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ON CONGRUENCES AND KERNELS OF SOME LEFT RESTRICTION SEMIGROUPS IN $\wp\mathfrak{I}_{\{11,12,13,14,15\}}$

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Abstract
This paper presents some left restriction semigroups LRS in partial transformation $\wp\mathfrak{I}_{\{X=11,12.13.14.15\}}$, computed the left congruences $\tilde{R}$-classes and enumerates all the kernels $K\left(\wp\mathfrak{I}_{\{11,12,13,14,15\}}\right)$ for each element of LRS in $\wp\mathfrak{I}_{X}$.

Key words: *Congruence, Kernels, Left Restriction semigroups , Partial Transformation*

Introduction

A compatible equivalence relation on a semigroup S is referred to as a Congruence. An equivalence R on a semigroup S is a congruence if and only if

$(a,b) \in R$ and $(c,d) \in R$ imply that $(ac,bd) \in R$ (Gonzalez, 2001).

Restriction semigroups are a generalization of inverse Semigroups. Restriction semigroups are semigroups with an additional unary operation $^+$ (Left) and $*$ (Right). A semigroup S is left restriction with respect to $E \subseteq E\,(S)$ if `

i. E is a subsemilattice of S
ii. Every element $a \in S$ is $\widetilde{R_E}$ -related to an element of E (denoted by a^+)
iii. $\widetilde{R_E}$ is a left congruence
iv. The left ample condition holds $\forall\, a \in S \; and \; e \in E,$

$$ae = (ae)^+a$$

Equivalently,

A semigroup S is a right restriction with respect to $E = E\,(S)$ if

i. E is a subsemilattice of S
ii. Every element $a \in S$ is $\widetilde{L_E}$ -related to an element of E (denoted by a^*)
iii. $\widetilde{L_E}$ is a right congruence
iv. The right ample condition holds $\forall\, a \in S \; and \; e \in E,$

$$ea = a(ea)^* \text{ (Gould, 2011)}$$

Left restriction semigroups have appeared at the convergence of several flow of research, they model unary semigroups of partial mappings on a set where the unary operation takes a map to the identity map on its domain

(Jones, 2012). Partial transformation semigroup $\wp\mathfrak{I}_X$ is a weakly left E-ample semigroup otherwise known as Left restriction semigroup.The identities that define a Left restriction semigroup S are

$$a^+ a = a, \qquad a^+ b^+ = b^+ a^+, (a^+ b)^+ = a^+ b^+, ab^+ = (ab)^+ a$$

We put

$$E = \{a^+ : a \in S\}$$

E is a semilattice known as the semilattice of projections of S (Zenab, 2014). Let $\wp T_X$ be a Left restriction semigroup with distinguished semilattice,

$$E = \{I_Y : Y \subseteq X\}$$

and with

$$\alpha^+ = 1_{dom\,\alpha}$$

S is Left restriction if and only if it embeds in some $\wp T_X$ in a way that preserves $^+$ where

$$\alpha^+ = \alpha\alpha^{-1} \qquad \text{(Hollings, 2009)}$$

Consider the semidirect product of a semilattice X and monoid S, then $X * S$ is a Left restriction semigroup with

$$(x, s)^+ = (x, 1) \;\; \forall\, (x, s) \in X * S$$

$X * S$ is a left restriction semigroup with distinguished semilattice

$$E = \{\,(e, 1) : e \in X\}$$

It can easily be seen that each element of E is an idempotent and

$$\begin{aligned}(e,1)\,(f,1) &= (e\,(1.f),1)\\ &= (ef,1)\\ &= (fe,1)\\ &= (f\,(1.e\,),1)\\ &= (f\,,1)\,(e,1)\end{aligned}$$

Since X is a semilattice , hence, E is a semilattice of $X * S$.
We claim that

$$(e,1)\widetilde{R_E}\,(e,1)$$

For $(e, s) \in X * S$. We have

$$(e,1)(e,s) = (e\,(1.e),s) = (e,s)$$

And for $(f,1)(e,s) = (e,s) \Longrightarrow (f\,(1.e),s) = (e,s)$

$$\begin{aligned}&\Longrightarrow (fe,s) = (e,s)\\ &\Longrightarrow fe = e\\ &\Longrightarrow (f,1)(e,1) = (fe,1) = (e,1)\end{aligned}$$

$$\therefore\;\; (e,s)\,\widetilde{R_E}\,(e,1)$$

For $(e,s) \in X * S$, $\widetilde{R_E}$ is a left congruence first, we note that for $(e,s)(f,t) \in X * S$,

$$\begin{aligned}(e,s)\widetilde{R_E}\,(f,t) &\Leftrightarrow (e,s)^+ = (f,t)^+\\ &\Leftrightarrow (e,1) = (f,1)\\ &\Leftrightarrow e = f\end{aligned}$$

For $(e,s),(f,t),(g,\mu) \in X * S$, we have

$$\begin{aligned}(e,s)\,\widetilde{R_E}\,(f,t) &\Longrightarrow e = f\\ &\Longrightarrow g(u.e) = g\,(u.f)\\ &\Longrightarrow g(u.e),us)\;\widetilde{R_E}\;g\,(u.f),ut)\\ &\Longrightarrow (g,u)(e,s)\;\widetilde{R_E}\;\;(g,u)(f\,t)\end{aligned}$$

So, $\widetilde{R_E}$ is a left congruence .
Equivalently,

$$a\,\tilde{\mathcal{R}}\,e \;\Longrightarrow aS = bS \;\;\forall\, a,b \in S$$

Hollings (2009) opined that the most natural way to represent restriction semigroup was through partial transformation. Examples of Left restriction semigroups have already been generated and computed from partial transformation $\wp T_X$ for orders $X = \{2,3,4,5,6,7,8,9,10\}$ and the algebraic properties , Left congruence $\widetilde{\mathcal{R}}-$ classes inherent and E_X semilatices of idempotent presented (Abubakar, et al., 2020).

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Kernels is an important property in algebra that defines quotient objects which also measures injectivity. Consider a function

$$f: G \rightarrow H$$

the kernel of f is a set of elements of G which map into the identity element $e \in G.$ That is, kernel

$$f = \{ a \in G : f(a) = e\}$$

Ker f ={ $g \in G : f(g) = e_H$} (Abubakar, et al., 2021)

In a semigroup S, the kernel are the set of elements mapped to the same elements. In this paper, Raf-baduT as designed in (Abubakar et al.,2021) was used to generate elements of partial transformation $\wp\mathfrak{I}_{\{11,12,13,14,15\}}$ .After the examples were obtained, the algebraic properties of left congruences , $\tilde{R}s$ –classes and from their semilattices of idempotents, the kernels were obtained.

Main Result

Computing the congruence class – $\tilde{R}$ -classes and the kernels

For $\wp\mathfrak{I}_{\{X=11\}}$

Table 1A **:** $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11\}}$: (1 3 6 7 8 X 9 X 10 X X)

+	A	B	C	D	F	G	H	I	J	K	L	M	N	O	E
A	B	C	D	D	E	G	H	I	J	K	L	M	N	O	E
B	C	D	D	D	E	G	H	I	J	K	L	M	N	O	E
C	D	D	D	D	E	G	H	I	J	K	L	M	N	O	E
D	D	D	D	D	E	G	H	I	J	K	L	M	N	O	E
F	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
G	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
H	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
I	H	E	E	E	E	E	E	E	E	E	E	E	E	E	E
J	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
K	I	H	E	E	E	E	E	E	E	E	E	E	E	E	E
L	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
M	J	E	E	E	E	E	E	E	E	E	E	E	E	E	E
N	K	I	H	E	E	E	E	E	E	E	E	E	E	E	E
O	L	E	E	E	E	E	E	E	E	E	E	E	E	E	E
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 3 & 6 & 7 & 8 & X & 9 & X & 10 & X & X \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 6 & X & 9 & X & X & 10 & X & X & X & X \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & X & X & 10 & X & X & X & X & X & X & X \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,

$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ X & 10 & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 11 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,

$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 10 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 9 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,

$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 8 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,

$L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 6 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $M = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 5 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,

$N = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 4 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $O = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 3 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,

$E = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,

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Left congruence $\tilde{R} - classes$
$\widetilde{R_D} = \widetilde{R_G} = \widetilde{R_H} = \widetilde{R_I} = \widetilde{R_J} = \widetilde{R_K} = \widetilde{R_L} = \widetilde{R_M} = \widetilde{R_N} = \widetilde{R_O} = \{A, B, C, D\}$,
$\widetilde{R_E} = \{A, B, C, D, F, G, H, I, J, K, L, N, O, E\}$

Kernels

$K_A = \{C, D\}_D ; \{F, G, H, J, L, E\}_E$; $K_B = \{B, C, D\}_D ; \{F, G, H, J, K, L, M, O, E\}_E$;

$K_C = \{A, B, C, D\}_D ; \{F, G, H, J, K, L, M, O, E\}_E$;$K_D = \{A, B, C, D\}_D ; \{F, G, H, J, K, L, M, N, O, E\}_E$

;$K_G = \{A, B, C, D\}_G ; \{F, G, H, J, K, L, M, N, O, E\}_E$; $K_H = \{A, B, C, D\}_H ; \{F, G, H, J, K, L, M, N, O, E\}_E$

$K_I = \{A, B, C, D\}_I ; \{F, G, H, J, K, L, M, N, O, E\}_E$; $K_J = \{A, B, C, D\}_J ; \{F, G, H, J, K, L, M, N, O, E\}_E$

;$K_K = \{A, B, C, D\}_K ; \{F, G, H, J, K, L, M, N, O, E\}_E$; $K_L = \{A, B, C, D\}_L ; \{F, G, H, J, K, L, M, N, O, E\}_E$

;$K_M = \{A, B, C, D\}_M ; \{F, G, H, J, K, L, M, N, O, E\}_E$;$K_N = \{A, B, C, D\}_N ; \{F, G, H, J, K, L, M, N, O, E\}_E$

$K_0 = \{A, B, C, D\}_O ; \{F, G, H, J, K, L, M, N, O, E\}_E$;$K_E = K_F = \{A, B, C, D, F, G, H, J, K, L, M, N, O, E\}_E$

Table 1B : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11\}}$ **:** $(\mathbf{1}\ \boldsymbol{X}\ \mathbf{6}\ \mathbf{7}\ \mathbf{8}\ \mathbf{2}\ \mathbf{9}\ \mathbf{4}\ \mathbf{10}\ \boldsymbol{X}\ \boldsymbol{X})$

+	A	B	C	D	F	G	H	I	J	K	L	M	N	E
A	B	C	D	F	G	G	I	J	E	K	L	M	M	E
B	C	D	F	G	G	G	J	E	E	K	L	M	M	E
C	D	F	G	G	G	G	E	E	E	K	L	M	M	E
D	F	G	G	G	G	G	E	E	E	K	L	M	E	E
F	E	E	E	E	E	E	E	E	E	E	E	E	M	E
G	G	G	G	G	G	G	E	E	E	K	L	M	M	E
H	G	G	G	G	G	G	E	E	E	K	L	M	E	E
I	E	E	E	E	E	E	E	E	E	E	E	E	E	E
J	E	E	E	E	E	E	E	E	E	E	E	E	E	E
K	E	E	E	E	E	E	E	E	E	E	E	E	E	E
L	E	E	E	E	E	E	E	E	E	E	E	E	E	E
M	E	E	E	E	E	E	E	E	E	E	E	E	E	E
N	L	E	E	E	E	E	E	E	E	E	E	E	E	E
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & X & 6 & 7 & 8 & 2 & 9 & 4 & 10 & X & X \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & X & 2 & 9 & 4 & X & 10 & 7 & X & X & X \end{pmatrix},$
$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & X & X & 10 & 7 & X & X & 9 & X & X & X \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & X & X & X & 9 & X & X & 10 & X & X & X \end{pmatrix},$
F= $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}, G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ X & 10 & X & X & X & X & X & X & X & X & X \end{pmatrix}$
$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ X & X & X & X & X & 10 & X & X & X & X & X \end{pmatrix} I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ X & X & 10 & X & X & X & X & X & X & X & X \end{pmatrix},$
$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 11 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}, K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 10 & X & X & X & X & X & X & X & X & X & X \end{pmatrix},$

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$$L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 9 & X & X & X & X & X & X & X & X & X & X & \end{pmatrix}, M = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 8 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$$

$$N = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}, E = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ X & X & X & X & X & X & X & X & X & X & X & \end{pmatrix}$$

Left congruences $\tilde{R}$ -classes

$\widetilde{R_G} = \{A, B, C, D\}$, $\widetilde{R_K} = \{A, B, C, D, G, H\}$, $\widetilde{R_L} = \{A, B, C, D, G, H\}$, $\widetilde{R_M} = \{A, B, C, D, G, H\}$, $\widetilde{R_E} = \{A, B, C, D, F, G, H, I, J, K, L, M, N, E\}$

Kernels

$K_A = \{G, H\}_G ; \{I, J, K, L, M, E\}_E$; $K_B = \{D, G, H\}_G ; \{I, J, K, L, M, N, E\}_E$

; $K_C = \{C, D, G, H\}_G ; \{I, J, K, L, M, N, E\}_E$; $K_D = \{B, C, D, G, H\}_G ; \{I, J, K, L, M, N, E\}_E$

;$K_F = \{A. B, C, D, G, H\}_G ; \{F, I, J, K, L, M, N, E\}_E$;$K_G = \{A, B, C, D, G, H\}_G ; \{F, I, J, K, L, M, N, E\}_E$

$K_H = \{C, D, F, G, H, I, J, K, L, M, N, E\}_E$;$K_I = \{B, C, D, F, G, H, I, J, K, L, M, N, E\}_E$;

$K_J = \{A, B, C, D, F, G, H, I, J, K, L, M, N, E\}_E$;$K_K = \{A, B, C, D, G, H\}_K ; \{F, I, J, K, L, M, N, E\}_E$

;$K_L = \{A, B, C, D, G, H\}_L ; \{F, I, J, K, L, M, N, E\}_E$; $K_M = \{A, B, C, D\}_M ; \{F, I, J, K, L, M, N, E\}_E$

$K_N = \{A, B, C, F, G\}_M ; \{D, H, I, J, K, L, M, N, E\}_E$; $K_E = \{A, B, C, D, F, G, H, I, J, K, L, M, N, E\}_E$

Table 1 C : $\wp T_{\{1,2,3,4,5,6,7,8,9,10,11\}}$: (1 2 X 7 11 X 3 4 X X X)

+	A	B	C	D	F	G	H	I	J	K	L	M	N	O
A	B	C	D	D	F	G	H	I	J	K	L	M	N	O
B	C	D	D	D	F	G	H	I	J	K	L	M	N	O
C	D	D	D	D	F	G	H	I	J	K	L	M	N	O
D	D	D	D	D	F	G	H	I	J	K	L	M	N	O
F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
G	F	F	F	F	F	F	F	F	F	F	F	F	F	F
H	F	F	F	F	F	F	F	F	F	F	F	F	F	F
I	F	F	F	F	F	F	F	F	F	F	F	F	F	F
J	N	K	O	F	F	F	F	F	F	F	F	F	F	F
K	O	F	F	F	F	F	F	F	F	F	F	F	F	F
L	F	F	F	F	F	F	F	F	F	F	F	F	F	F
M	G	F	F	F	F	F	F	F	F	F	F	F	F	F
N	K	O	F	F	F	F	F	F	F	F	F	F	F	F
O	F	F	F	F	F	F	F	F	F	F	F	F	F	F

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & X & 7 & 11 & X & 3 & 4 & X & X & X \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & X & 3 & X & X & X & 7 & X & X & X \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & X & X & X & X & X & 3 & X & X & X \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix},$$

$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ X & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix}, G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 11 & 2 & X & X & X & X & X & X & X & X & \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 10 & 2 & X & X & X & X & X & X & X & X & \end{pmatrix}, I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 9 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix},$$

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$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 8 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix}, K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix},$
$L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 6 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix}, M = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 5 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix}$
$N = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 4 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix}, O = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 3 & 2 & X & X & X & X & X & X & X & X & X \end{pmatrix}$

Left congruences $\tilde{R} - classes$

$\widetilde{R_D} = \widetilde{R_G} = \widetilde{R_H} = \widetilde{R_I} = \widetilde{R_J} = \widetilde{R_K} = \widetilde{R_L} = \widetilde{R_M} = \widetilde{R_N} = \widetilde{R_O} = \{A, B, C, D\},$

$\widetilde{R_F} = \{A, B, C, D, F, G, H, I, J, K, L, M, N, O\}$

Kernels

$K_A = \{C, D\}_D$; $\{F, G, H, I\}_F$; $K_B = \{B, C, D\}_D$; $\{F, G, H, I, K, L, M, O\}_F$

; $K_C = \{A, B, C, D\}_D$; $\{F, G, H, I, J, K, L, M, N, O\}_F$; $K_D = \{A, B, C, D\}_D$; $\{F, G, H, I, J, K, L, M, N, O\}_F$;

$K_F = \{A, B, C, D, F, G, H, I, J, K, L, N, O\}_F$;$K_G = \{A, B, C, D\}_G$; $\{F, G, H, I, J, K, L, N, O\}_F$

$K_I = \{A, B, C, D\}_I$; $\{F, G, H, I, J, K, L, M, N, O\}_F$; $K_J = \{A, B, C, D\}_J$; $\{F, G, H, I, J, K, L, M, N, O\}_F$;

$K_L = \{A, B, C, D\}_L$; $\{F, G, H, I, J, K, L, M, N, O\}_F K_M = \{A, B, C, D\}_M$; $\{F, G, H, I, J, K, L, M, N, O\}_F$;
$K_N = \{A, B, C, D\}_N$; $\{F, G, H, I, J, K, L, M, N, O\}_F$;$K_O$=$\{A, B, C, D\}_O$; $\{F, G, H, I, J, K, L, M, N, O\}_F$

For $\wp\mathfrak{I}_{\{X=12\}}$
Table 2 A : $\wp T_{\{1,2,3,4,5,6,7,8,9,10,11,12\}}$: ($\mathbf{2\ 3\ X\ 9\ 12\ X\ 5\ X\ 6\ X\ 4\ X}$)

+	A	B	C	D
A	B	C	E	E
B	C	E	E	E
C	E	E	E	E
D	E	E	E	E

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 2 & 3 & X & 9 & 12 & X & 5 & X & 6 & X & 4 & X \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & X & X & 6 & X & X & 12 & X & X & X & 9 & X \end{pmatrix},$
$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ X & X & X & X & X & X & X & X & X & X & 6 & X \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 12 & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$

Left congruences $\tilde{R} - classes$
$\tilde{R}$= { }

Kernel
$K_A = \{C, D\}_E$;$K_B = \{B, C, D\}_E$;$K_C = \{A, B, C, D\}_E$;$K_D = \{A, B, C, D\}_E$

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Table 2 B : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12\}}$: $(\mathbf{2\ 7\ 10\ 9\ 12\ }\boldsymbol{X}\mathbf{\ 8\ }\boldsymbol{X}\mathbf{\ 6\ }\boldsymbol{X}\mathbf{\ 4\ }\boldsymbol{X})$

+	A	B	C	D	F
A	B	C	E	E	D
B	C	E	E	E	E
C	E	E	E	E	E
D	E	E	E	E	E
F	E	E	E	E	E

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 2 & 7 & 10 & 9 & 12 & X & 8 & X & 6 & X & 4 & X \end{pmatrix}$, B= $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 7 & 8 & X & 6 & X & X & X & X & X & X & 9 & X \end{pmatrix}$,
$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 8 & X & X & X & X & X & X & X & X & X & 6 & X \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 12 & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,
$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ X & 12 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$

Left congruences $\tilde{R} - classes$
$\tilde{R}$= { }

Kernel
$K_A = \{C, D, E\}_E$; $K_B = \{B, C, D, F\}_E$; $K_C = K_D = \{A, B, C, D, F\}_E$;$K_F = \{B, C, D, F\}_E$

Table 2C : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12\}}$: (2 5 10 9 12 X 1 X 6 X 4 3)

+	A	B	C	D	F	G
A	B	C	D	F	G	E
B	C	D	F	G	E	E
C	D	F	G	E	E	E
D	F	G	E	E	E	E
F	G	E	E	E	E	E
G	E	E	E	E	E	E

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 2 & 5 & 10 & 12 & X & 1 & X & 6 & X & 4 & 3 & \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 5 & 12 & X & 6 & 3 & X & 2 & X & X & X & 6 & X \end{pmatrix}$,
$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 12 & 3 & X & X & 10 & X & 5 & X & X & X & 6 & X \end{pmatrix}$,$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 10 & X & X & X & X & 12 & X & X & X & X & X \end{pmatrix}$,
$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 10 & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$,$G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ X & X & X & X & X & X & X & 10 & X & X & X & X \end{pmatrix}$

Left congruences $\tilde{R} - classes$

$\tilde{R}$= { }

Kernel
$K_B = \{F, G\}_E$; $K_C = \{D, F, G\}_E$; $K_D = K_F = \{C, D, F, G\}_E$; $K_G = \{A, B, C, D, F, G\}_E$

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For $\wp\mathfrak{I}_{\{X=13\}}$

Table 3A : $\wp T_{\{1,2,3,4,5,6,7,8,9,10,11,12,13\}}$: (3 4 X 5 10 X 9 X 6 1 X 11 X)

+	A	B	C	D	F	G	H	I	J	K	L
A	B	C	D	F	E	H	I	J	K	E	G
B	C	D	F	E	E	I	J	K	E	E	H
C	D	F	E	E	E	J	K	E	E	E	I
D	F	E	E	E	E	K	E	E	E	E	J
F	E	E	E	E	E	E	E	E	E	E	K
G	E	E	E	E	E	E	E	E	E	E	E
H	E	E	E	E	E	E	E	E	E	E	E
I	E	E	E	E	E	E	E	E	E	E	E
J	E	E	E	E	E	E	E	E	E	E	E
K	E	E	E	E	E	E	E	E	E	E	E
L	E	E	E	E	E	E	E	E	E	E	E

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 3 & 4 & X & 5 & 10 & X & 9 & X & 6 & 1 & 11 & X & X \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & 5 & X & 10 & 1 & X & 6 & X & X & 3 & X & X & X \end{pmatrix}$$
$$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & 10 & X & 1 & 3 & X & X & X & X & X & X & X & X \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & 1 & X & 3 & X & X & X & X & X & X & X & X & X \end{pmatrix},$$
$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & 3 & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}, G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 13 & X & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$$
$$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & X & X & X & X & X & X & 13 & X & X & X \end{pmatrix}, I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & X & 13 & X & X & X & X & X & X & X & X \end{pmatrix},$$
$$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 13 & X & X & X & X & X & X & X & X & X \end{pmatrix}, K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & 13 & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix},$$
$$L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & 13 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$$

Left congruences $\tilde{R} - classes$

$\tilde{R}$= { }

Kernel

$K_A = \{D,F,G,H,I,J,K,L\}_E$; $K_B = \{D,F,G,H,I,J,K,L\}_E$; $K_C = \{C,D,F,G,H,I,J,K,L\}_E$

;$K_D = \{B,C,D,F,G,H,I,J,K,L\}_E$ $K_F = \{A,B,C,D,F,G,H,I,J,K,L\}_E$;$K_G = \{F,G,H,I,J,K,L\}_E$

;$K_H = \{D,F,G,H,I,J,K,L\}_E$;$K_I = \{C,D,F,G,H,I,J,K,L\}_E$

$K_J = \{B,C,D,F,G,H,I,J,K,L\}_E$;$K_K = \{A,B,C,D,F,G,H,I,J,K,L\}_E$;$K_L = \{G,H,I,J,K,L\}_E$

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Table 3B : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12,13\}}$: (3 6 X 4 10 X 8 X 2 1 X 13 X)

+	A	B	C	D	F	G	H	I	J	K	L	M	N
A	B	C	D	D	G	H	D	J	D	F	M	N	D
B	C	D	D	D	H	D	D	D	D	G	N	D	D
C	D	D	D	D	D	D	D	D	D	H	D	D	D
D	D	D	D	D	D	D	D	D	D	D	D	D	D
F	D	D	D	D	D	D	D	D	D	D	D	D	D
G	D	D	D	D	D	D	D	D	D	D	D	D	D
H	D	D	D	D	D	D	D	D	D	D	D	D	D
I	D	D	D	D	D	D	D	D	D	D	D	D	D
J	D	D	D	D	D	D	D	D	D	D	D	D	D
K	D	D	D	D	D	D	D	D	D	D	D	D	D
L	F	D	D	D	D	D	D	D	D	D	D	D	D
M	G	D	D	D	D	D	D	D	D	D	D	D	D
N	H	D	D	D	D	D	D	D	D	D	D	D	D

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 3 & 6 & X & 4 & 10 & X & 8 & X & 2 & 1 & X & 13 & X \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 4 & 1 & X & X & X & 6 & 3 & X & X & X \end{pmatrix}$,
$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 4 & 3 & X & X & X & X & X & X & X & X \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 4 & X & X & X & X & X & X & X & X & X \end{pmatrix}$,
$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 13 & X & X & 4 & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 4 & X & X & X & X & X & 13 & X & X & X \end{pmatrix}$
$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & X & 4 & 13 & X & X & X & X & X & X & X \end{pmatrix}$, $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & 13 & X & 4 & X & X & X & X & X & X & X & X & X \end{pmatrix}$,
$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 4 & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $K = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & 13 & 4 & X & X & X & X & X & X & X & X & X \end{pmatrix}$,
$L = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 12 & X & X & 4 & X & X & X & X & X & X & X & X & X \end{pmatrix}$, $M = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 4 & X & X & X & X & X & 12 & X & X & X \end{pmatrix}$

$$N = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ X & X & X & 4 & 12 & X & X & X & X & X & X & X & X \end{pmatrix}$$

Left congruences $\tilde{R} - classes$

$\widetilde{R_D} = \{A, B, C, D, F, G, H, I, J, K, L, M, N\}$ }

Kernel

$K_A = \{C, D, F, G, H, I, J, K\}_D$; $K_B = \{B, C, D, F, G, H, I, J, K, L, M, N\}_D$
; $K_C = \{A, B, C, D, F, G, H, I, J, K, L, M, N\}_D$;$K_D = \{A, B, C, D, F, G, H, I, J, K, L, M, N\}_D$
$K_F = \{C, D, F, G, H, I, J, K, L, M, N\}_D$;$K_G = \{B, C, D, F, G, H, I, J, K, L, M, N\}_D$
;$K_H = \{A, B, C, D, F, G, H, I, J, K, L, M, N\}_D$,$K_I = \{B, C, D, F, G, H, I, J, K, L, M, N\}_D$
;$K_J = \{A, B, C, D, F, G, H, I, J, K, L, M, N\}_D$;$K_K = \{D, G, H, I, J, K, L, M, N\}_D$
$K_L = \{C, D, F, G, H, I, J, K, L, M, N\}_D$;$K_M = \{B, C, D, F, G, H, I, J, K, L, M, N\}_D$
;$K_N = \{B, C, D, F, G, H, I, J, K, L, M, N\}_D$

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Table 3C : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12,13\}}$: $(\mathbf{3\ X\ 1\ 5\ 7\ X\ 8\ 12\ 11\ X\ X\ X})$

+	A	B	C	D	F	G
A	B	C	D	F	G	F
B	C	D	F	G	F	G
C	D	F	G	F	G	F
D	F	G	F	G	F	G
F	G	F	G	F	G	F
G	F	G	F	G	F	G

$A=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\3&X&1&5&7&X&8&12&11&X&X&X&X\end{pmatrix}$, $B=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\1&X&3&7&8&X&12&X&X&X&X&X&X&X\end{pmatrix}$,
$C=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\3&X&1&8&12&X&X&X&X&X&X&X&X&X\end{pmatrix}$, $D=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\1&X&3&12&X&X&X&X&X&X&X&X&X\end{pmatrix}$,
$F=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\3&X&1&X&X&X&X&X&X&X&X&X&X\end{pmatrix}$, $G=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\1&X&3&X&X&X&X&X&X&X&X&X&X\end{pmatrix}$

Left congruences $\tilde{R}-classes$

$\widetilde{R_F}=\{B,G\}\quad \widetilde{R_G}=\{B,D,G\}\ \}$

Kernel

$K_A=\{D,G\}_F\ ;K_B=\{C,F\}_F\ ;\ \{D,G\}_G;K_B=\{C,F\}_F\ ;\ \{D,G\}_G;\ K_C=\{B,D,G\}_F\ ;\ \{C,F\}_G;$

$K_D=\{A,C,F\}_F\ ;\ \{B,D,G\}_G,K_F=\{A,C,F\}_G\ ;\ \{B,D,G\}_F;K_G=\{A,C,F\}_F\ ;\ \{B,D,G\}_G$

For $\wp\mathfrak{I}_{\{X=14\}}$

Table 4A : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}}$: $(\mathbf{4\ X\ 3\ 5\ 14\ X\ 9\ 8\ 11\ 7\ X\ 1\ X\ X})$

+	A	B	C	D	F	G	H	I	J
A	B	C	D	F	F	D	I	F	F
B	C	D	F	F	F	F	F	F	F
C	D	F	F	F	F	F	F	F	F
D	F	F	F	F	F	F	F	F	F
F	F	F	F	F	F	F	F	F	F
G	F	F	F	F	F	F	F	F	F
H	F	F	F	F	F	F	F	F	F
I	F	F	F	F	F	F	F	F	F
J	F	F	F	F	F	F	F	F	F

$A=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13&14\\4&X&3&5&14&X&9&8&11&7&X&1&X&X&5\end{pmatrix}$, $B=\begin{pmatrix}1&2&3&4&5&6&7&8&10&11&12&13&14\\5&X&3&14&X&X&11&8&X&9&X&4&X&X\end{pmatrix}$
$C=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13&14\\14&X&3&X&X&X&X&8&X&11&X&5&X&X\end{pmatrix}$, $D=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13&14\\X&X&3&X&X&X&X&8&X&X&X&14&X&X\end{pmatrix}$
$F=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13&14\\X&X&3&X&X&X&X&8&X&X&X&X&X&X\end{pmatrix}$. $G=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13&14\\14&X&3&X&X&X&X&8&X&X&X&X&X&X\end{pmatrix}$
$H=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13&14\\13&X&3&X&X&X&X&8&X&X&X&14&X&X\end{pmatrix}$, $I=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13&14\\X&X&3&X&X&X&X&8&X&X&X&13&X&X\end{pmatrix}$

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$$J = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ X & 14 & 3 & X & X & X & X & 8 & X & X & X & X & X & X \end{pmatrix}$$

Left Congruences $\tilde{R} - classes$

$\tilde{R} = \{ \ \} \}$

Kernel

$K_A = \{D,F,G,H,I,J\}_F$; $K_B = \{C,D,F,G,H,I,J\}_F$; $K_C = \{B,C,D,F,G,H,I,J\}_F$; $K_D = \{A,B,C,D,F,G,H,I,J\}_F$; $K_F = \{A,B,C,D,F,G,H,I,J\}_F$
$K_G = \{B,C,D,F,G,H,I,J\}_F$; $K_H = \{B,C,D,F,G,H,I,J\}_F$; $K_I = \{A,B,C,D,F,G,H,I,J\}_F$; $K_J = \{A,B,C,D,F,G,H,I,J\}_F$

Table 4B : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}}$**: 4 2 X 12 X 7 X X 10 X 3 5 X X**

+	A	B	C	D	F
A	B	C	D	D	D
B	C	D	D	D	D
C	D	D	D	D	D
D	D	D	D	D	D
F	D	D	D	D	D

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 4 & 2 & X & 12 & X & 7 & X & X & 10 & X & 3 & 5 & X & X \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 12 & 2 & X & 5 & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$$
$$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 5 & 2 & X & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ X & 2 & X & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$$
$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 14 & 2 & X & X & X & X & X & X & X & X & X & X & X & X \end{pmatrix}$$

Left Congruences $\tilde{R} - classes$

$\tilde{R} = \{ \ \}$

Kernel
$K_A = \{C,D,F\}_D$; $K_B = \{B,C,D,F\}_F$; $K_C = K_D = K_F = \{A,B,C,D,F\}_D$

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Table 4C : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}}$: $(\mathbf{4\ 1\ X\ 11\ 12\ 7\ X\ X\ 2\ X\ 8\ 5\ X\ 6})$

+	A	B	C	D	F	G	H
A	B	C	D	F	G	H	G
B	C	D	F	G	H	G	H
C	D	F	G	H	G	H	G
D	F	G	H	G	H	G	H
F	G	H	G	H	G	H	G
G	H	G	H	G	H	G	H
H	G	H	G	H	G	H	G

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 4 & 1 & X & 11 & 12 & 7 & X & X & 2 & X & 8 & 5 & X & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 11 & 4 & X & 8 & 5 & X & X & X & 1 & X & X & 12 & X & 7 \end{pmatrix}$
$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 8 & 11 & X & X & 12 & X & X & X & 4 & X & X & 5 & X & X \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ X & 8 & X & X & 5 & X & X & X & 11 & X & X & 12 & X & X \end{pmatrix}$
$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ X & X & X & X & 12 & X & X & X & 8 & X & X & 5 & X & X \end{pmatrix}$. $G = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ X & X & X & X & 5 & X & X & X & X & X & X & 12 & X & X \end{pmatrix}$
$$H = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ X & X & X & X & 12 & X & X & X & X & X & X & 5 & X & X \end{pmatrix}$$

Left Congruences $\tilde{R} - classes$

$\widetilde{R_G} = \{ B, D, G \}$

Kernel
$K_A = \{F, G\}_G$; $K_B = \{D, G\}_G$; $K_C = \{C, F, H\}_G$; $\{D, G\}_H$; $K_D = \{B, D, G\}_G$; $\{C, F, H\}_H$; $K_F = \{A, C, F\}_G$; $\{B, D, G\}_H$; $K_G = \{A, C, F, H\}_H$; $\{B, D, G\}_G$
$$K_H = \{A, C, F\}_G; \{B, D, G\}_H$$

For $\wp\mathfrak{I}_{\{X=15\}}$

Table 5 A : $\wp\boldsymbol{T}_{\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}}$: $(\mathbf{5\ X\ 1\ \ 3\ X\ 6\ X\ X\ 9\ 13\ 2\ 7\ 11\ X\ 4})$

+	A	B	C	D	F
A	B	C	D	F	F
B	C	D	F	F	F
C	D	F	F	F	F
D	F	F	F	F	F
F	F	F	F	F	F

$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 5 & X & 1 & 3 & X & 6 & X & X & 9 & 13 & 2 & 7 & 11 & X & 4 \end{pmatrix}$,
$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ X & X & 5 & 1 & X & 6 & X & X & 9 & 11 & X & X & 2 & X & 3 \end{pmatrix}$$
$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ X & X & X & 5 & X & 6 & X & X & 9 & 2 & X & X & X & X & 1 \end{pmatrix}$,

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$$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ X & X & X & X & X & 6 & X & X & 9 & X & X & X & X & X & 5 \end{pmatrix}$$

$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ X & X & X & X & X & 6 & X & X & 9 & X & X & X & X & X & X \end{pmatrix}$$

Left Congruences $\tilde{R} - classes$

$\tilde{R} = \{ \ \}$

Kernel

$K_A = \{D, F\}_F$; $K_B = \{C, D, F\}_F$; $K_C = \{B, C, D, F\}_F$; $K_D = K_F = \{A, B, C, D, F\}_F$

Table 5 B : $\wp T_{\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}}$: $\mathbf{51X3X810X613X7XX4}$ }

+	A	B	C
A	B	C	E
B	C	E	E
C	E	E	E

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 5 & 1 & X & 3 & X & 8 & 10 & X & 6 & 13 & X & 7 & X & X & 4 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 12 & 13 & 14 & 15 \\ X & 5 & X & X & X & X & 13 & X & 8 & X & X & 10 & X & X & 3 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ X & X & X & X & X & X & X & X & X & X & X & 13 & X & X & X \end{pmatrix}$$

Left Congruences $\tilde{R} - classes$

$\tilde{R} = \{ \ \}$

Kernel

$K_B = \{B, C\}_E$; $K_C = \{A, B, C\}_E$

Table 5 C : $\wp T_{\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}}$: $(\mathbf{5\ 2\ X\ 6\ 8\ 9\ X\ 11\ X\ 12\ X\ X\ 13\ X\ 15})$

+	A	B	C	D	F
A	B	C	D	D	D
B	C	D	D	D	D
C	D	D	D	D	D
D	D	D	D	D	D
F	D	D	D	D	D

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 5 & 2 & X & 6 & 8 & 9 & X & 11 & X & 12 & X & X & 13 & X & 15 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 8 & 2 & X & 9 & 11 & X & X & X & X & X & X & X & 13 & X & 15 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 11 & 2 & X & X & X & X & X & X & X & X & X & X & 13 & X & 15 \end{pmatrix},$$

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$$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ X & 2 & X & X & X & X & X & X & X & X & X & X & 13 & X & 15 \end{pmatrix}$$
$$F = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 14 & 2 & X & X & X & X & X & X & X & X & X & X & 13 & X & 15 \end{pmatrix}$$

Left Congruences $\tilde{R} - classes$
$\tilde{R} = \{ \ \}$
Kernel
$K_A = \{C, D, F\}_D \quad ; K_B = \{B, C, D, F\}_D ; K_C = K_D = K_F = \{A, B, C, D, F\}_D$

Summary and Conclusion

This work examined the Left congruences the $\tilde{R} - classes$ and the corresponding kernels of $\wp\mathfrak{I}_{\{X=11,12.13.14.15\}}$.In all, fifteen Partial transformations were computed using Raf-BaduT, for For $\wp\mathfrak{I}_{\{11\}}$ in 1A, eleven (11) $\tilde{R} - classes$ were generated , incidentally the $\tilde{R} - classes$ were generated by the same elements {A, B, C, D} while $\widetilde{R_E}$ was for all the elements, 1B had five (5) $\tilde{R} - classes$, 1C had eleven (11) $\tilde{R} - classes$. For $\wp\mathfrak{I}_{\{12\}}$ in 2A, 2B and 2C all had empty $\tilde{R} - classes$.For $\wp\mathfrak{I}_{\{13\}}$ in 3A had empty $\tilde{R} - class$, 3B had only one (1) $\tilde{R} - class$, 3C had two (2) $\tilde{R} - classes$.For $\wp\mathfrak{I}_{\{14\}}$ in 4A and 4B had empty $\tilde{R} - class$ and 4C had two (2) $\tilde{R} - classes$.For $\wp\mathfrak{I}_{\{15\}}$ in 5A, 5B and 5C all had $\tilde{R} - class$.The kernels were all distinct and followed a sequence of one- more element addition for subsequent kernels.

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