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COMPARATIVE STUDY ON METHODS OF GENERATING RANDOM VARIABLES FROM THE POWER LINDLEY DISTRIBUTION

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Abstract

In this paper we considered some methods of generating random variables from the power Lindley distribution using the Composition, Lambert W function and Numerical methods. In order to compare these methods with respect to their ability to generate random samples from the distribution, a Monte Carlo simulation study was conducted to show the performance and asymptotic behavior of the maximum likelihood estimates (MLEs) of the power Lindley distribution. Three quantities such as the average bias, average mean square error and coverage probability of 95% confidence interval for the parameter estimates were computed for each method and the results show that the Lambert W function and the Numerical methods which gave same result are more efficient in generating random samples from the Power Lindley Distribution than the one based on composition method.

Keywords: *Inverse transform, Lambert W function, Maximum Likelihood Estimates, Numerical, Power Lindley distribution*

Introduction

Lifetime distribution is one of the statistical models used in modeling real lifetime data based on survival times. Many lifetime distributions have been proposed in literature to model real lifetime data. These include the exponential distribution, Gumbel distribution, gamma distribution, Burr distribution, Weibull distribution, Lindley distribution, etc. In an attempt to increase the flexibility of these distributions, a wide range of generalization of these distributions have been proposed. Some of those generalizations are found in the works of Ghitany et al., (2013), Ekhosuehi and Opone (2018), Ekhosuehi et al., (2020), Opone and Ekhosuehi (2020), Opone et al. (2020), Opone and Iwerumor (2021), Tuoyo et al., (2021), Opone and Osemwenkhae (2022), Chesneau et al., (2022), Akata et al., (2023), among others. For every proposed statistical distribution, it is important to show the performance and asymptotic behavior of the parameters estimates of the proposed distribution through a Monte Carlo simulation study. To achieve this, the first procedure in the simulation study is to generate random samples from the proposed distribution. In probability distribution theory, the inverse transform method obtained by inverting the cumulative distribution function of the probability distribution is seen as the most explicit and convenient way of generating random samples from a known probability distribution (i.e., $x = F^{-1}(u)$). But due to the complexity of the cumulative distribution function (cdf) of some distributions, the inverse transform method cannot be achieved. Thus, addressing this pitfall, several methods of generating random samples from a known probability distribution which include the composition, convolution, acceptance/rejection, Lambert *W* function, numerical method, etc., have been introduced as alternatives to the inverse transform method.

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In this paper, we shall conduct a comparative study on the methods of generating random samples from the power Lindley distribution. The remaining sections of this paper are organized as follows: Section 2 presents the material and methods of the study which give a detailed discussion of three methods of generating random samples from a known probability distribution (Composition, Lambert *W* function and Numerical method). Some basic properties of the power Lindley distribution are discussed in Section 3. Section 4 presents a Monte Carlo simulation study to examine the performance and asymptotic behavior of the MLEs of the power Lindley distribution. We shall be generating random samples from the distribution considering the three methods under study, compute the quantities (Average bias, Average mean square error and Coverage probability of 95% confidence interval for the MLE) for each method and compare the estimates obtained. Finally, in Section 5, we give a concluding remark.

2. Materials and method

Let X be a random variable with probability density function (pdf), $f(x)$. Then, the cumulative distribution function (cdf), $F(x)$ which takes on any value between 0 and 1 is given by:

$$F(x) = \int_{-\infty}^{x} f(y)dy \qquad (2.1)$$

Now, if u is a uniform random number which takes on value between 0 and 1, then u has the pdf

$$f(u) = \begin{cases} 1, & 0 < u < 1 \\ 0, & otherwise \end{cases} \qquad (2.2)$$

It is straight forward as in Rubinstein (2008) that combining Eq. (2.1) and (2.2) result to an expression of the quantile function of a known distribution

$$u = F(x) \qquad (2.3)$$

Using this Eq. (2.3), we formulate the following methods of generating random sample from any given probability distribution function.

2.1 Lambert W function

If the expression in Eq. (2.3) has a close form solution, we can derive an explicit expression for the variable x such that:

$$x = F^{-1}(u) \qquad (2.4)$$

Generating random number from any distribution requires that a uniform number u is generated and then used to generate the random variable x. This method of generating random number using Eq. (2.4) is called the inverse transform method. On the other hand, for some probability distributions, Eq. (2.4) may not exist in a close form solution, thus the Lambert W function is employed as an alternative means to express such function in a closed form. The Lambert W function is a complex function defined as the solution of the equation

$$W(x)\exp(W(x)) = x \qquad (2.5)$$

where x is a complex number. Using the idea in Eq. (2.5), Jodra (2010) proposed an explicit form of the quantile function of the Lindley distribution which is also applicable for all Lindley family of distributions.

2.2 Composition method

This method is based on the fact that the density function can be decomposed into mixture of density functions. This is actually necessary when it is not possible to make use of the inverse method. Okwuokenye and Peace (2016) and Rubinstein (2008) gave the illustration of the composition method as follows:
Assume that the density function $f(x)$ can be expressed as a weighted sum of k different densities such that:

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$$f(x)=\sum_{i=1}^{k} p_i f_i(x),\ p_i>0 \tag{2.6}$$

where $\sum_{i=1}^{k} p_i = 1$ and $f_i(x)$ are the component density functions.

Many classes of Lindley density functions belong to this set that contain mixture densities. These classes of Lindley density function $f(x)$, can be decomposed into a mixture of two density functions such that:

$$f(x)=ph(x)+(1-p)g(x) \tag{2.7}$$

where p is a mixing proportion and $h(x)$ and $g(x)$ are separate known density functions which can be handled independently using inverse method. Then, the following steps can be used in generating random variable from the density $f(x)$:

Step1: generate $u_i \sim U(0,1),\ i=1,2,\cdots,n$

Step2: generate $v_i \sim h(x),\ i=1,2,\cdots,n$

Step3: generate $w_i \sim g(x),\ i=1,2,\cdots,n$

Step4: if $u_i \le p$, then set $x_i = v_i$ otherwise $x_i = w_i,\ i=1,2,\cdots,n$

2.3 Numerical method

Now, there are situations where the inverse method and its alternative (Lambert *W* function) fail to give an explicit form of the quantile function. In this regard, we can make use of numerical method of solution. This method simply involves solving the non-linear system of equation in Eq (2.3) numerically. This method involves locating the value of x that satisfies the following equation when a random number $u \sim U(0,1)$ is generated:

$$u-F(x)=0 \tag{2.8}$$

The algorithm for generating random variable using this numerical method is given as follows:
Step 1: generate random number $u \sim U(0,1)$
Step 2: choose initial value and then solve the function in equation (2.8) for fixed values of the parameter(s) and the generated u, numerically till convergence;
Step 3: If the sample space for the value of the generated random variable is satisfied, set $x = x^*$ otherwise reject;
Step 4: return to step 1.

3. The power Lindley distribution

Ghitany et al., (2013) introduced a shape parameter to the one parameter Lindley distribution reported in the work of Lindley (1958) by considering the power transformation $X=T^{1/\alpha}$ to obtain a probability density function (pdf) $f(x)$ of the form

$$f(x) = \frac{\alpha\beta^2(1+x^\alpha)x^{\alpha-1}e^{-\beta x^\alpha}}{1+\beta},\quad x>0,\ \alpha,\beta>0 \tag{3.1}$$

and the corresponding cumulative distribution function (cdf) of the power Lindley distribution is given as

$$F(x) = 1-\left[1+\frac{\beta x^\alpha}{1+\beta}\right]e^{-\beta x^\alpha},\qquad x>0,\ \alpha,\beta>0 \tag{3.2}$$

The pdf in Eq. (3.1) which is a two-component mixture of Weibull distribution and generalized gamma distribution can be expressed as:

$$f(x)=pf_W(x)+(1-p)f_{GG}(x)$$

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where $f_W(x)=\alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}$ and $f_{GG}(x)=\alpha\beta^2 x^{2\alpha-1}e^{-\beta x^{\alpha}}$ are the pdf of the Weibull distribution (with shape parameter α and scale parameter β) and generalized gamma distribution (with shape parameters 2, α and scale parameter β) respectively and $p=\dfrac{\beta}{\beta+1}$ is the mixing proportion. Figure 1 shows the plots of the density function of the power Lindley distribution for various choices of the parameters.

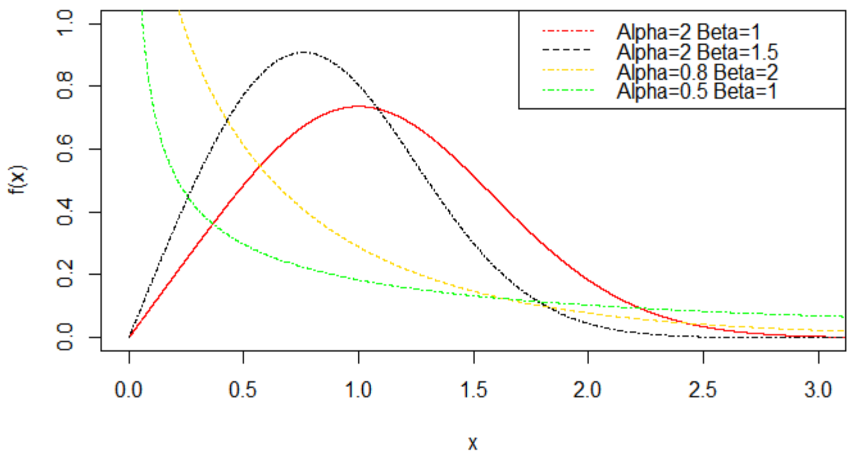

Figure 1: Density plots of the power Lindley distribution

3.2 Quantile function of the power Lindley distribution

Let X be a random variable with the cumulative distribution function $F(x)$, the quantile function of X can be obtained as $Q_X(u)=F^{-1}(u)$. Jodra (2010) derived a general form of the quantile function of the Lindley family of distributions in terms of the Lambert W function.

Thus, given the cdf in Eq. (3.2), the q^{th} quantile function of the power Lindley distribution is defined as

$$1-\frac{\left[1+\beta+\beta x^{\alpha}\right]e^{-\beta x^{\alpha}}}{1+\beta}=u, \qquad (3.3)$$

$$-\left[1+\beta+\beta x^{\alpha}\right]e^{-\beta x^{\alpha}}=(1+\beta)(u-1), \qquad (3.4)$$

multiplying both sides by $e^{-(1+\beta)}$, we have

$$-\left[1+\beta+\beta x^{\alpha}\right]e^{-(1+\beta+\beta x^{\alpha})}=-(1+\beta)(1-u)e^{-(1+\beta)}.$$

Clearly, we observe that $-\left[1+\beta+\beta x^{\alpha}\right]$ is the Lambert W function of the real argument $-(1+\beta)(1-u)e^{-(1+\beta)}$.
Thus, we have

$$W_{-1}\left[-(1+\beta)(1-u)e^{-(1+\beta)}\right]=-\left[1+\beta+\beta x^{\alpha}\right]$$

$$\beta x^{\alpha}=-1-\beta-W_{-1}\left[-(1+\beta)(1-u)e^{-(1+\beta)}\right]$$

$$x=\left\{-1-\frac{1}{\beta}-\frac{1}{\beta}W_{-1}\left[\frac{-(1+\beta)(1-u)}{e^{(1+\beta)}}\right]\right\}^{1/\alpha} \qquad (3.5)$$

where $u\in(0,1)$, is uniformly generated random number.

Consequently, some quantiles from the power Lindley distribution are generated using Eq. (3.5) for fixed u as shown in Table 1.

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Table 1: Quantiles of the power Lindley distribution for some selected parameters.

u	$(\alpha=0.5,\beta=3)$	$(\alpha=1.5,\beta=1)$	$(\alpha=2,\beta=0.8)$	$(\alpha=1,\beta=2)$
0.1	0.00217	0.34340	0.52599	0.07804
0.2	0.00961	0.55132	0.74440	0.16322
0.3	0.02425	0.73554	0.91871	0.25757
0.4	0.04906	0.91347	1.07576	0.36406
0.5	0.08902	1.09523	1.22778	0.48721
0.6	0.15303	1.29061	1.38388	0.63454
0.7	0.25922	1.51352	1.55465	0.82005
0.8	0.45243	1.79121	1.75864	1.07489
0.9	0.89550	2.20390	2.04778	1.49744

3.3 Maximum Likelihood Estimation

Let $(x_1, x_2, ..., x_n)$ be random samples from the power Lindley distribution, then the log-likelihood function is given by,

$$\ell(x,\eta) = \sum_{i=1}^{n} \log\left[\frac{\alpha\beta^2}{1+\beta}(1+x^\alpha)x^{\alpha-1}e^{-\beta x^\alpha}\right],$$

$$= n\log\alpha + 2n\log\beta + \sum_{i=1}^{n}\log(1+x_i{}^\alpha) + (\alpha-1)\sum_{i=1}^{n}\log(x_i) - \beta\sum_{i=1}^{n}x_i{}^\alpha - n\log(1+\beta) \qquad (3.6)$$

The estimates $\hat{\alpha}$ and $\hat{\beta}$ of the parameters α and β are obtained by taking the first derivative of Eq. (3.6) with respect to the parameters and equate to zero.

Hence, we have

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\frac{x^\alpha\log(x_i)}{(1+x_i^\alpha)} + \sum_{i=1}^{n}\log(x_i) - \beta\sum_{i=1}^{n}x^\alpha\log(x_i) = 0$$

$$\frac{\partial\ell}{\partial\beta} = \frac{2n}{\beta} - \sum_{i=1}^{n}x_i^\alpha - \frac{n}{(1+\beta)} = 0$$

The maximum likelihood estimator $\hat{\eta}$ of η can also be solved by an iterative scheme such as Newton Raphson method given by;

$$\hat{\eta} = \eta_k - H^{-1}(\eta_k)U(\eta_k), \qquad \hat{\eta} = (\hat{\alpha}, \hat{\beta})^T \qquad (3.7)$$

where $U(\eta_k)$ is the score function and $H(\eta_k)$ is the Hessian matrix defined as

$$H(\eta_k) = \begin{pmatrix} \frac{\partial^2\ell}{\partial\alpha^2} & \frac{\partial^2\ell}{\partial\alpha\partial\beta} \\ \frac{\partial^2\ell}{\partial\beta\partial\alpha} & \frac{\partial^2\ell}{\partial\beta^2} \end{pmatrix}$$

where, $\frac{\partial^2\ell}{\partial\alpha^2} = \sum_{i=1}^{n}\frac{x_i^\alpha\log^2(x_i)}{(1+x_i^\alpha)^2} - \beta\sum_{i=1}^{n}x_i^\alpha\log^2(x_i) - \frac{n}{\alpha^2}$, $\quad \frac{\partial^2\ell}{\partial\beta^2} = \frac{n}{(1+\beta)^2} - \frac{2n}{\lambda^2}$

$$\frac{\partial^2\ell}{\partial\alpha\,\partial\beta} = \frac{\partial^2\ell}{\partial\beta\,\partial\alpha} = -\sum_{i=1}^{n}x_i^\alpha\log(x_i)$$

4. Monte Carlo Simulation Study This Section investigates the asymptotic behavior of the maximum likelihood estimates of the parameters of power Lindley distribution through a Monte Carlo simulation study.

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Random samples from the power Lindley distribution are generated using the three methods of generating random samples. Algorithms for the three methods considered are given as follow:

Method I (Lambert W Function)

Step I. Generate $U_i \sim$ Uniform (0,1), $i = 1,2,..., n$

Step II.set $x_i = \left[-1 - \frac{1}{\beta} - \frac{1}{\beta} W_{-1}\left[\frac{-(1+\beta)(1-u_i)}{e^{(1+\beta)}} \right] \right]^{1/\alpha}, \quad i = 1, 2,..., n$

where $W_{-1}(.)$ denotes the negative branch of the Lambert W function. The values of x_i generated from Step II forms the random sample from the power Lindley distribution.

Method II (Composition)

Step I. Generate $U_i \sim$ uniform (0,1), $i = 1,2,..., n$

Step II.Generate $X_i \sim$ Weibull (α, β), $i = 1,2,..., n$

Step III. Generate $Y_i \sim$ generalized-gamma ($2, \alpha, \beta$), $i = 1,2,..., n$

Step IV. if $U_i \leq p = \frac{\beta}{\beta+1}$, then set $Z_i = X_i$, else $Z_i = Y_i$

Z_i obtained from Step IV form the random samples generated from the power Lindley distribution.

Method III (Numerical)

Step I. Generate $U_i \sim$ uniform (0,1), $i = 1,2,..., n$

Step II.solve the system of nonlinear equation numerically:

$$(1+\beta)(1-u_i) - \left[1 + \beta + \beta x_i^{\alpha}\right] e^{-\beta x_i^{\alpha}} = 0$$

The values of x_i from Step II form the random sample from the power Lindley distribution.

Similarly, an algorithm for the Monte Carlo simulation study is given by the following steps.

Step I. Choose the value N (i.e. number of Monte Carlo simulation);

Step II.choose the values $\eta_0 = (\alpha_0, \beta_0)$ corresponding to the parameters of the power Lindley distribution (α, β);

Step III. generate a sample of size *n* from the power Lindley distribution;

Step IV. compute the maximum likelihood estimates $\hat{\eta}_0$ of η_0;

Step V. repeat steps (3-4), N-times;

Step VI. compute the desired quantities.

The Monte Carlo simulation study is repeated 1000 times for different sample sizes n = 25, 50, 75, 100 for the following parameter values ($\alpha = 0.2, \beta = 4$), ($\alpha = 1.5, \beta = 1$) and ($\alpha = 0.9, \beta = 0.35$). Using the R statistical software, we computed the following three quantities;

(i) Average bias of the estimates $\hat{\eta}_i$ of the parameter η_0 given by

$$\text{AB}(\hat{\eta}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\eta}_i - \eta_0) \; ;$$

(ii) Average mean square error of the estimates $\hat{\eta}_i$ of the parameter η_0 given by

$$\text{AMSE}(\hat{\eta}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\eta}_i - \eta_0)^2 ;$$

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(iii) Coverage probability of 95% the confidence interval for the estimates $\hat{\eta}_i$ given by; CP($\hat{\eta}$) =

$$\frac{1}{N}\sum_{i=1}^{N} I\left[\hat{\eta}_i - 1.96\sqrt{\text{var}(\hat{\eta}_i)} < \eta_0 < \hat{\eta}_i + 1.96\sqrt{\text{var}(\hat{\eta}_i)}\right];$$

where $I(.)$ is an indicator function

Table 2: Monte Carlo Simulation Result for Average Bias of the MLE for α and β

Parameters	n	Method I		Method II		Method III	
		$AB(\hat{\alpha})$	$AB(\hat{\beta})$	$AB(\hat{\alpha})$	$AB(\hat{\beta})$	$AB(\hat{\alpha})$	$AB(\hat{\beta})$
$\alpha = 0.9$, $\beta = 0.35$	25	0.0335	−0.0035	0.1181	0.0899	0.0335	−0.0035
	50	0.0125	0.0017	0.1396	0.0954	0.0125	0.0017
	75	0.0143	−0.0017	0.1381	0.0897	0.0143	−0.0017
	100	0.0120	−0.0024	0.1397	0.0881	0.0120	−0.0024
$\alpha = 1.5$, $\beta = 1$	25	0.0802	−0.0017	0.2781	0.0800	0.0802	−0.0017
	50	0.0405	0.0077	0.3091	0.0920	0.0405	0.0077
	75	0.0290	0.0011	0.3149	0.0843	0.0290	0.0011
	100	0.0255	−0.0096	0.3134	0.0713	0.0255	−0.0096
$\alpha = 0.2$, $\beta = 4$	25	0.0122	0.4168	0.0287	−0.3902	0.0122	0.4168
	50	0.0015	0.2155	0.0334	−0.5354	0.0015	0.2155
	75	0.0035	0.1235	0.0347	−0.6061	0.0035	0.1235
	100	0.0023	0.0785	0.0353	−0.6301	0.0023	0.0785

Table 3: Monte Carlo Simulation Result for Average Mean Square Error of the MLE for α and β

Parameters	n	Method I		Method II		Method III	
		$AMSE(\hat{\alpha})$	$AMSE(\hat{\beta})$	$AMSE(\hat{\alpha})$	$AMSE(\hat{\beta})$	$AMSE(\hat{\alpha})$	$AMSE(\hat{\beta})$
$\alpha = 0.9$, $\beta = 0.35$	25	0.0204	0.0093	0.0360	0.0287	0.0204	0.0093
	50	0.0088	0.0049	0.0292	0.0199	0.0088	0.0049
	75	0.0054	0.0029	0.0252	0.0148	0.0054	0.0029
	100	0.0040	0.0023	0.0239	0.0127	0.0040	0.0023
$\alpha = 1.5$, $\beta = 1$	25	0.0717	0.0337	0.1112	0.0506	0.0717	0.0337
	50	0.0297	0.0173	0.1091	0.0311	0.0297	0.0173
	75	0.0192	0.0111	0.1082	0.0219	0.0192	0.0111
	100	0.0128	0.0089	0.1047	0.0166	0.0128	0.0089
$\alpha = 0.2$, $\beta = 4$	25	0.0013	1.3168	0.0013	0.9737	0.0013	1.3168
	50	0.0005	0.5189	0.0013	0.5597	0.0005	0.5189
	75	0.0004	0.2701	0.0013	0.5099	0.0004	0.2701
	100	0.0003	0.1915	0.0013	0.5000	0.0003	0.1915

Table 4: Monte Carlo Simulation Result for Coverage Probability of 95% CI for the MLE of α and β

Parameters	n	Method I		Method II		Method III	
		$CP(\hat{\alpha})$	$CP(\hat{\beta})$	$CP(\hat{\alpha})$	$CP(\hat{\beta})$	$CP(\hat{\alpha})$	$CP(\hat{\beta})$
$\alpha = 0.9$, $\beta = 0.35$	25	0.942	0.9100	0.682	0.854	0.942	0.9100
	50	0.94	0.932	0.508	0.774	0.94	0.932
	75	0.96	0.95	0.42	0.712	0.96	0.95
	100	0.96	0.938	0.316	0.64	0.96	0.938
$\alpha = 1.5$, $\beta = 1$	25	0.946	0.934	0.62	0.924	0.946	0.934
	50	0.956	0.948	0.336	0.89	0.956	0.948
	75	0.952	0.938	0.184	0.872	0.952	0.938
	100	0.954	0.94	0.09	0.878	0.954	0.94
$\alpha = 0.2$, $\beta = 4$	25	0.96	0.974	0.794	0.766	0.96	0.974
	50	0.962	0.952	0.536	0.674	0.962	0.952
	75	0.962	0.954	0.332	0.57	0.962	0.954
	100	0.942	0.964	0.198	0.434	0.942	0.964

4.1 Results and Discussion

Tables 2, 3 and 4 respectively present the Monte Carlo simulation results for the average bias, average mean square error and coverage probability of 95% confidence interval of the maximum likelihood estimate of the parameters of power Lindley distribution. From Table 2, we observed that the average bias of the estimate $\hat{\alpha}$ is positively biased, while the estimate $\hat{\beta}$ could either be positively or negatively biased. From Table 3, the average mean square error of the estimates decreases as the sample size n increases. Also, from Table 4, the coverage probabilities of the confidence interval are close to the nominal level of 95%. The assertions from Tables 2, 3 and 4 are what one should expect in satisfying the consistency properties of the maximum likelihood estimates (MLEs).

Generally, it is noteworthy to state that the method having the least estimated average mean square error and a closer coverage probability to the nominal level of 95% is regarded as the appropriate method to be used in generating random sample from a known probability distribution. Thus, we can conclude that the Lambert *W* function and the numerical methods are the best methods of generating random samples from the power Lindley distribution.

5. Conclusion

This paper investigates three methods of generating random variables from a known probability distribution using the power Lindley distribution as a case study. Three quantities (Average Bias, Average Mean Square Error and Coverage Probability) were computed for each method using Monte-Carlo simulation and the results obtained were compared. From our results in Tables 2, 3 and 4, it was clearly evident that the Lambert *W* function method and the Numerical method which gives same result demonstrates more efficiency in generating random samples from the Power Lindley distribution than the one based on composition method. It is also important to note that these methods may be applicable to some generalized forms of the Lindley family of distributions.

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