COLLECTIVE TRANSPORTATION SYSTEM MANAGEMENT IN ALGIERS AREA: A GAME THEORETIC APPROACH

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ABSTRACT

In Algiers area, several collective transportation means such as: public buses, private buses, tramway, metro, train, etc… are available; nevertheless ,the lack of coordination among them arises some relevant questions concerning the profitability of these means, as well as the satisfaction of travellers. In order to answer these questions, a statistical survey is conducted by the urban dynamics team of our research centre. Furthermore, the present work aims to figure out optimal strategies of both public authority and the private transport union, taking into account the travellers' reactions. In this direction, a three-step game is considered. To compute a solution to the game under consideration, an ABC algorithm combined to TOPSIS method are used. The whole methodology is applied to Algiers area and bring solutions that motivate the private operators to cooperate with the authorities' plans. In addition, the developed algorithm has the advantage to be flexible, since it allows multiple optimal solutions.

KEY WORDS: Collective transport; game theory; Algiers area.

JEL CLASSIFICATION : C72; L62.

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إدارة نظام النقل الجماعي في منطقة الجزائر: منهج نظرية الألعاب

ملخص

تتوفر في منطقة الجزائر العاصمة العديد من وسائل النقل الجماعي مثل: الحافلات العامة، والحافلات الخاصة، والترام، والمترو، والقطار، إلخ. ومع ذلك، فإن الافتقار إلى التنسيق فيما بينها يثير بعض الأسئلة ذات الصلة فيما يتعلق بربحية هذه الوسائل، فضلاً عن إرضاء المسافرين. للإجابة على هذه الأسئلة، يتم إجراء مسح إحصائي بواسطة فريق الديناميكيات الحضرية في مركز الأبحاث لدينا. علاوة على ذلك، يهدف العمل الحالي إلى معرفة الاستراتيجيات المثلى لكل من السلطة العامة واتحاد النقل الخاص، مع مراعاة ردود فعل المسافرين. في هذا الاتجاه، يتم النظر في لعبة من ثلاث خطوات. لحساب حل للعبة قيد الدراسة، يتم استخدام خوارزمية ABC مدمجة مع طريقة TOPSIS. يتم تطبيق المنهجية بأكملها في منطقة الجزائر العاصمة وتقديم الحلول التي تحفز المشغلين الخاصين على التعاون مع خطط السلطات. بالإضافة إلى ذلك، تتمتع الخوارزمية المطورة بميزة كونها مرنة، لأنها تتيح العديد من الحلول المثلى.

كلمات مفتاحية: النقل الجماعي؛ نظرية اللعبة؛ منطقة الجزائر العاصمة

GESTION DU SYSTÈME DE TRANSPORT COLLECTIF DANS LA WILAYA D'ALGER : APPROCHE PAR LA THÉORIE DES JEUX

RÉSUMÉ

Dans la région d'Alger plusieurs moyens de transports collectifs tels que : bus publics, bus privés, tramway, métro, train, etc. sont disponibles ; néanmoins le manque de coordination entre eux pose des questions pertinentes quant à la rentabilité de ces moyens, ainsi qu'à la satisfaction des voyageurs. Afin de répondre à ces questions, une enquête statistique est menée par l'équipe de dynamique urbaine de notre centre de recherche. En outre, le présent travail vise à déterminer les stratégies optimales de l'autorité publique et du syndicat des transports privés, en tenant compte des réactions des voyageurs. Dans ce sens, un jeu en trois temps est envisagé. Pour calculer une solution au jeu considéré, un algorithme ABC combiné à la méthode TOPSIS est utilisé. Toute la méthodologie est appliquée à la région d'Alger et apporte des solutions qui motivent les opérateurs privés à coopérer avec les plans des autorités. De plus, l'algorithme développé a l'avantage d'être flexible, puisqu'il permet de multiples solutions optimales.

MOTS CLÉS : Transport collectif ; la théorie des jeux ; Région d'Alger.

INTRODUCTION

Collective transports play a major role in the mobility of citizen, as well as in the economy of the city. Indeed, a powerful collective transportation system, that meets citizen's daily requirement would encourage them to use these modes rather than the personal vehicle; hence, the decrease of traffic congestion and its drawbacks (economic losses, pollution, etc).

With about three million inhabitants concentrated in1190Km^2, Algiers is the most populated city in Algeria. Three main studies concerning the inhabitants' mobility were conducted over these last decades. The two first studies were carried before the introduction of the tram and metro, hence there were three major transportation means (buses, personal vehicle and taxis), in addition to the train and the walking. The first study was carried in 1990 (C.E.M.A. & B.E.T.U.R., 2004) and showed that 53% of the inhabitants use collective transportation, whereas 40% use personal vehicle, 6% use taxis and only 1% use other means (train and walking). The second one, held in 2004 (C.E.M.A. & B.E.T.U.R., 2004), showed an increase of the collective transportation use, indeed the percentage of the inhabitants using this mean reached 65%. However, the use of the personal vehicle decreased to 29%, the use of taxis decreased slightly to 5% whereas the use of the other means remains the same. The third study was conducted in 2011 (Baouni., Bakour., & Berchache., 2013) after the introduction of the tram and the metro. In the last one, the personal vehicle appears as the most used transportation means, with about 53,71% of inhabitants using it whereas the combination bus-metro is used by 22% of inhabitants and the bus-tram combination is used by 18%. In the present study, it is found that the percentage of inhabitants using collective transportation has decreased to 34% since then. These results can be explained by the lack of coordination among the collective transportation modes, which induces the users' dissatisfaction on the one hand, and the government policies regarding the allocation of bank credit easily for a car acquisition on the another hand.

Problem statement

The articles 14,18 and 21 described in the decree of August 11, 2007 of the Algerian constitution, define the conditions that collective transport mode must satisfy. In general, these conditions stipulate that the collective transport mode must assume a continue service in terms of timing, frequencies, itineraries and stopover points. Furthermore, the travelers must be boarded/land at determined stopovers; also, the health and safety measures must be respected. In Algiers, the collective transport is a public service executed by private or public operators, both controlled by the Transport Organizing Authority (TOA). Thereby, this authority manages the transit network and supplies metro, tramway, taxis, suburban trains and buses. The TOA is also responsible of the contract needed by the private operators to ensure the bus transportation service. These agreements must meet the strategic state's objectives. The most important risk factors to be included in such contract are those concerning the underlines objectives, the supply evaluation, the risk repartition, the financial sustainability and conflicts management (Bray & Mulley, 2013). Hence, the TOA can face several difficulties when making calls for tenders. Indeed, unclear description of the objectives and the expected results, mis-evaluated supplies or risk and responsibilities repartition among the contracting parties are the most relevant ones (Bray & Mulley, 2013).

The achievement of optimal performance is strongly related to what is called efficient contract. This latter must be evaluated following procedures' criteria and assessed results of an integrated strategy at tactical and operational level. Defining clear objectives at strategic level leads to tactical planning of the system and involves a service provision that meets these objectives. Furthermore, an efficient negotiated contract and the trust among the operators and regulators remain very important criteria (Stanley & Longva, 2010). In another side, questions about the relationship between the performance of the operators and the contract's agreements are addressed in the literature. In this direction, some authors showed that a negotiated contract brings more efficient results than a competitive scheme (Hensher, 2007; Mellish, 2009). Indeed, lack of transparency in the service market attribution on the one hand, and the limited capacity of the TOA to control whether the operators respect the contract's agreements on the other hand makes the competitive scheme less performant than the negotiated one (Yvrande-Billon, 2008). That is why sanctions such as warnings, penalties or contract's cancellation must be applied.

The private operators aim essentially to make their investment profitable. Indeed, without any authority's subvention, these operators prefer to maximize their own gain instead of assuring a good quality of service. On the other side, the public operators are supplied by the authority's subvention; hence, their first objective is to provide a high quality of service to the travelers, without carrying about the financial

profitability. Thereby, there are initially two different operating ways of the Algiers' public transport system; that obviously lead to an unbalanced supply/ demand law. Furthermore, the private operators respect their own rules established in a self-organized manner.
The main problem is then the design of an optimal plan which includes the number of stopovers between each origin-destination pair, the frequencies and the fares of each transportation mode, that increase the satisfaction of the travelers and encourage them to use collective transport on one hand, and on the other hand, maximize both profits of public authority and private operators.

Related works

Such plans are traditionally addressed in the so-called Urban Transportation Network Design Problems (UNTDP) (Farahani, Miandoabchi, Szeto, & Rashidi, 2013), especially, in the Urban Transit Network Design Problems (UTNDP). These later include: 1-strategic decisions, such as routes design, 2- tactical decisions, such as frequencies settings and timetabling, and 3-operational decisions, such as vehicles scheduling and crew scheduling (Magnanti & Wong, 1984; Guihaire & Hao, 2008).

In order to evaluate and manage public transport, several case studies were conducted. Rietveld et al. (Rietveld, Bruinsma, & Van Vuuren, 2001) studied the reliability of public transport means in Netherland and provide an estimation of both the unreliability transport chain in the multimodal case and the costumer evaluation of this unreliability. Cullinane (Cullinane, 2002) demonstrated among a survey of 389 university students in Hong-Kong that a good public transport impact car ownership, such that 65% of the asked students were unlikely to buy a car, since the Hong-Kong public transport is plentiful and cheap. Zhan Guo et al. (Guo & Wilson, 2011) proposed a path choice-based method to study the transfer cost along a used sequence of transportation modes of passengers' trip. The proposed approach is then applied to the London Underground and showed that the public transport system can be improved by a better understanding of transfer behaviour. Cascetta et al. (Cascetta & Cartenì, 2014) studied the effects of service quality including different quality indicators on different public transport policies in Campania, Italy. Irtema et al. (Irtema, Ismail, Borhan, Das, & Alshetwi, 2018) conducted a survey of 412 public transport passengers to examine the relationship between their behavioural intention and several factors, such as quality of service, perceived value, involvement and satisfaction. The authors showed that the aforementioned factors affect positively the behavioural intension of the passengers. Murray et al. (Murray & Wu, 2003) proposed two approaches based on the p-median problem to find the optimal number of stop stations and their geographical localisation in Columbus, Ohio. In order to fix optimal itineraries, frequencies and capacities of the Sandiago de Chile public transport system, which is composed by buses and metro, De Cea Ch et al. (de Cea Ch & Malbran, 2008) considered a bi-level program. The first level aims to determine the optimal itineraries and the second one concerns the corresponding frequencies and capacities. The obtained solution consists of a central corridor of a metro line completed by a high-capacity buses fleet, in addition to a fleet of buses with lower capacities serving the influence area of some main corridor. Other mathematical approaches are used to treat the problem of public transport design problems. According to Van Nes et al. (Van Nes, Hamerslag, & Immers, 1988) these approaches can be classified into six categories: 1- analytical methods used to derive optimal relations of the public transport parameters; 2- models used to construct the public transport network i.e. the selection of the links; 3- models that determine the routes without considering the frequencies; 4- models that determine the corresponding frequencies of a given set of routes; 5- models that determine the routes and the frequencies sequentially and 6- models that determine the routes and the frequencies simultaneously. In this work the authors focused on the two last categories. In the same direction, Cancela et al. (Cancela, Mauttone, & Urquhart, 2015) proposed a mixed integer linear program aiming to define the number and the itineraries of bus routes and their frequencies taking into account the interests of both users and operators, the users' behaviour, and constraints regarding transfer, infrastructure and bus capacities. Mahdavi Moghaddam et al. (Mahdavi Moghaddam, Rao, Tiwari, & Biyani, 2019) also modelled the problem of finding optimal bus routes and their corresponding frequencies as weighted sum multi-objective genetic optimization in order to minimize costs of both users and operator.

Another important parameter in the public transport management is the ticket price (fares). In this direction, Borndörfer et al. (Borndörfer, Karbstein, & Pfetsch, 2012) presented different mathematical models to optimize the fares and applied the resulting models to compute optimal fare of Potsdam transit network. Kaddoura et al. (Kaddoura, Kickhöfer, Neumann, & Tirachini, 2015) proceeded to a microscopic simulation to estimate optimal bus fares. Other studies considered the optimization of both frequencies and fare (Huang, Liu, Liu, & Chen, 2016; Chien & Tsai, 2007). Chien et al. (Chien & Tsai, 2007) formulated the problem of maximizing the profit for a transit route as a mathematical program, where the profit is considered as the difference between the revenue (demand multiplied by the fares) and the supplier cost (fleet size multiplied by the operating cost per hour). The authors developed a method based on the gradient to solve the built program and considered a case study of Newark, NJ, USA. Huang et al. (Huang, Liu, Liu, & Chen, 2016) proposed a three-party game where there are two leaders and one follower. The first leader is the authority who aims to maximize the total social welfare and the second one is a transit company who aims to maximize its profits. The follower represents a passenger who aims to minimize his total travel time. The authors considered non-linear fare function, based on the Euclidian distance between each Origin-Destination pair and used Artificial Bee Colony algorithm to solve the game.

Our contribution

In the present work, the fares, the number of stopovers and the frequencies of each collective transportation mode linking a given Origin-Destination are computed in order to optimize the management of transit network such as the Algiers one. In this direction, a three-step game is considered. On the first step of the game, a player representing the authority fixes the aforementioned parameters. In addition, to the probability of inspecting the private operators. On the second step, a player representing the private operators' union chooses whether to cooperate with the first player and applies the fixed parameters, or disobeys by selecting his own frequencies and stopovers number and keeping the same fares. If the second player disobeys and gets controlled by the first player, he will pay a penalty. The third step is added in order to take into account the passengers' satisfaction, hence a fictive player choosing the distribution of the passengers among the different modes, based on their satisfaction is considered. To solve the game under consideration an Artificial Bee Colony Algorithm (ABC) combined to the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) are used.

The rest of this paper is outlined as follows, section II is devoted to the game modelling approach, the case study is then exposed in section III. Finally, a conclusion is drawn in section IV.

1- GAME MODELLING APPROACH

Nomenclature:

$\boldsymbol{A_1}$	**Origin-Destination matrix**
$\boldsymbol{M_T = \{M_{T1}; M_{T2}; \ldots; M_{Tm}\}}$	Vector of m transportation means
$\boldsymbol{T}$	A $(m \times \lvert A_1 \rvert)$matrix with$T_{j,i}$ The fare of a trip through the$\boldsymbol{i^{th}}(\boldsymbol{O-D})$pair using the$\boldsymbol{j^{th}}$transportation means.
$\boldsymbol{C}$	A $(m \times \lvert A_1 \rvert)$matrix with$C_{j,i}$ The cost of a trip through the$\boldsymbol{i^{th}}(\boldsymbol{O-D})$pair using the$\boldsymbol{j^{th}}$transportation means.
$\boldsymbol{L}$	A $(m \times \lvert A_1 \rvert)$matrix with $L_{j,i}$ The number of trips needed to reach a destination **D** from an origin **O** using the$\boldsymbol{j^{th}}$transportation means.
$\boldsymbol{F_p}$	A $(m \times \lvert A_1 \rvert)$matrix with$F_{pj,i}$ The frequency of the$\boldsymbol{j^{th}}$transportation meanthrough the$\boldsymbol{i^{th}}(\boldsymbol{O-D})$pair.
$\boldsymbol{P}$	A $(\lvert A_1 \rvert \times m)$matrix with $P_{i,j}$ The percentage of users travelling through the$\boldsymbol{i^{th}}(\boldsymbol{O-D})$pair using the$\boldsymbol{j^{th}}$transportation means.
$\boldsymbol{\rho}$	Vector of m probabilities, such as $\boldsymbol{\rho_i}$is theprobability that the private buses

	operating through the $i^{th}(O-D)$ pair get controlled by the authority

Source: defined by the authors

1.1- The players set

The players set J represents the important actors of the problem under consideration, hence

$J = \{J_1: authority;\ J_2: private\ operators\ union;\ J_3: fictive\ player\}$

As the focus of the present study is at a macroscopic level, The fictive player is used to represent the dispersion of the passengers among the different transportation means.

1.2- The strategies sets

The player J_1 has the power to choose for each $(O-D)$pair, the frequencies, the stopovers number and the fare of each transportation mean. Consequently, the k^{th} strategy $x^{1k} \in X^1$of the player J_1 is a n-uplet $\left(F_p; L; T; \rho\right)_k$ of the above defined vectors and matrices.

The player J_2chooses to cooperate with the first player (J_1) and follows the managed plan, or disobey to the authority by choosing other frequencies and stopovers number. Thereby, his strategy set $X^2 = \{cooperate; (\widetilde{F_p}; \tilde{L})\}$.In other words, if the private operators' union chooses to cooperate, they will adopt the frequencies$Fp_{j^*,i}$, the stopover numbers $L_{j^*,i}$ and the fare $T_{j^*,i}$ designed by the authority to the private buses. However, if they decide not to cooperate, they will choose the frequencies$\tilde{F}p_{j^*,i}$and the stopover numbers $\tilde{L}_{j^*,i}$. Only the fare $T_{j^*,i}$are assumed to be respected any way. The choice of the variables to violate are chosen adequately to reality observations in Algiers area.

Furthermore, the frequencies, the fare and the stopover numbers must be bounded. Their lower and upper bounds are defined as follows.

- The frequencies $Fp_{j,i}$ can be computed by the following formula,

$$Fp_{j,i} = \sum_{k=1}^{w_j} \frac{D}{D_t(j,k)}, \quad (1)$$

where D is the duration of the studied period, w_j is the number of vehicles of type j and $D_t(j,k)$ is the duration of a tour using the k^{th} vehicle of type j.

The duration of a tour using the k^{th} vehicle of the j^{th} transportation mean is given by the sum of the travel times between the stations and the time of boarding/landing passengers, i.e.:

$$D_t(j,k) = tp(i) + \sum_{h=1}^{n_s} t_{ed}(h)\,, \tag{2}$$

where $tp(i)$ is the travel time among the i^{th} $(O-D)$ pair, n_s is the number of stations concerned by the j^{th}transportation mean and t_{ed} is the duration time of boarding/landing passengers at station h. Hence, the frequencies $Fp_{j,i}$ depend on the number of vehicles w_j of type j, the travel time $tp(i)$ through the i^{th} $(O-D)$ pair, the number of stations n_s and the duration $t_{ed}(h)$of boarding/landing passengers at the h^{th} station. As w_j and n_s are constants hence the lower bound of a frequency $Fp_{j,i}$ is computed using the maximum travel time $tp(i)$and the maximum durations time $t_{ed}(h)$ of boarding/landing passengers. Inversely, its upper bound is computed using the minimum travel time $tp(i)$ and the minimum duration time $t_{ed}(h)$of boarding/landing passengers.

- Stopover numbers: to determine the lower and upper bounds of the stopover numbers, the geographical localization of the origins and destinations are taken into account. Hence, the lower bounds are equal to 1 for all the $(O-D)$ pairs. The upper bounds are equal to 1 if the origin is neighbor the destination, and equal to the number of stations belonging to the shortest path linking the origins to the destination otherwise.
- Fare: the lower bounds of the fare $T_{j,i}$ are the costs $C_{j,i}$ and their upper bounds are chosen in such away that the passengers could use the corresponding modes without devoting more than 5% of their revenues to the transportation.

The player J_3 chooses the percentage of users selecting each transportation means. Hence, the k^{th} strategy x^{3k} of the player J_3 is a $(|A_1| \times m)$ matrix$P$, with $P_{i,j}$ the percentage of passengers traveling through the i^{th} $(O-D)$ pair using the j^{th} transportation mean. Thereby, these strategies must satisfy the following constraints.

$$\sum_{j=1}^{m} P_{i,j} = 100, \forall i = 1, \dots, |A_1|,$$
$$0 < P_{i,j} < 100$$

1.3- The utility functions

1.3.1. The utility function of the player J_1

The utility of the player $\boldsymbol{J_1}$ represents the benefits of the authority (public services) i.e the difference between his revenue and his cots. Consequently, the utility function f^1 of the player ($\boldsymbol{J_1}$) is given as follows.

$$f^1(x^1,x^2,x^3) = \sum_{i=1}^{|A_1|} \sum_{j=1, j\neq j^*}^{m-1} P_{i,j}(T_{j,i}L_{j,i}) + (1-\alpha)\pi \sum_{i=1}^{|A_1|} \rho_i - \sum_{i=1}^{m-1} \sum_{j=1}^{|A_1|} Fp_{i,j}C_{i,j}$$

with π the penalty that the player J_2 must pay if he does not cooperate with the player J_1, and

$$\alpha = \begin{cases} 1, & if\ the\ player\ J_2\ cooperate; \\ 0, & otherwise. \end{cases}$$

The whole problem of the player J_1 is then written as follows,

$$\max_{x^1} f^1(x^1,x^2,x^3) = \sum_{i=1}^{|A_1|} \sum_{j=1, j\neq j^*}^{m-1} P_{i,j}(T_{j,i}L_{j,i}) + (1-\alpha)\pi \sum_{i=1}^{|A_1|} \rho_i - \sum_{i=1}^{m-1} \sum_{j=1}^{|A_1|} Fp_{i,j}C_{i,j}$$
$$s.t \left| \begin{array}{c} l_{Fp} \leq Fp \leq U_{Fp}, \\ C \leq T \leq U_T \quad , \\ l_L \leq L \leq U_L, \\ L_{j,i} \in \mathbb{N}, Fp_{j,i} \in \mathbb{N}, T_{j,i} \geq 0;\ \forall i = 1, \dots, |A_1|, \forall\, j = 1, \dots, m. \end{array} \right. \qquad (4)$$

with l_{Fp} and l_L the lower bounds of the frequencies, respectively the stopover numbers. U_{Fp}, U_T, and U_L the upper bounds of the frequencies, respectively the fare and the stopover numbers.

1.3.2. The utility function of the player J_2

The utility of the player J_2 represents its global benefits. These benefits are divided into two parts following his strategy i.e., if he decides to cooperate with the player J_1or not. If he cooperates, he will

- respect the frequencies fixed by the authority, i.e., the private operators will not stay at the different stations longer than what they must do in order to achieve the trip in the estimated duration,
- respect the stopover numbers, i.e., they do the trip in its globality and will not divide the itinerary.

In this case, his benefits are given by:

$$\sum_{i=1}^{|A_1|} P_{i,j}\,(T_{j^*,i}L_{j^*,i}) - \sum_{i=1}^{|A_1|} C_{j^*,i}Fp_{j^*,i}.$$

If he does not cooperate, he will choose his own frequencies and stopover numbers, and takes the risk of paying the penalty. In this case his benefits are given by

$$\sum_{i=1}^{|A_1|} P_{i,j^*}(T_{j^*,i}\tilde{L}_{j^*,i}) - \sum_{i=1}^{|A_1|} C_{j^*,i}\tilde{F}p_{j^*,i} - \pi\sum_{i=1}^{|A_1|}\rho_i$$

The whole utility function of the player J_2 is then written as follows,

$$f^2(x^1,x^2,x^3) = \alpha\left[\sum_{i=1}^{|A_1|} P_{i,j}\,(T_{j^*,i}L_{j^*,i}) - \sum_{i=1}^{|A_1|} C_{j^*,i}Fp_{j^*,i}\right] + (1-\alpha)\left[\sum_{i=1}^{|A_1|} P_{i,j^*}(T_{j^*,i}\tilde{L}_{j^*,i}) - \sum_{i=1}^{|A_1|} C_{j^*,i}\tilde{F}p_{j^*,i} - \pi\sum_{i=1}^{|A_1|}\rho_i\right]$$

Where:

$$\alpha = \begin{cases} 1, & if\ the\ player\ J_2\ cooperate; \\ 0, & otherwise. \end{cases}$$

ρ_i is a probability that the private operators' union get caught by the authority through the i^{th} $(O-D)$ pair and pay a penalty π. $\tilde{L}_{j^*,i}$and$\tilde{F}p_{j^*,i}$ are respectively the stopover numbers and the frequencies chosen by the player J_2.

The number of links $\tilde{L}_{j^*,i}$ and the frequencies $\tilde{F}p_{j^*,i}$ are also bounded by the lower bounds l_L (respectively l_{Fp}) and the upper bounds U_L (respectively U_{Fp}) defined before.

The whole problem of the second player (J_2) is then given by the following system:

$$\max_{x^2} f^2(x^1,x^2,x^3) = \alpha \left[\sum_{i=1}^{|A_1|} P_{i,j}\,(T_{j^*,i} L_{j^*,i}) - \sum_{i=1}^{|A_1|} C_{j^*,i} Fp_{j^*,i} \right] + (1-\alpha) \left[\sum_{i=1}^{|A_1|} P_{i,j^*}\big(T_{j^*,i} \tilde{L}_{j^*,i}\big) - \sum_{i=1}^{|A_1|} C_{j^*,i} \tilde{F}p_{j^*,i} - \pi \sum_{i=1}^{|A_1|} \rho_i \right]$$

$$s.t \left| \begin{array}{c} l_{Fp} \le \tilde{F}p \le U_{Fp}, \\ l_L \le \tilde{L} \le U_L \\ \tilde{L}_{j,i} \in \mathbb{N}, \tilde{F}p_{j,i} \in \mathbb{N}, \forall i = 1, \dots, |A_1|, \forall\, j = 1, \dots, m. \\ \alpha \in \{0,1\}\,. \end{array} \right. \quad ,(5)$$

1.3.3. The utility function of the player J_3

The utility of the player J_3 represents the satisfaction of the passengers using the different common transportation means. The satisfaction S_{ij} of a passenger traveling through the $i^{th}\,(O-D)$ pair using the j^{th} transportation means decreases with the increase of the travel cost $L_{j,i}T_{j,i}$and increases with the increase of the frequency $Fp_{j,i}$, hence this satisfaction could be calculated as follows,

$$S_{ij} = \frac{Fp_{ji}}{L_{ji}T_{ji}} \quad , \qquad (6)$$

Then, the utility of the player J_3 is given by:

$$f^3(x^1,x^2,x^3) = \sum_{i=1}^{|A_1|} \sum_{j=1}^{m} P_{ij} S_{ij}\,, \qquad (7)$$

Hence, the whole problem of the player J_3 is given by:

$$\max_{x^3} f^3(x^1,x^2,x^3) = \sum_{i=1}^{|A_1|} \sum_{j=1}^{m} P_{ij} S_{ij}$$

$$s.t \left| \begin{array}{l} \sum_{j=1}^{m} P_{ij} = 100, \forall\, i = 1, \dots, |A_1| \\ 0 < P_{ij} < 100, \forall i = 1, \dots, |A_1| \\ \qquad\qquad , \forall\, j = 1, \dots, m. \end{array} \right. ,(8)$$

In order to compute a solution to the game under consideration an Artificial Bee Colony (ABC) algorithm is considered. ABC algorithms are bio-inspired and focus on honey bee behavior when looking for food (Rajasekhar, Lynn, Das, & Suganthan, 2017). They were introduced by Karaboga (Karaboga, 2005) and then widely studied and applied to several optimization problem (Tsai, Pan, Liao, & Chu, 2008; Akay & Karaboga, 2015; Patel, Tiwari, & Patel, 2016).

2- ALGIERS CASE STUDY

The above methodology is applied to Algiers area, where the needed data were collected via a household survey conducted by our team in November, 2019. The survey contains a household file dedicated to general information of the house, in addition to an individual file for each family member where their whole displacements are described. Following the most important Origin-Destination flows, a set of six $(O - D)$ pairs are selected to be presented herein. The data concerning these pairs are summarized in Table 1. The lower and upper bounds are calculated using the aforementioned formulas.

Table 1. Data of the selected (O-D) pairs

	Nb private bus (w_1)	Nb public bus (w_2)	Nb stations (n_s)	Boarding/landing duration (t_{ed})	Minimum travel time (t_p) min	Maximum travel time (t_p) max	Travel costs in Algerian Dinars
$1^{st}(O-D)$ pair	0	4	11	1	22	25	31
$2^{nd}(O-D)$ pair	5	0	3	1	25	43	41
$3^{rd}(O-D)$ pair	24	6	5	1	15	18	29
$4^{th}(O-D)$ pair	72	2	14	1	25	62	31
$5^{th}(O-D)$ pair	56	2	4	1	19	58	27.5
$6^{th}(O-D)$ pair	27	4	3	1	20	21	25

Source: Household transport survey, CREAD, 2019

As the number of private buses is greater than the number of public buses, except for the first (O-D) pair, where the number of private buses is

0, it is interesting to see whether an increased number of public buses will be advised. In this direction, the application is executed, firstly, with the real number of public buses, and secondly with an increased number (see Table 2). the increased number is constructed such that, it is doubled for the first (O-D) pair, since there are no private buses, and set to the same number of the private bus for the second (O-D) pair. For the other pairs, this number is increased to reach the half of the private buses number.

Table 2. Public buses variation

	Number of public buses	
	Real	Increased
$\mathbf{1^{st}(O-D)}$ **pair**	4	8
$\mathbf{2^{nd}(O-D)}$ **pair**	0	5
$\mathbf{3^{rd}(O-D)}$ **pair**	6	12
$\mathbf{4^{th}(O-D)}$ **pair**	2	36
$\mathbf{5^{th}(O-D)}$ **pair**	2	28
$\mathbf{6^{th}(O-D)}$ **pair**	4	13

Source: computed by the authors

2.1- The obtained results

Using the data of Table 1, nineteen solutions bringing the same objective functions values are obtained. In each of them, the obtained strategy of the private operator is to cooperate. The other variables (fares, frequencies and stopovers' number) differ slightly from a solution to another. A sample of three solutions is figured out in Tables 3, 4, 5 and 6.

Table 3. The obtained fares of the different (O-D) pairs in Algerian Dinars

	Solution 1		**Solution 2**		**Solution 3**	
Fares	Private	public	Private	public	Private	public
$\mathbf{1^{st}(O-D)}$ **pair**	-	26	-	27	-	26
$\mathbf{2^{nd}(O-D)}$ **pair**	41	-	41	-	41	-
$\mathbf{3^{rd}(O-D)}$ **pair**	29	25	29	25	29	25
$\mathbf{4^{th}(O-D)}$ **pair**	32	27	31	25	31	27
$\mathbf{5^{th}(O-D)}$ **pair**	28.5	21	28.5	20	27.5	22
$\mathbf{6^{th}(O-D)}$ **pair**	25	20	25	20	25	20

Source 1: computed by the authors

Table 4. The obtained frequencies for the different (O-D) pairs

Frequencies	Solution 1		Solution 2		Solution 3	
	Private	public	Private	public	Private	public
$\mathbf{1^{st}(O-D)}$ **pair**	-	7	-	7	-	7
$\mathbf{2^{nd}(O-D)}$ **pair**	7	-	9	-	10	-
$\mathbf{3^{rd}(O-D)}$ **pair**	64	16	66	16	67	16
$\mathbf{4^{th}(O-D)}$ **pair**	65	2	79	2	91	2
$\mathbf{5^{th}(O-D)}$ **pair**	83	3	120	3	107	4
$\mathbf{6^{th}(O-D)}$ **pair**	69	10	69	10	69	10

Source 2: computed by the authors

Table 5. The obtained links' number for the different (O-D) pairs

Links	Solution 1		Solution 2		Solution 3	
	Private	public	Private	public	Private	public
$\mathbf{1^{st}(O-D)}$ **pair**	-	2	-	2	-	1
$\mathbf{2^{nd}(O-D)}$ **pair**	2	-	2	-	2	-
$\mathbf{3^{rd}(O-D)}$ **pair**	1	2	1	2	2	2
$\mathbf{4^{th}(O-D)}$ **pair**	1	1	2	2	2	2
$\mathbf{5^{th}(O-D)}$ **pair**	2	1	1	2	2	2
$\mathbf{6^{th}(O-D)}$ **pair**	1	1	1	2	1	1

Source 3: Computed by the authors

Table 6. The obtained control probabilities for the different (O-D) pairs

Probabilities	Solution 1		Solution 2		Solution 3	
	Private	public	Private	public	Private	public
$\mathbf{1^{st}(O-D)}$ **pair**	-	0.27	-	0.72	-	0.32
$\mathbf{2^{nd}(O-D)}$ **pair**	0.58	-	0.48	-	0.59	-
$\mathbf{3^{rd}(O-D)}$ **pair**	0.10	0.83	0.68	0.83	0.02	0.83
$\mathbf{4^{th}(O-D)}$ **pair**	0.96	0.59	0.54	0.77	0.17	0.95
$\mathbf{5^{th}(O-D)}$ **pair**	0.89	0.72	0.60	0.95	0.65	0.14
$\mathbf{6^{th}(O-D)}$ **pair**	0.90	0.30	0.74	0.42	0.81	0.66

Source 4: Computed by the authors

The obtained results when the number of public buses is increased (as shown in table 2) are given in the following table (table 7).

Table 7. The obtained results when increasing the public buses number

	Fares (DA)		Frequencies		Links	
	Private	public	Private	public	Private	public
$\mathbf{1^{st}(O-D)}$ **pair**	-	25	-	14	-	2
$\mathbf{2^{nd}(O-D)}$ **pair**	42	36	7	9	2	1
$\mathbf{3^{rd}(O-D)}$ **pair**	29	25	65	31	1	1
$\mathbf{4^{th}(O-D)}$ **pair**	31	26	83	37	1	1
$\mathbf{5^{th}(O-D)}$ **pair**	27.5	20	125	54	1	2
$\mathbf{6^{th}(O-D)}$ **pair**	25	20	68	33	1	2

Source 5: Computed by the authors

Table 8. The obtained control probabilities when increasing the public buses number

	Private	public
$\mathbf{1^{st}(O-D)}$ **pair**	-	0.45
$\mathbf{2^{nd}(O-D)}$ **pair**	0.90	0.28
$\mathbf{3^{rd}(O-D)}$ **pair**	0.75	0.18
$\mathbf{4^{th}(O-D)}$ **pair**	0.06	0.36
$\mathbf{5^{th}(O-D)}$ **pair**	0.29	0.57
$\mathbf{6^{th}(O-D)}$ **pair**	0.07	0.61

Source 6: Computed by the authors

2.2- Discussion

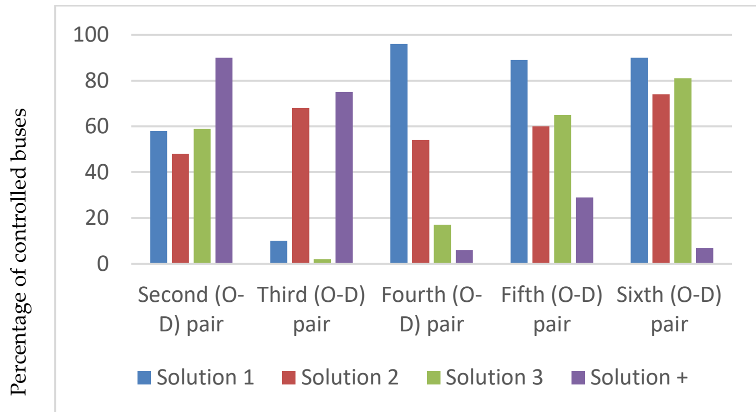
When considering the actual number of public buses, the developed algorithm allows us to obtain several solutions with the same objective functions' values. This multiplicity is an advantage since it allows some flexibility to the authority when choosing the plan to adopt. For instance, the public authority can choose the plan to adopt according to its control capacity. Indeed, when comparing the obtained control probabilities of the three solutions of the fourth(O-D) pair (where the number of private buses is the most important), we see that:

- The percentage of the private buses that must be controlled, according to the first solution, is about 96% (9 buses all over 10 must be controlled), whereas this percentage is about 54% (one bus over two must be controlled) respectively, 17% (2 buses over 10 must be controlled) according to the second respectively, the third solution. As the number of the private buses of the fourth (O-D) pair equals 72 buses, the third solution is the one which requires the minimum number of controllers.

Furthermore, the strategy of the player J_2 (private operator's union) is to cooperate in all cases (in each solution). Even when considering other penalties' values the strategy of this player still to cooperate. This result meets the one of Bjørnskau and Elvik (Bjørnskau & Elvik, 1992) in such away a player's compliance to a rule designed by an authority is related to the control level instead of the amount of the penalty.

When using the increased number of public buses (given in Table 2), the obtained results show, as expected, an increasing in public buses frequencies, but do not considerably affect the private operators' union strategies. This means that the considered increase of the public buses is not enough. Nevertheless, one can remark that the control probabilities of the private operators of both the second and the third (O-D) pairs are greater than one obtained in the first case (with initial public buses' number) (see Figure 1). Indeed, in the case of the second (O-D) pair the number of public buses equals the number of private buses, this leads to a severe concurrence between the two players, and the private operators can be tempted to deviate from the authorities' plans. It is then clear that a high level of control is the unique mean to incite the private operators to cooperate. Due to the compensation, the high level of control in the second and the third (O-D) pairs is sufficient to incite the private operators to cooperate and then induces low control probabilities in the other (O-D) pairs.

Figure 1. The obtained control probabilities of the private buses with the obtained solutions within the initial public buses' number (solution1, solution2, and solution 3) and the one obtained when increasing the public buses' number (solution +)



Source 7: Computed by the authors

CONCLUSION

In the present work, a methodology based on game theory is presented, in order to optimize public transportation management in cities such Algiers. To solve the obtained game, an ABC algorithm is developed. Nevertheless, the developed algorithm leads to incomparable effective solutions. By consequent, TOPSIS method is used to choose the best solution.

The developed method is applied to Algiers area and showed interesting results. In fact, the developed algorithm brings several solutions without a difference in the objective functions' values. This multiplicity constitutes an advantage since it allows the authority to choose the optimal plan that mostly meets his goals. In addition to the performance requirements to be met by operators, the organizing authority must provide user an affordable ticket price to ensure the attractiveness of the public transport system.

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