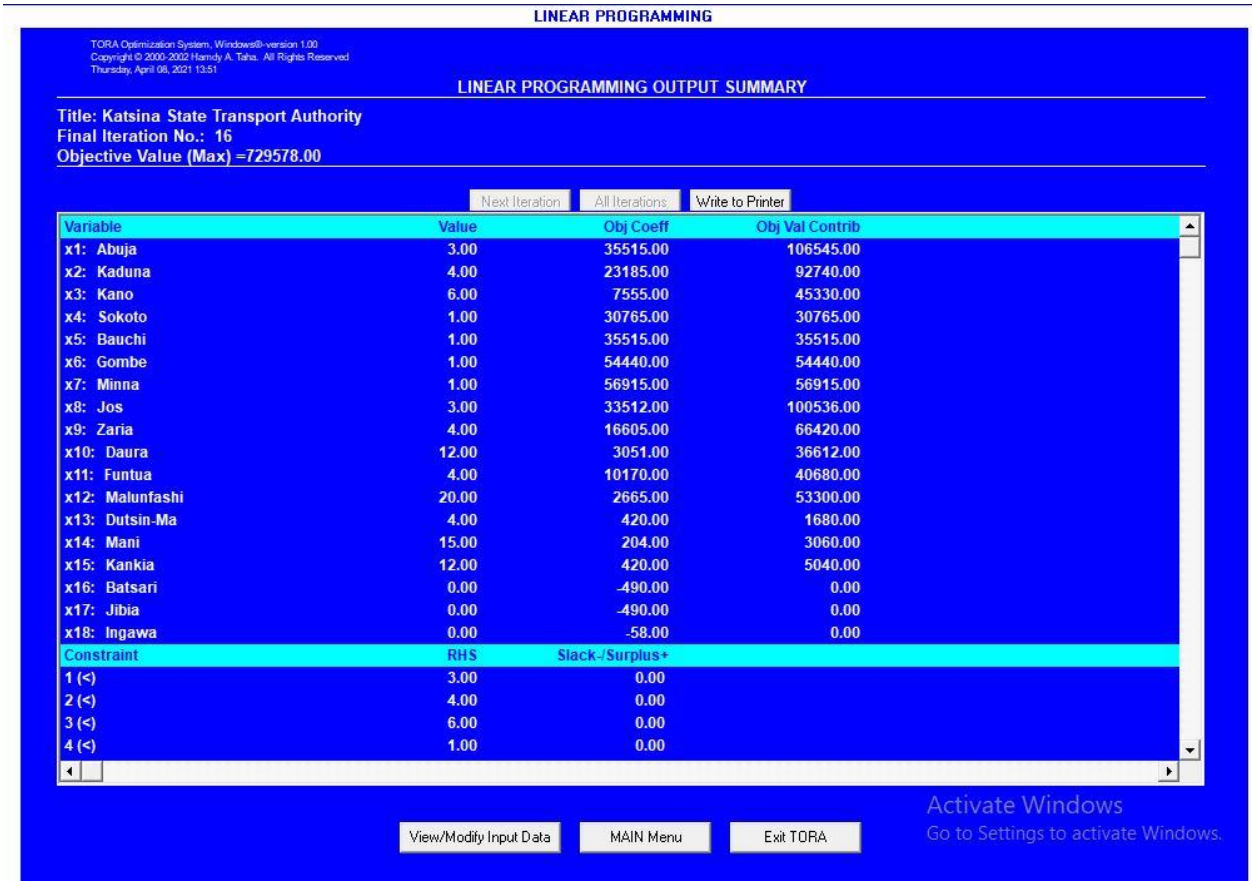

Figure1. Linear Programming Output Summary

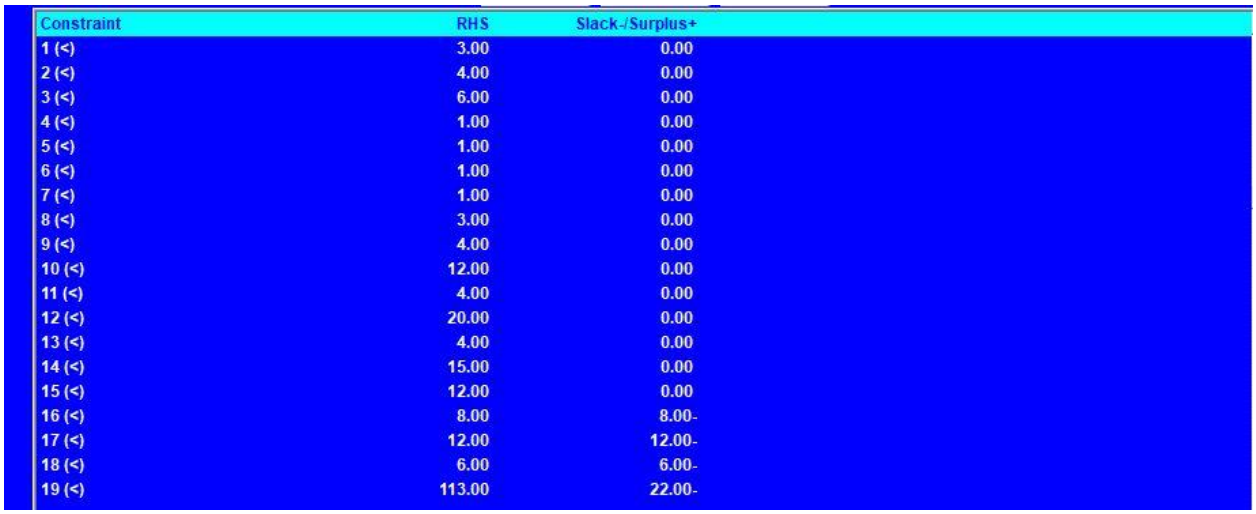

Figure2. Linear Programming Output Summary

Figures 1 and 2 above, variable is the name of the route. For example, x_1 represents Katsina to Abuja route. Value is the number of buses to be allowed to ply Katsina-Abuja. Objective coefficient is the contribution made per bus. Objective value contribution is the result of multiplying 'value' and the 'objective coefficient'. $x_{16}, x_{17},$ and x_{18} are zero because the buses are running at lost, therefore, no bus is recommend to be allocate to those routes. The optimum solution is reached at 16th iterations with the objective value of ₦729, 578 is obtained as can be seen on TORA window.

Sensitivity Analysis

Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1: Abuja	35515.00	0.00	infinity	0.00
x2: Kaduna	23185.00	0.00	infinity	0.00
x3: Kano	7555.00	0.00	infinity	0.00
x4: Sokoto	30765.00	0.00	infinity	0.00
x5: Bauchi	35515.00	0.00	infinity	0.00
x6: Gombe	54440.00	0.00	infinity	0.00
x7: Minna	56915.00	0.00	infinity	0.00
x8: Jos	33512.00	0.00	infinity	0.00
x9: Zaria	16605.00	0.00	infinity	0.00
x10: Daura	3051.00	0.00	infinity	0.00
x11: Funtua	10170.00	0.00	infinity	0.00
x12: Malunfashi	2665.00	0.00	infinity	0.00
x13: Dutsin-Ma	420.00	0.00	infinity	0.00
x14: Mani	204.00	0.00	infinity	0.00
x15: Kankia	420.00	0.00	infinity	0.00
x16: Batsari	-490.00	-infinity	0.00	490.00
x17: Jibia	-490.00	-infinity	0.00	490.00
x18: Ingawa	-58.00	-infinity	0.00	58.00
Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<)	3.00	0.00	25.00	35515.00
2 (<)	4.00	0.00	26.00	23185.00

Figure3. Linear Programming Output Summary

x18: Ingawa	-58.00	-infinity	0.00	58.00
Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<)	3.00	0.00	25.00	35515.00
2 (<)	4.00	0.00	26.00	23185.00
3 (<)	6.00	0.00	28.00	7555.00
4 (<)	1.00	0.00	23.00	30765.00
5 (<)	1.00	0.00	23.00	35515.00
6 (<)	1.00	0.00	23.00	54440.00
7 (<)	1.00	0.00	23.00	56915.00
8 (<)	3.00	0.00	25.00	33512.00
9 (<)	4.00	0.00	26.00	16605.00
10 (<)	12.00	0.00	34.00	3051.00
11 (<)	4.00	0.00	26.00	10170.00
12 (<)	20.00	0.00	42.00	2665.00
13 (<)	4.00	0.00	26.00	420.00
14 (<)	15.00	0.00	37.00	204.00
15 (<)	12.00	0.00	34.00	420.00
16 (<)	8.00	0.00	infinity	0.00
17 (<)	12.00	0.00	infinity	0.00
18 (<)	6.00	0.00	infinity	0.00
19 (<)	113.00	91.00	infinity	0.00

View/Modify Input Data | MAIN Menu | Exit TORA

Activate Windows
Go to Settings to activate Windows.

Figure4. Linear Programming Output Summary

Figures 3 and 4 present the sensitivity analysis report. The sensitivity analysis was carried out to know the extent to which any of the data (that is, the per unit profit of the variable) of the company can change without affecting the primal solution. The result of the sensitivity analysis also showed the dual price also known as the shadow price enable the company to know the cost of acquiring an additional unit of any of their resources in case there is a need.

Table 9: Current Schedule versus recommended schedules

Inter- State Routes	KTSTA current schedule	Recommended schedule	Intra-State Routes	KTSTA current schedule	Recommended schedule
Katsina-Abuja	2	3	Katsina-Daura	12	12
Katsina-Kaduna	3	4	Katsina-Funtua	4	4
Katsina-Kano	6	6	Katsina-Malumfashi	20	20
Katsina-Sokoto	1	1	Katsina-Dutsin-Ma	4	4
Katsina-Bauchi	1	1	Katsina-Mani	15	15
Katsina-Gombe	1	1	Katsina-Kankia	12	12
Katsina-Minna	1	1	Katsina-Batsari	8	0
Katsina-Jos	2	3	Katsina-Jibia	12	0
Katsina-Zaria	3	4	Katsina-Ingawa	6	0

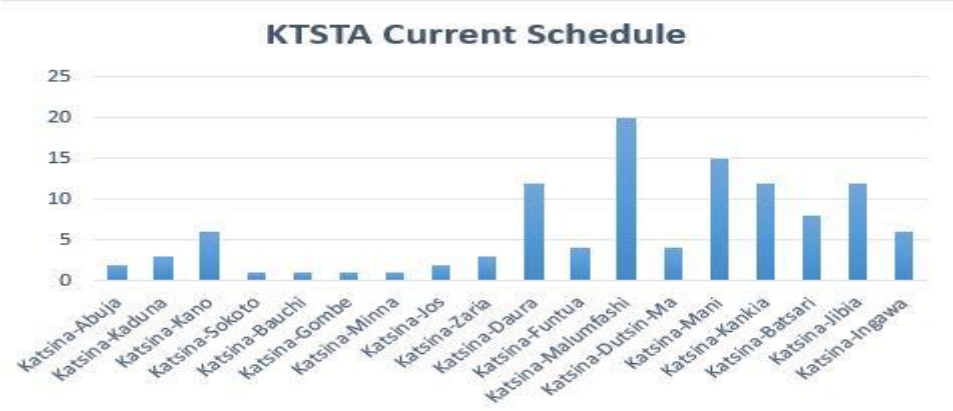

Figure 5: KTSA current schedule

Figure 6: KTSA recommended schedule

Figures 5 and 6 show the current and recommend schedule for the KTSTA respectively. It is observed from the Figure 6 that in the recommend chart, Katsina to Batsari, Katsina to Jibia and Katsina to Ingawa are zeros, which implies no allocation made for these routes in the recommendation schedule as against 8, 12 and 6 respectively in the current intuitive way of scheduling. This is justifiable because KTSTA is recording losses plying those routes.

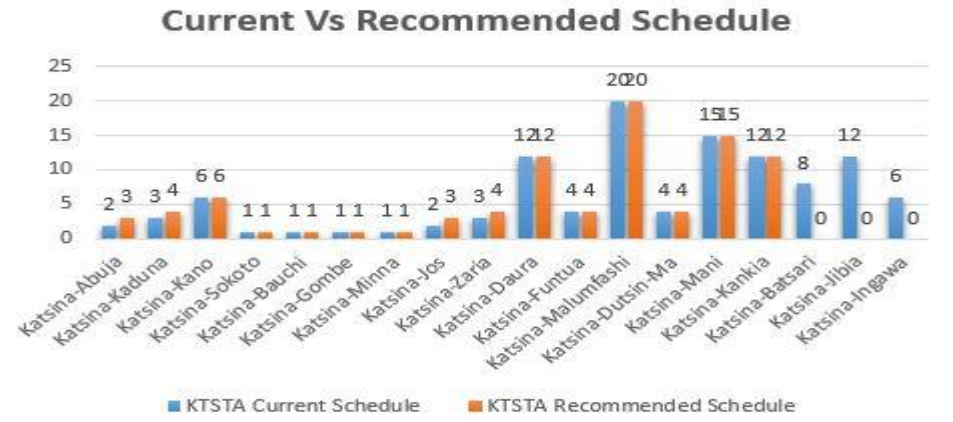

Figure 7: KTSTA current versus recommended schedules

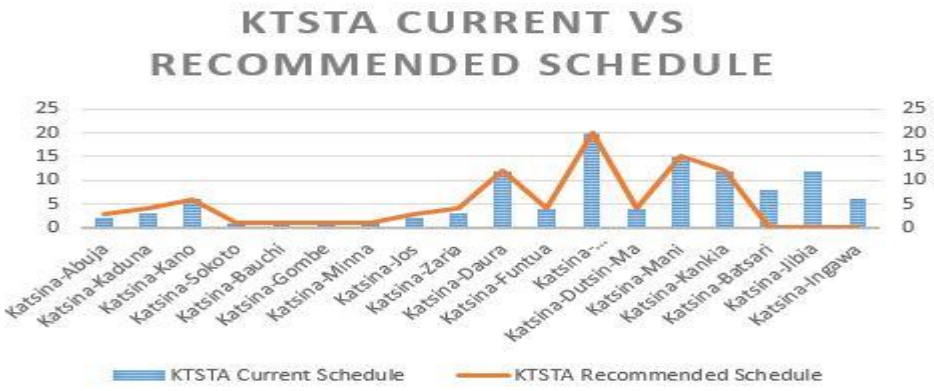

Figure 8: KTSTA current Versus recommended schedules

Figures 7 and 8 represent the combination of current and recommendation charts for clearer illustration of the current and recommendation schedules. Recommendation suggests either increase or decrease or no change in the current intuitive way of scheduling for optimum profit to the organization. Recommendation chart suggest increase in allocation of buses in the following routes: Katsina-Abuja from 2 to 3, Katsina -Kaduna from 3 to 4, Katsina- Jos from 2 to 3, and Katsina -Zaria from 3 to 4 for better profit to KTSTA. While no change in the allocation of buses was made to Katsina – Kano, Katsina –Sokoto, Katsina- Bauchi, Katsina-Gombe, Katsina-Minna, Katsina- Daura, Katsina-Funtua, Katsina- Malumfashi, Katsina-Dutsin-Ma, Katsina-Mani, and Katsina-Kankia. However, recommendation suggest drastic decrease in the allocation buses, in fact zero allocation in the followings routes: Katsina-Batsari, Katsina –Jibia and Katsina-Ingawa. This zero allocation is due to losses recorded on the routes as shown in Table 6.

CONCLUSION

The current KTSTA schedule seen in Table 7 yields a daily contribution of ₦584,652.00. Our recommended schedule will yield ₦729,578.00 when implemented. Furthermore, consideration of sensitivity analysis will yield more profit from the recommended solution obtained when more resources are deployed to the appropriate routes. The organization will be getting additional ₦144,926.00 daily which translate to 24.8% daily profit increase and over ₦4,000,000.00 monthly when implemented. Especially, at this current unstable economic situation, this recommendation will help in boosting the state internally generated revenue (IGR).

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