FINANCIAL MARKETS INTERACTION: AN APPLICATION OF PROBABILISTIC MARKOV MODEL AND ARDL APPROACH

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ABSTRACT

In the context of the last world financial crisis, the interconnection between the different markets is arising as a serious problem ; which made a situation of uncertainty about the trend and direction of causality among these markets.This paper examined the causality and long-run relationships between *Oil Price, Gold price and Dollar index using ARDL* approach and a *Probabilistic Markov Models* over the period (*1986-2016*).The findings reveal that: **i)** A positive trend of Gold prices; which can take a *big bull market* . **ii**) A symmetric dynamic structure of Oil prices . **iii)** Stable, long-run relationship exists between Oil prices and Gold prices. **iv)** The evidence also suggests that the US dollar index as indicator has no effect on Oil prices. **v)** Model stability results also reveal that after incorporating the CUSUM and CUSUMSQ tests, Oil price function is stable over the period (**1986-2016)**.

KEY WORDS

Markov Models, ARDL, Oil, Gold, Dollar index.

JEL CLASSIFICATION: C51, E41, F37.

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INTERACTION DES MARCHÉS FINANCIERS: APPLICATION DU MODÈLE DE MARKOV ET DE L'APPROCHE ARDL

RÉSUMÉ

Dans le contexte de la dernière crise financière mondiale, l'interconnexion entre les différents marchés devient un problème grave; Cette étude examinait la relation de causalité et les relations à long terme entre le prix du pétrole, le prix de l'or et l'indice du dollar en utilisant une approche **ARDL** et un modèle de **Markov** probabiliste sur la période (**1986- 2016**). Les résultats révèlent que: **i)** il y a une tendance positive des prix de l'or; **ii**) il y a aussi une structure dynamique symétrique des prix du pétrole **iii)** Il existe une relation stable à long terme entre les prix du pétrole et les prix de l'or **iv)** l'indice du dollar américain en tant qu'indicateur n'a aucun effet sur les prix du pétrole. **v)** Les résultats de stabilité du modèle révèlent également qu'après l'intégration des tests CUSUM et CUSUMSQ, la fonction de prix du pétrole est stable sur la période (**1986-2016**).

MOTS CLÉS

Modèles de Markov, ARDL, pétrole, or, indice du dollar.

JEL CLASSIFICATION: C51, E41, F37.

التزابط بين الأسواق المالية: تطبيق لنماذج ماركوف ومقاربة **ARDL**

ملخص:

في سياق الأزمة المالية العالمية الأخيرة، برز الترابط بين الأسواق المالية المختلفة كمشكلة خطيرة، مما خلق حالة من عدم التأكد بشأن اتجاه السببية بين هذه الأسواق. في هذه الورقة ، تناولنا لدراسة وتحليل العلاقة السببية والعلاقات طويلة الأجل بين أسعار النفط ،سعر الذهب ومؤشر الدولار باستخدام مقاربةARDL ونماذج ماركوف ، وذلك خلال الفترة 1986-2016. النتائج كشفت عن: **1**) اتجاه إيجابي لأسعار الذهب، والتي يمكن أن تأخذ منحى تصاعدي. **2**) هيكل ديناميكي متماثل لأسعار النفط خلال فترة الدراسة. **3**) توجد علاقة مستقرة وطويلة المدى بين أسعار النفط وأسعار الذهب. تشير التقديرات أيضًا إلى أن مؤشر الدولار الأمريكي ليس له تأثير واضح على أسعار النفط. **4**) ثبات النموذج وذلك بالاعتماد على اختبارات CUSUM و CUSUMSQ.

كلمات مفتاحية:

نماذج ماركوف ، ARDL ، النفط ، الذهب ، مؤشر الدولار.

تصنيف جال: C51،E41،F37.

INTRODUCTION

Studying the nature of financial and economic crisis has a great importance in the last decade. This crisis is one outcome of the financialisation of contemporary capitalism, Lapavitsas, (2009). In this setting, the relationship between the different markets occupied a priority. Gold; oil, dollar and Dow Jones index are playing an important role in this area.

Oil prices are considered as a leading index in the world economy, Mo et al, (2017). A rise of oil prices can cause inflation, which lead to a rise of gold prices because investors believe that gold can hedge against inflation. The US dollar is the main currency in the international oil market and the volatility of the US dollar exchange rates will have a direct impact on oil-exporting countries. Gold is dominated by the US dollar in the international market, and gold is the best commodity used to hedge against the recession to preserve capital. The balance of the relationship among oil prices, gold market and the US dollar will have an important impact (*positive or negative*) on the international financial markets and the world economy.

Modelling and analysing the interconnections among these three markets have been convoluted and can be specified by deterministic or **stochastic** models. Using various methods, the relations between these variables have been analysed in literature. Sujit and Kumar, **(2011**) established the dynamic relationship among gold price, Oil price with exchange rate and stock index using vector autoregressive technique. Moreover, their degree of correlation is greatly positive Simakova, (2011). Akram, (2009**)** uses quarterly data during 1990-2007 and a structural VAR model to examine the relationships between real oil prices, interest rates, and the US dollar exchange rate. The analysis consists of a vector autoregressive (VAR) model that includes the real oil price, a measure of global activity, real short term U.S. interest rates, and the effective real exchange rate for the US dollar. **Akram, (2009)** revealed that real oil prices increase in response to negative interest rate shocks and that oil prices tend to display overshooting behaviour in response to such shocks.

Markov Chain, as a statistical technique, is widely used to study the dynamic movement of time series. In practice, it is often useful to use a homogeneous Markov chain. Moreover, the assumption that the process is homogeneous simplifies the methods used to fit the model Ocana, (2002), Richard, (2016) used the Markov chain for modelling the West Texas Intermediate (WTI) oil price movement. He was interesting in predicting the random walk that WTI price is likely to take between an initial time and the long term foreseeable future than upon attempting to estimate the size of each on those steps (*i.e.*) the magnitude of the price change at each point in time).

For the Autoregressive Distributed Lag (ARDL) modeling approach, we consider Oil price as a dependent variable, because, the oil price is one of the world's most influential global economic indicators. Also, it is precisely observed by policy-makers, producers, consumers and financial market participants Mostafaei, (2011).The current state of knowledge on the important factors influencing oil prices (*Economic factors*: supply and demand, Speculation, Anticipations of investors... *Geopolitical Crisis* in: Iran, Iraq, Libya...etc) have been identified in relevant venues, including recent academic literature, government reports, policy debate, and industry analysis**,** Xiang, (2010).

Sujit and Kumar, (2011) , tried to capture dynamic and stable relationship among Gold price, Oil price, Exchange Rate and Stock Market Returns; using vector autoregressive and cointegration technique. The results show that exchange rate is highly affected by changes in other variables. However, stock market has fewer roles in affecting the exchange rate. In addition, they tested two models and one model suggests that there is weak long term relationship among variables. The variations of these variables are vital issues of macro-economy. Aguiar and Soares, (2010), used (cross) wavelet analysis to decompose the time-frequency effects of oil price changes on the macro-economy. They argue that the relation between oil prices and industrial production is not clear-cut.

Recently, Arfaoui and Ben Rejeb, (2017), examined, in a global perspective, the oil, gold, US dollar and stock-prices interdependencies and to identify instantaneously direct and indirect linkages among them. **Chen** and Xu, (2019) tried to forecast and analyze the correlation between oil and gold prices.

In this study, we are interested in the interconnections between these three financial Markets (Oil market, Gold Market and US dollar index).We: first, studied and analyzed the dynamic nature of each variable by using the probabilistic Markov chain, second, we are trying to assess the long-run relationship of oil prices, gold prices and the US dollar index by using the Autoregressive Distributed Lag (ARDL) bounds test approach; we think this long memory analysis can provide a good description of many highly persistent financial time series.

The rest of the article is divided as follows, in the first section: the theoretical principle of two techniques used is presented: the of Markov chain and that also of ARDL approach, in the second section: we presented the data used, the application of these two techniques, the estimation results and the analysis and discussion, finally a conclusion which summarizes the essential points of this work.

1- METHODS

1.1- MARKOV MODELS

1.1.1. Definitions and Representations

Markov chains represent a class of stochastic processes of great interest for the wide spectrum of practical applications. In particular, discrete time Markov chains (**DTMC**) permit to model the transition probabilities between discrete states by the aid of matrices.

Definition1. State space: The set of possible states $= \{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_r\}$; can be finite or countable and it is named the state space of the chain.

Definition2. Discrete time Markov chains: it is a sequence of random variables $\mathbf{X_1}, \mathbf{X_2}, \dots, \mathbf{X_n}$ characterized by the *Markov property* (also known as memoryless property, *see Equation1*). The Markov property states that the distribution of the forthcoming state $\mathbf{X_{n+1}}$ depends only on the current state $X_t$and does not depend on the previous ones$\mathbf{X_{n-1}}, \mathbf{X_{n-2}}, \mathbf{2}\ \mathbf{X_1}$.

$$\mathbb{P}(\mathbf{X_{n+1}} = \mathbf{e_{n+1}} \backslash \mathbf{X_0} = \mathbf{e_0}, \mathbf{X_1} = \mathbf{e_1}, \dots, \mathbf{X_n} = \mathbf{e_n}) = \mathbb{P}(\mathbf{X_{n+1}} = \mathbf{e_{n+1}} \backslash \mathbf{X_n} = \mathbf{e_n}) \mathbf{(1)}$$

Definition3.Transition probability: The chain moves from one state to another (this change is named either *'transition'* or *'step'*) and the probability $\mathbf{p_{ij}}$ to move from state $\mathbf{e}_i$ to state $\mathbf{e}_j$in one step is called transition probability:

$$\mathbf{p_{ij}} = \mathbb{P}\ (\mathbf{X_{n+1}} = \mathbf{e_j} \backslash \mathbf{X_n} = \mathbf{e_i}\ ) \qquad \mathbf{(2)}$$

Definition4. Time-homogeneous Assumption: Time homogeneity implies no change in the underlying transition probabilities as time goes on.

$$\mathbf{p_{ij}} = \mathbb{P}\big(\mathbf{X_t} = \mathbf{e_j} \backslash \mathbf{X_{t-1}} = \mathbf{e_i}\big) = \mathbb{P}\big(\mathbf{X_s} = \mathbf{e_j} \backslash \mathbf{X_{s-1}} = \mathbf{e_i}\big)\ ; \mathbf{s} \neq \mathbf{t} \mathbf{(3)}$$

Equation **(4)**shows the maximum likelihood estimator (MLE) of the $\mathbf{p_{ij}}$entry, where the $\mathbf{n_{ij}}$ element consists in the number sequences $(\mathbf{X_{t+1}} = \mathbf{e_j}, \mathbf{X_t} = \mathbf{e_i})$ found in the sample, that is:

$$\mathbf{p_{ij}} = \frac{\mathbf{n_{ij}}}{\sum_{i=1}^{n} \mathbf{n_{ij}}} \mathbf{(4)}$$

For the application of Discrete Markov chain on time series, a categorization of these time series must be applied; the three time series: (Oil price, Gold price and USA dollar index) would be transformed in a sequence data; that represent a random distribution of sequences (later called chain states). For this purpose, we have proceeded as follow:

Let $\boldsymbol{X_t}$ denotes a time series, for$\boldsymbol{t} = \mathbf{1}, \dots, \boldsymbol{n}$, we make a first order differentiation of $\boldsymbol{X_t}$, and we find a new series $\boldsymbol{Z_t} = \Delta(\boldsymbol{X_t}) = \boldsymbol{X_t} - \boldsymbol{X_{t-1}}$. So the question is how we can create a sequence data using our new series $\boldsymbol{Z_t}$? The response to this question is to study the movements of $\boldsymbol{Z_t}$, (*i.e.*) At the end of any given month, for example, the Oil price could have moved up, moved down or stayed the same. Thus, we create another new series $\mathbf{Y_t}$ by supposing that it represents the *three possible states* of each $\boldsymbol{Z_t}$movement. Let $\mathbf{E} = \{\mathbf{S}, \mathbf{L}, \mathbf{H}\}$ denote the state space, with: **S**: "**stable»; L : "Low" ; H : "High".** The mathematical link between $\boldsymbol{Z_t}$ movements and state space $\mathbf{E} = \{\mathbf{S}, \mathbf{L}, \mathbf{H}\}$is defined as:

$$\mathbf{Y_t} = \begin{cases} \mathbf{S: stable} & \mathbf{if,} \ -1 \leq \boldsymbol{X_t} - \boldsymbol{X_{t-1}} \leq 1 \\ \mathbf{L: Low} & \mathbf{if,} \ -1 > \boldsymbol{X_t} - \boldsymbol{X_{t-1}} \\ \mathbf{H: High} & \mathbf{if,} \ \boldsymbol{X_t} - \boldsymbol{X_{t-1}} > 1 \end{cases}$$

So, the new data frame $\mathbf{Y_t}$for the three time series will take the form: **(S, L, H, S, S, L, L ...etc)**

1.2- Autoregressive distributed Lag Approach (ARDL)

Since our intention is to detect the long run relationship between Oil price, Gold price and the US dollar index, the appropriate technique to be used is error correction modeling and cointegration analysis. In applying any cointegration technique, the first exercise is to determine the degree of integration of each variable in the model. This, of course, will depend on which unit root test one can use. To avoid this difficulty and pre-testing of unit roots, **Peseran, (1996), Peseran** ***et al*****, (2001)** introduced a relatively new cointegration test - known as Autoregressive Distributed Lag (ARDL) approach.

In reality, ARDLs are standard least squares regressions, which include lags of both the dependent variable and independents variables as regressors **Greene, (2008**). Although, ARDL models have been used, particularly in econometrics for decades, they have gained importance in last few years as an approach of examining long-run and cointegrating relationships between variables (Pesaran and Shin, 1999).In its (explicit) general form, an ARDL model is written as follow:

$$\boldsymbol{y_t = \alpha_0 + \sum_{i=1}^{p} \gamma_i y_{t-i} + \sum_{i=1}^{q} \underline{\delta_i X_{t-i}} + \varepsilon_t} \qquad (5)$$

With: $\underline{\boldsymbol{\delta_i X_{t-i}}}$: is a matrix notation of independent variables

This test has several advantages, one of its most notable features is that the existence of the long run relationship is tested without any *a* *priori* knowledge of the order of the time series (*i.e.)* stationary at level$I(0)$ or stationary after first differentiation:$I(1)$ of the possibility of cointegration. Since the power of existing unit root tests to identify the order of integration, in particular, whether $I(0)$ or $I(1)$ is always questionable, hence their test may be useful**, Nagayasu, (2012).** Another useful feature of this test is that it does not matter whether the explanatory variables are exogenous or not, **Pesaran, (1996).**Following the notation in Equation (5)**,** we replace directly the dependent variable $y_t$by the oil prices (in US dollar) denoted by:$Oil_t$and regressors X_{it} variablesby: lagged in level i of oil prices :Oil_{t-i}, Gold prices and its lagged values with max level q_1 **labeled by:**$Gold_{t-i}$ and finally US dollar index and its lagged values Usd_{t-i} with a max level q_2 in equation form:

$$Oil_t = \alpha_0 + \sum_{i=1}^{p} \gamma_i Oil_{t-i} + \sum_{i=1}^{q_1} \delta_i Gold_{t-i} + \sum_{i=1}^{q_2} \delta_i Usd_{t-i} + \varepsilon_t \qquad (6)$$

With:

- α_0: Intercept term;
- $\gamma_i, \delta_i, \delta_i$**:** short term coefficients
- p, q_1, q_2**:** lags of variables.
- ε_t :a noise process supposed$iid\ (0, \sigma_\varepsilon^2)$

1.1. **Long-run Relationships Specification**

We know that an ARDL model estimates the dynamic relationship between a dependent variable (*i.e.*) in this case Oil_t and explanatory variables : $Gold_t$; Usd_t, it is possible to transform the model in equation (6)into a long-run representation, by which the long run effect of explanatory variables on dependent variable is determined, the long-run estimated coefficients is calculated by:

$$\widehat{\boldsymbol{\beta}}_{k,Long} = \frac{\sum_{i=1}^{q}\theta_{k,i}}{1-\sum_{i=1}^{p}\phi_i} \quad (7)$$

We obtain

$$\Delta(OIL)_t = \alpha_0 + \sum_{i=1}^{p}\phi_i\Delta(OIL)_{t-i} + \sum_{i=1}^{q_1}\theta_i\Delta(GOLD)_{t-i} + \sum_{i=1}^{q_2}\vartheta_i\Delta(USD)_{t-i} + \beta_1 GOLD_{t-1} + \beta_2 USD_{t-1} + \varepsilon_t \quad (8)$$

With: Δ first difference operator; α_0: interceptterm;ϕ_i,θ_i,$\vartheta_i$short-term effects;β_1, β_2: **long-term dynamics of the model;** ε_t : **error term** , a noise process supposed $iid\ (0, \sigma_\varepsilon^2)$.

2- CASE STUDIES AND DISCUSSION

The present study is based on monthly data covering the period from 1986 through 2016, which includes the break points in Oil price as well. **LOIL is** the Oil price series in logarithm. **LUSINDEX** is US Dollar index. **LGOLD** is the Once Gold price. The data were obtained from the *World Bank Database*, Energy International Agency and the *Gold World Council*.

2.1- Probabilistic Markov Chain Application

An objective of this part of study is to be able to make better predictions about the future path of the three variables: Oil price, Gold price and US dollar index. First, we have to check if the sequences of the three variables follow a Markov Chain. See equation (1).

Table 1. **Results of Estimated Probabilistic Markov Chain**

Variables	$\widehat{\alpha}_{ij}$	$\widehat{\Sigma}_{\widehat{\alpha}_{ij}}$
Oil Price	$M_{oil} = \begin{matrix} & S & L & H \\ S & 0.47 & 0.25 & 0.28 \\ L & 0.23 & 0.41 & 0.36 \\ H & 0.31 & 0.23 & 0.46 \end{matrix}$	$\widehat{\Sigma}_{M_{oil}} = \begin{pmatrix} 0.06 & 0.04 & 0.04 \\ 0.04 & 0.06 & 0.06 \\ 0.04 & 0.04 & 0.04 \end{pmatrix}$

Gold Price	$M_{gold} = \begin{matrix} & S & L & H \\ S & 0.09 & 0.5 & 0.41 \\ L & 0.12 & 0.49 & 0.39 \\ H & 0.06 & 0.39 & 0.55 \end{matrix}$	$\hat{\Sigma}_{M_{gold}} = \begin{pmatrix} 0.05 & 0.12 & 0.11 \\ 0.03 & 0.05 & 0.05 \\ 0.01 & 0.05 & 0.06 \end{pmatrix}$
US dollar index	$M_{usd} = \begin{matrix} & S & L & H \\ S & 0.43 & 0.27 & 0.3 \\ L & 0.38 & 0.37 & 0.25 \\ H & 0.5 & 0.26 & 0.24 \end{matrix}$	$\hat{\Sigma}_{M_{usd}} = \begin{pmatrix} 0.05 & 0.04 & 0.04 \\ 0.06 & 0.06 & 0.05 \\ 0.07 & 0.05 & 0.05 \end{pmatrix}$

Source: Edited by Authors using Output of R estimation.

To test this hypothesis, we use the chi-square independence test, whose general hypothesis is given by:

$$\begin{cases} H_0: All\ states\ are\ independent \\ H_1: At\ least\ two\ states\ are\ dependents \end{cases}$$

The test statistic is defined by:

$$\chi_c^2 = \sum_i \sum_j \frac{\left(n_{ij} - \frac{n_{i.}n_{.j}}{n}\right)^2}{\frac{n_{i.}n_{.j}}{n}}$$

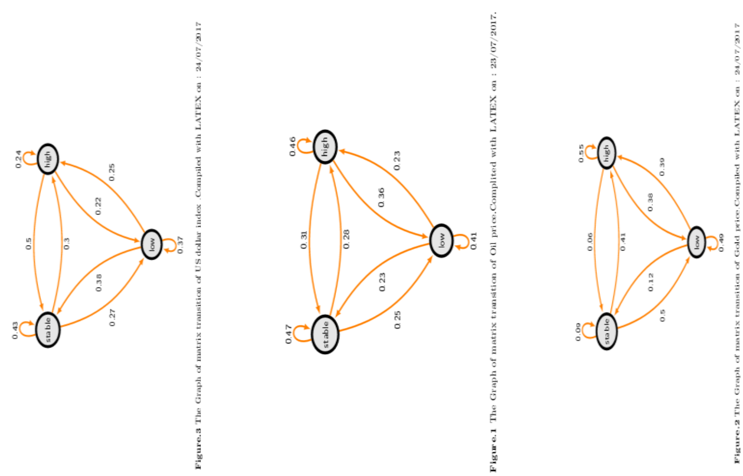
With: $\boldsymbol{n_{ij}}$:number of ordered sequences of type$(\boldsymbol{i}, \boldsymbol{j})$, for *example* the number of sequences (Stable, High).$\boldsymbol{n_{i.}}$: Partial sum of sequences (transitions) from state $\boldsymbol{i}$ to others states.$\boldsymbol{n_{.j}}$: Partial sum of sequences (transitions) from others states to state$\boldsymbol{j}$. this statistics has an asymptotic $\boldsymbol{\chi^2} -$ distribution with $(\boldsymbol{m} - \mathbf{1})^{\mathbf{2}}$ degrees of freedom; m: number of states, here $\boldsymbol{m} = \mathbf{3}$.

After testing such hypothesis, using a $\boldsymbol{\alpha} = \mathbf{0.05}$ significance level, we found that the observed values of the sample statistics for the three contingency tables are higher than $(\boldsymbol{\chi^2_{0.05}}(\mathbf{4}) = \mathbf{9.49})$, tabulated value. Therefore, we reject the null hypothesis that states are independent. Finally, it approved that a state transition chain of Oil prices, Gold prices and US dollar index follows a Markov chain. In "**markov chain**" R package, we can easily apply the command **"verify Markov Property"** on sequences data.

As a first note of the transition matrices estimated in the table above, an asymmetric structure of the dynamics is well confirmed; it is more likely for the chain to go from a **stable state** to a **higher state** value than to a **lower state**. In addition, and for the Oil price and Gold price once it is positive, the chain is more likely to stay positive than go negative, but for the US dollar index, once reached the **High state**, it is likely to stay there. From the estimates in the table, and for the oil price matrix, it can be seen that in the short term (*at least for three months*), prices remain the same with almost equal probability. For example, if the price is stable in period $\boldsymbol{t}$, it will also be stable in the next period with probability $\widehat{\boldsymbol{\alpha}}_{ss} = \mathbb{P}(\boldsymbol{Oil_t} = \boldsymbol{S} \setminus \boldsymbol{Oil_{t-1}} = \boldsymbol{S}) = \mathbf{0.47}.$

Figure1. **Graphs of Transition Matrix.(a): Us dollar index. (b): Oil Price. (c): Gold Price.** ***for finest presentation, they are compiled with LATEX editor.***

(a) **(b)** **(c)**



Source: Edited by Authors using Output of R estimation and Latex graphic.

2.2. The stationary law of Markov Chain

We first estimate all the transition probability matrices $\widehat{\boldsymbol{\alpha}}_{ij}$ and we also have the estimates of the stationary probability distributions of the three variables, which is considered as a long-term behaviour of the three variables: [1]

$$\hat{\boldsymbol{\pi}}_{oil} = (0.344 \quad 0.287 \quad 0.369)^t$$
$$\hat{\boldsymbol{\pi}}_{gold} = (0.089 \quad 0.446 \quad 0.465)^t$$
$$\hat{\boldsymbol{\pi}}_{usd} = (0.433 \quad 0.298 \quad 0.269)^t$$

In practice, the estimation of limit probability of a Markov Chain, is based *on ergodic theorem*, see **(R.Douc & P.Soulier, 2007)**. A Markov chain is *ergodic*, if all states in a chain are recurrent, aperiodic, and communicate with each other, the chain is said to be **ergodic.** $\boldsymbol{\pi_i}$ is usually interpreted as the *fraction of time* spent in each state$\mathbb{E}_\mathbf{i}$. For example, taking the oil Markov chain :**34.4%** of time (time here equal **359 Months**), the oil price was in the low state, **28.7%** of time in a stable state and **36.9%** of time in the High state. We can talk here, that the probability stationary distribution of oil price, is symmetric about stat $= \mathbf{S}$, so the probability of a run of positive changes$\mathbb{E} = \mathbf{H}$, is nearly equal the probability of a negative change $\mathbb{E} = \mathbf{L}$.

For the US dollar index, the highest probability is $\mathbb{P}(\boldsymbol{usd_t} = \boldsymbol{L}) = \mathbf{0.43}$, over the all period of estimation, **43%** of months, the dollar return is negative, which in fact a tendency of a future weak dollar. A weak dollar implies a protective investment in what has been the world's currency for millennia, namely gold and silver, the loss of confidence in the fiduciary money can only rekindle the need to hold money. Gold and silver, as long as the dollar weakens, this fear will drive capital away from gold and silver.

Taking the case of gold price vector, which is reputed to be the most advanced macroeconomic indicator.$\mathbb{P}(\boldsymbol{gold_t} = \boldsymbol{L}) = \mathbf{0.08}$ $\boldsymbol{Vs}\mathbb{P}(\boldsymbol{gold_t} = \boldsymbol{H}) = \mathbf{0.46}$) makes main evidence that the big bull market for gold has already begun, and the future tendency behaviour is an important key for best understanding this market. In comparison, with oil price, we can conclude that it is nearly the same

[1] With « **markov chain** » package in R , the limit distribution is estimated by the command: "**steady states**".

positive trend, unless that the oil price, is more volatile than the gold price. However, this similitude trend provides us a basic idea about an existence of a long-run relationship between the two variables. **Shahriar and Erkan , (2010) ,** made an investigation into the relationship between *gold price* and other key influencing variables, such as oil price and global inflation over the period (**1968-2010) .**They applied a modified econometric version of the long-term trend reverting jump and dip diffusion model for forecasting natural-resource commodity prices.

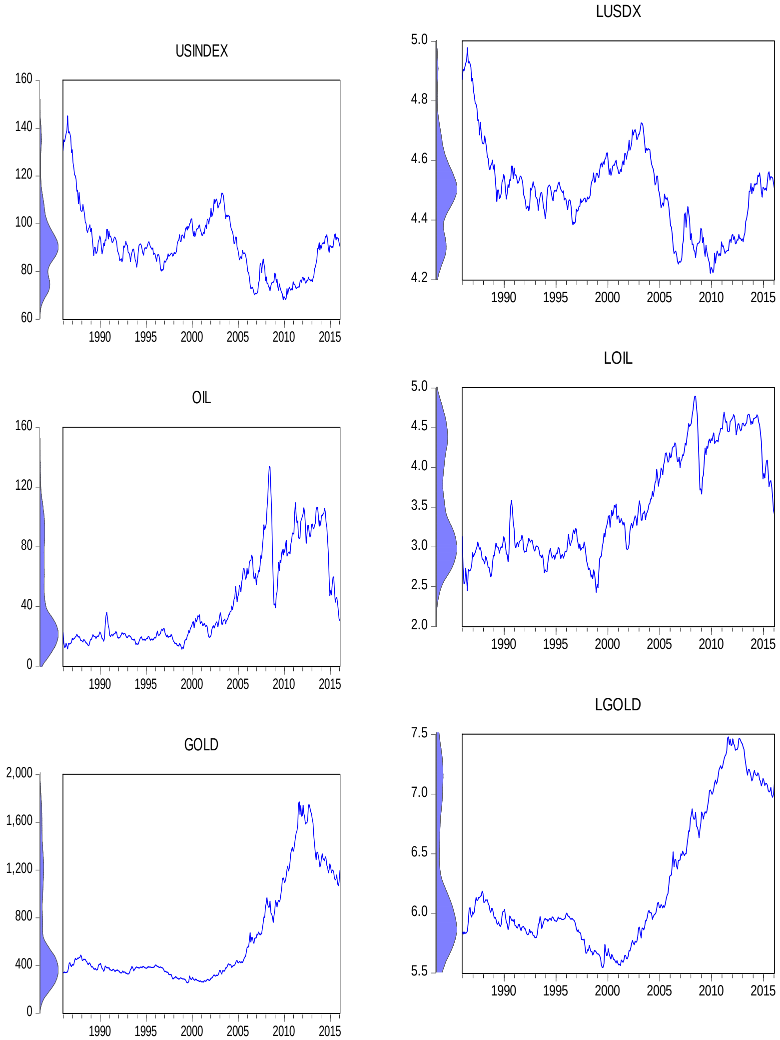
Until this level of study we analysed just the behaviour of the variables, but how about the relationship between those variables, is a Markov Models can give us a complete response? TO answer this question, in the next section, we deal with the ARDL cointegration approach.

3.2. ARDL Results and Application

From Figure 2, we see a no stationary of these time series, especially for Oil price and Gold Price series. *Oil price* is characterised by a high volatility and a presence of a *break point*, this is so clear ,if we put its graph separated with a kernel density in the axis broads, which help us to confirm the heterogeneity of this series. We used the Augmented Dickey Fuller (ADF) and KPSS tests to check that none of the series we're working with are integrated of second order: $\mathbf{I(2)}$. Applying the ADF test to the levels of Oil price, Gold price and US dollar index, the *p-values* are 0.53 and 0.10 respectively. Applying the test to the first-differences of the series, the *p-values* are both **0.00**. Obviously, neither series is I(2).

Figure2. **Monthly trend of Oil, Gold price and Dollar index from January 1986 to March 2016.**

Trend Graph of Variables in Level	**Trend Graph of variables in logarithm, $ln(X)$**



Source: Edited by Authors using Output of Eviews9.

Closer inspection of this figure shows, that the evolution of oil and gold prices has followed a similar path. We started here to analyse the correlation matrix between the three variables, we can see that the linear correlation between the Oil price and Dollar index is negative, the same correlation is between the Gold price and Dollar index. In contrast, there is a strong positive correlation between the Oil price and the Gold price. As a primarily result, how can we explain that?

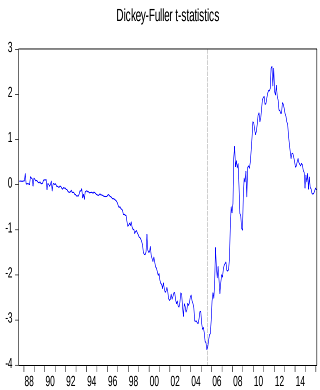
Correlation	LUSDX	LOIL	LGOLD
LUSDX	1.000000		
LOIL	-0.614236	1.000000	
LGOLD	-0.563606	0.840657	1.000000

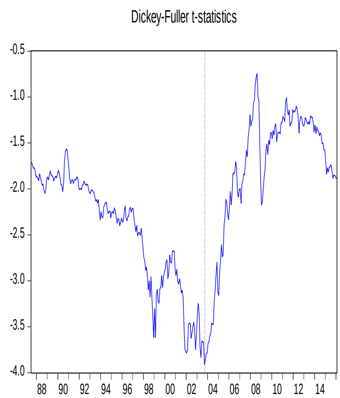
There are many reasons why the value of the dollar influences Oil prices, Gold Price (*generally*, commodities prices. The primary reason is that the dollar is the benchmark pricing mechanism for most commodities because the U.S. currency is the reserve currency of the world. Therefore, when it comes to international trade for raw materials, the dollar is the exchange mechanism in many, if not most cases. When the value of the dollar drops, it costs more dollars to buy commodities.

At the same time, it costs a lesser amount of other currencies when the dollar in moving lower. Commodity prices do not necessarily tick higher for every tick lower in the Dollar Index, but there is a strong inverse relationship over the long-haul. Individual commodities have fundamental supply and demand characteristics so they move one way or another at times despite the direction of the U.S. currency. For this reason, we applied the ARDL approach to check out the long-run relationship between the Oil price, Gold price and Dollar Index.

Figure3. **Test statistic graph of variables, based on Dickey-Fuller min-Test with an innovation outlier Break Type.**

Gold: 2005, M 07.	**Oil: 2003, M 09.**





Source: Edited by Authors using Output of Eviews9.

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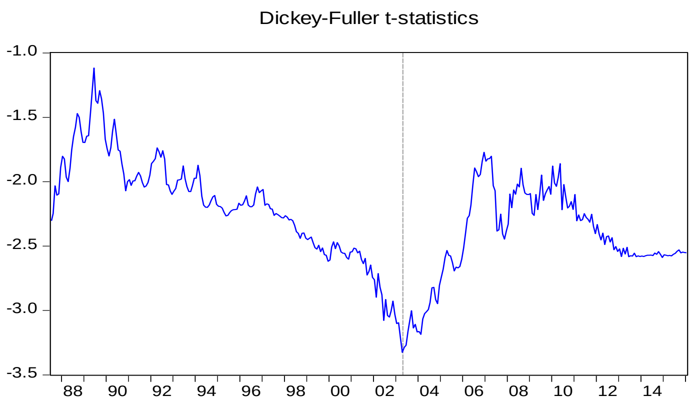


Source: Edited by Authors using Output of Eviews9.

When we see the trend graphs of variables, clearly, there is a break point in each one, *see the figures below*, so the estimation results will be biased. To go further analysis, and by taking the Oil Price as dependent variable we performed two ARDL models , the first for the period : (**1986, 2003M09)** and the second through the period (**2003M10, 2016M02**.) We can use the technique of Marko Switching regression Models, as a result, we elaborate two models for two regimes. *For more details*, see**Perlin, (2007).**

2.3.1. Estimated ARDL Results

To specify an ARDL model, we must determine the lags of each variable should be included (*i.e. in the equation* **(5)** *above,* $\boldsymbol{q_1}$*and* $\boldsymbol{q_2}$).We know that an ARDL model can be estimated via least squares regression. Standard *Akaike, Schwarz* and *Hannan-Quinn* information criteria may be used for model selection. We select the model **ARDL (2, 0, 0)** for the period (**1986-2003)**, and **ARDL (1, 0, 1)** for the period **(2003-2016).**

Table2. **Short Run Estimation Results of ARDL Model over the two periods**

	1986-2003		**2003-2016**	
Variables(*Dependent variable is Oil price*)	Coefficient	Prob.*	Coefficient	Prob.*
DOIL(-1)	**0.285183**	**0.0001**	**0.422827**	**0.0000**
DOIL(-2)	**-0.159606**	**0.0162**	**-**	**-**
DGOLD	**0.026266**	**0.0142**	**0.018064**	**0.0907**
DUSINDEX	-0.007249	0.8953	-1.55E-05	0.6474
DUSINDEX(-1)	-	-	5.33E-05	0.8018
C	0.053567	0.6490	-0.114833	0.8431

Source: Elaborated by Authors.

We perform a "*Bounds Test*" to see if there is evidence of a long-run relationship between the variables. The *F-statistic* test is used for testing the existence of long-run relationship. When long-run relationship exist, *F* test indicates which variable should be normalized. The null hypothesis for no cointegration among variables in equation (3) is $\boldsymbol{H_0}$: $\boldsymbol{\beta_1 = \beta_2 = 0}$ against the alternative hypothesis $\boldsymbol{H_1}$: $\boldsymbol{\beta_1 \neq \beta_2 \neq 0}$.

Table3. **Bounds test results**

Dependent variable	5% critical value (Pesaran,2001)		(Narayan,2005)		1986-2003	2003-2016
	I(0)	I(1)	I(0)	I(1)		
Oil	3.79	4.85	4.07	5.19	5.9	5.03
Gold	3.79	4.85	4.07	5.19	6.3	7.09
US dollar index	3.79	4.85	4.07	5.19	2.8	2.7

** *denotes the rejection of the null hypothesis of no cointegration at the* 5% *significance level*

Source: Elaborated by Authors

If the computed *F-statistic* falls below the lower bound we would conclude that the variables are $\boldsymbol{I}(\mathbf{0})$, so no cointegration is possible, by definition. If the F-statistic exceeds the upper bound, we conclude that we have cointegration. Finally, if the F-statistic falls between the bounds, the test is inconclusive.The $\boldsymbol{F_c} = \mathbf{39.3},$ which is higher than the critical value of **Peseran *et al*, (2001)**table. Therefore, we conclude that we have cointegration.

Table 4. **Long-run Coefficients over the two periods**

	1986-2003		2003-2016	
Variable	Coefficient	Prob.*	Coefficient	Prob.*
DGOLD	**0.030038**	**0.0166**	0.031119	0.1030
DUSINDEX	-0.008290	**0.8954**	0.000071	0.5083
C	0.061259	0.6487	-0.162670	0.8431

Source: Elaborated by Authors.

In this view, an error correction model which estimates the speed of adjustment to equilibrium in a cointegrating relationship. The long-run coefficients, at the table above of the output, show that the long-run impact of a change in D(DUSINDEX) on DOIL has essentially no lagged-effects. The gold price of the period$\boldsymbol{t-2}$,is positively correlated to the price of oil, with an estimated coefficient $\hat{\boldsymbol{\theta}} = \mathbf{0.026}.$

Table 5. **Cointegrating Form and speed of adjustment**

	1986-2003		2003-2016	
Variable	**Coefficient**	**Prob.***	**Coefficient**	**Prob.***
D(DGOLD)	0.026266	0.0142	0.017961	0.0989
D(DUSINDEX)	-0.007249	0.8953	-0.000013	0.7150
CointEq(-1)	-0.874424	0.0000	-0.577173	0.0000

Source: Elaborated by Authors.

An increase in Gold prices may lead to an increase in oil prices, in contrast an increase by 1% in US dollar index makes a decrease in oil price by 17%. *Essentially if oil prices rise users substitute gas for oil placing upward pressure on gas prices (a demand effect), and we note here the effect of speculation.*

We can interpret the result above as: variations by one unite (*i.e.* by **1 $**) per month of Gold price, other things being equal can make a change of Oil price by **0.28 $** over the period **1986-2003**, and a change by **0.42$** over the period **2003-2016**. On the long-run relationship, the effect of gold price on oil price is the same over the two periods, *see* **Table.4**. Although, we can add other information: both *Gold* and *Oil* are traded in *US dollars*. Therefore, their prices depend on the strength of the US dollar. The strength of the dollar is determined by inflation. We can therefore argue that the prices of the two materials have a similar tendency, not because of influence from one another, but because their prices are influenced by a common and major factor, inflation. See **Dan, (2017).**

Another important link between *gold* and *oil* is *inflation*. As crude oil prices rise, inflation also rises. Gold is known to be a good hedge against inflation. The value of gold only increases when inflation rises. Over 60% of the time, gold and crude oil have a direct relationship. The above chart shows historical prices for both dollar-denominated assets. Gold and crude oil are further related in that a rise in the price of oil dampens economic growth due to its excessive industrial use.

Dampened economic growth adversely affects most industries. This can lead to a fall in equity markets, which boosts the demand for alternative assets such as gold. Gold's magnetic force is in play here. The recessionary phase would be good for precious metal lovers by pushing gold prices higher.

By the error correction model, we can conclude that, the movements of gold price can suggest us to predict the fluctuations of oil price. *See* **Table 5.**

- **Autocorrelation of Errors terms**

The test of **LM** or Breusch-Godfrey **(Breusch, 1978) (Godfrey, 1978)**, is especially applied where lagged values of dependant variable are included in the models as independent variables. The statistics test equals $\mathbf{LM} = \boldsymbol{n} * \boldsymbol{R}^2 \sim \chi^2_p$, with $\boldsymbol{p}$ is the number of lags in the error term.

Table 6. **Breusch-Godfrey Serial Correlation LM Test**

Test's Results / **Period**	1986-2003	2003-2016
F-statistic	1.004159	3.975704
Prob. F-statistic	0.3681	0.0208
Prob. Chi-Square(2)	0.4296	0.0183

Source: Elaborated by Authors.

For the first period (**1986-2003)**, we reject the null hypothesis (H0) stating that there is no autocorrelation of errors, and we accept it over the second period (**2003-2016).**

- **Stability of the Models**

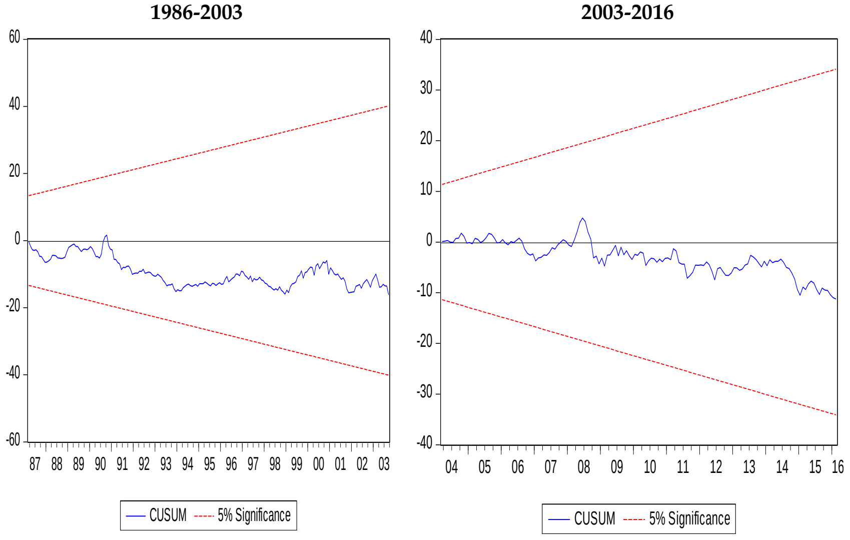
To test a structural stability of the model there are different tests based on recursive residuals. The two most important are the *CUSUM* and the *CUSUM-OF-SQUARES*. With the data ordered chronologically, rather than according to the value of an explanatory variable. This option plots the cumulative sum together with the 5% critical lines. The test finds parameter instability if the cumulative sum goes outside the area between the two critical lines.The CUSUM test

(**Brown, Durbin, and Evans, 1975)** is based on the cumulative sum of the recursive residuals. The test statistics is done by:

$$\mathbf{R_t} = \sum_{i=k+1}^{T} \frac{r_i}{\hat{\sigma}} \qquad (\mathbf{8})$$

For $\boldsymbol{i} = \boldsymbol{k} + \mathbf{1}, \ldots, \boldsymbol{T}$, where $\boldsymbol{r_i}$ is the recursive residual defined above, and $\hat{\sigma}$ is the standard deviation of the recursive residuals.

Figure 4. **The graph of CUSUM stability test for the two periods.**



Figures 4 plots the CUSUM statistics for Equation (**8**). It can be seen that the plot of CUSUM stays within the critical 5% bounds that confirms the long-run relationships among variables and thus shows the stability of coefficients.

CONCLUSION

This paper sheds light on recent world financial crisis and the implicit interconnection between the markets. We investigate whether a change in a market contributes to a change in other(s) market(s). We analyse the behaviour of real prices of crude **Oil**, **Gold** price and **US dollar index**. The study is based on **Probabilistic Markov Models**, and **ARDL** bounds test, estimated on monthly data over the period **1986-2016**. The findings reveal: **i)** A positive trend of Gold prices **ii)** Despite the high volatility of Oil prices, and by using Markov model, we get a stable- symmetric structure of Oil prices. **iii)** There is a high correlation between Oil and Gold prices at around **84%**. **IV)** Moreover, our results suggest that Oil prices increase significantly in response to reductions in gold prices .**V)** The evidence also suggests that the US dollar index as an indicator has no effect on Oil prices.

A vital policy implication of this study is that complex and danger relationships among financial markets should be taken in consideration on any governmental decisions, policies and strategies relative to the opening of the national financial market.Due to endogeneity in the relationships that may exist across macroeconomic variables and between macroeconomic factors and energy prices, and despite all the sophistically statistical models and methods, researchers often face significant challenges when empirically modelling their relationships .Under this problem, we think that a meta-regression analysis would be a good method to calibrate all these divergent results.

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