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Mathematical Assesement of Crime Dynamic with Public Education and Police Intervation Via: ABC Fractional Operator

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ABSTRACT

In this work, a fractional mathematical model of crime dynamic with police and public education is designed and used to access the impacts of the intervention in controlling crime spread. The model's positivity and boundedness were established, demonstrating its epidemiological well-posture. The model's asymptotically stable local and global states were established using the basic reproduction number that was established. When the basic reproduction number (R_0), less than unity, indicate that crime will be reduced or eliminated in the community; a reproduction number greater than unity indicates that crime will continue in the community. The model was validated using yearly data from 2002-2021, sourced from Macrotrends. The simulation of the fractional crime model combined with public education and police presence, have shown to be effective in lowering crime rates.

Keyword: Crime, Fractional, Stability Analysis, Modelling, Police, Dynamics, Public Education.

INTRODUCTION

A criminal act is defined as a purposeful behaviour that is clearly characterized as being against the law, socially damaging, and penalized (Edge *et al*., 2022). Financial, violent, property, and moral crimes are among the most common types of crime that significantly affect many cultures in developing countries, particularly those in Africa and Latin America (González-Parra *et al*., 2018). Although crime is a complex phenomenon, one of the main risk factors for its rise, especially in urban areas, has been identified as unemployment (Block & Block, 1984; Garside, 2015; Ajayi & Adefolaju, 2013; Ayodele, 2015; Squires *et al*., 2008; McMillon *et al*., 2014; Nyabadza *et al.,* 2017). According to a UN assessment, young people who join criminal gangs account for 40% of the unemployed because they cannot find gainful jobs. Poverty, unemployment, unstable families, political unrest, and demographic shifts are the main causes of crimes. Emerging economies have challenges with the jobless rate among youth and the rise in inequality. Crime affects progress and society, and social structure has an impact on how it spreads. Understanding the nature of crime and its contributing factors is necessary to develop strategies to lessen its prevalence and effects (Pasour *et al.,* 2008; Shukla *et al.,* 2013). Crime encompasses illegal behaviours that are punished by victims and criminal institutions. It is a social status issue. Contact with criminal groups influences the occurrence of more crime, which is determined by motivated offenders, suitable targets, and a dearth of guardians. The existence of a crime-free equilibrium cannot be achieved, even with the premise of contagion relaxed, according to a mathematical model of crime dynamics (McMillon *et al*., 2014). Differential equations, for instance, have been used to show how drug addiction has spread like wildfire (Behrens *et al.,* 1999; Kwofie *et al.,* 2023; Atangana & Igret, 2021). Gonzalez-Parra *et al.* (2018) found that fear can reduce criminal intention by diminishing anticipation of benefits in their model of peer pressure on college-age bulimia. The rational decision-making theory proposed by Becker, which is the basis for this technique (Arora *et al.,* 2018), serves as an inspiration. Crime is concentrated where there is a greater financial gain from committing the crime than there is an opportunity cost (Partohaghighi *et al*., 2022). The factors Wang *et al*. (2023) considered while assessing group crime activities. Wang *et al.* (2023) evaluated group crime activity by considering a variety of costs related to various societal strata and potential offenders. Mataru *et al.* (2023) developed a mathematical model to address unemployment-related criminality in developing nations. The model, based on Lipschitz condition and epidemiological principles, suggests employing employment tactics and vocational training to reduce crime. Chikore *et al.* (2023) developed a dynamic system analysing criminal activity and policing efficiency, demonstrating bi-stability and potential for predictive use with real data. A fractional mathematical model for the transmission of crime is put forth by Bansal *et al.* (2024), who divide society into six clusters: the judiciary, law enforcement, convicted, free, and vulnerable. Fractional-order differential and integral operators have been shown to be valuable in more recent studies, which have also shown how well they work when simulating complicated real-world processes (Atangana, 2021; Podlubny, 1999). Non-integer-order models have been found to be more effective than integer-order systems in specifying dynamical behaviours. A fractional-order crime transmission model taking into account five classes—law-abiding citizens, non-jailed criminals, incarcerated criminals, prison-released individuals, and recidivists—was presented by Arora, Mathur, and Tiwari in 2023. It measures local stability, establishes the proper length of incarceration, and analyses the effects of recidivism on society. Nwajeri *et al*. (2023) presented a novel co-dynamic fractional order system to mitigate the destruction brought about by drug trafficking and money laundering. The system includes those who commit both crimes and those who are vulnerable due to their connections to drug trafficking. The stability framework maintains its stability when all conditions are met, and the model contributes to our understanding of how these illegal actions function in society. Partohaghigh et al. (2022) use three operators—Atangana-Baleanu-Caputo, Caputo, and Caputo-Fabrizio derivatives—to study the application of fractional derivatives in the construction of non-integer criminal system models. In the USA, they develop approximate solutions by numerical approaches using real beginning conditions for subgroups. Rahat *et al.* (2023) used fractional order calculus and Newton's polynomial to study the population dynamics of financial crime. They created financial crime equilibria, computed reproduction number, and presented a fractional-order model. Public education campaigns have the potential to increase knowledge of the negative effects of criminal behaviour, inform those who may be at danger of committing crimes about the resources that are available to them, and encourage constructive alternatives to illegal activity. This operator's capacity to identify detailed and non-local patterns in a system's behaviour is what motivates its use in the analysis of the complex linkages between crime rates, public education levels, and police intervention techniques., we develop a mathematical model to explore the dynamic of crime transmission in terms of the Atangana Beleanu fractional operator with police innervation and public education.

Preliminary

Definition 1. (Atangana *et al.,* 2020)

The fractional integral associated with the new fractional derivative with non-local kernel is defined as:

$$ {}_{a}^{ab}I_{t}^{\alpha}\left[U\left(t\right)\right]=\frac{1-\alpha}{M\left(\alpha\right)}U\left(t\right)+\frac{\alpha}{M\left(\alpha\right)\overline{)\alpha}}\int_{0}^{t}U\left(\tau\right)\left(t-\tau\right)^{\alpha-1}d\tau \cdot \quad (1)$$

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Definition 2 (Atangana & Baleanu, 2016).

Let $U \in C(a,b)$, $a < b$, $\alpha \in [0,1]$, then

$$ {}_{a}^{ab}I_t^{\alpha}\left[{}_{0}^{abc}D_t^{\alpha}U(t)\right] = U(t) - U(a). \quad (2)$$

Theorem 3 (Thabet & Baleanu, 2018; Caputo & Fabrizio, 2015)

The general transform of the Atangana-Baleanu fractional integral of the function $f(t)$ is given as

$$L\left\{{}_{0}^{abc}I_t^{\alpha}\left[U(t)\right]\right\}(p) = \frac{B(\alpha)}{1-\alpha}\frac{p^{\alpha}L\{U(t)\}(p) - p^{\alpha-1}U(0)}{p^{\alpha}+\dfrac{\alpha}{1-\alpha}}. \quad (3)$$

Lemma 1 (Hossein & Moshen, 2018)

Let $x(t) \in {}^{+}$ be a continuous and differentiable function. Then for any time instant $t \geq 0$

$$ {}_{0}^{abcC}D_t^{\alpha}\left(\Phi(t) - \Phi^{*} - \Phi^{*}\ln\frac{x(t)}{\Phi^{*}}\right) \leq \left(1 - \frac{\Phi(t)}{\Phi^{*}}\right){}_{0}^{abc}D_t^{\alpha}(t). \quad (4)$$

Model Description

The model describe the total population $N(t)$ of individuals in a society subdivided into different class at time t, as follows: susceptible population $S(t)$, population of the individual exposed to the crime $E(t)$, population of those committing the crime $D(t)$, population of individual arrested by security agents as a result of the crime committed and sentence to prison $J(t)$, population of individual acquainted by the court or those that completed their sentence period $R(t)$ while $P(t)$ is the police population . Observe that the population is open, as demographic measures are considered and the recruitment rate in the susceptible population is the constant π . The manifestation of a crime remains unveil when it is been detected, and individuals in a crime society are assume to be susceptible until are exposed to the crime by means of interaction with criminals, and as a result one may developed an interest in the crime and will therefore migrate to the exposed class at a rate λ , where $\lambda = \frac{(1-\xi\chi)D}{N}$, χ and ξ are proportion of the population of the community that are educated and the education efficacy respectively . Individuals in the exposed population may decide to join the criminals at the rate α , and can consequently may die as a result of the crime at a rate d , or be apprehended by a honest and dedicated security personals at a rate ϑ . Individual who completed their sentence are release at the rate γ. Acquitted Individual may decide to go back committing the crime at the rate $\eta\varepsilon$ and in all the classes, μ is the natural death rate. Λ is the recruitment rate into the police force, and τ is the retirement rate.

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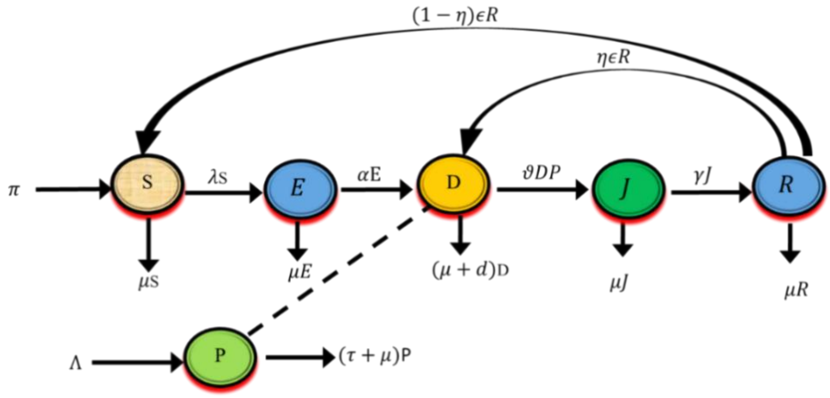

Figure 1. Schematic Diagram of the Model

$$\left.\begin{aligned}
{}^{abc}_{\ 0}D_t^{\sigma}S(t) &= \pi + (1-\varepsilon)\eta R - \left(\frac{\beta(1-\chi\xi)D}{N} + \mu\right)S \\
{}^{abc}_{\ 0}D_t^{\sigma}E(t) &= \frac{\beta(1-\chi\xi)DS}{N} - (\alpha+\mu)E \\
{}^{abc}_{\ 0}D_t^{\sigma}D(t) &= \alpha E + \eta\varepsilon R - (d+\mu+\vartheta P)D \\
{}^{abc}_{\ 0}D_t^{\sigma}J(t) &= \vartheta DP - (\gamma+\mu)J \\
{}^{abc}_{\ 0}D_t^{\sigma}R(t) &= \gamma J - (1+\mu)R \\
{}^{abc}_{\ 0}D_t^{\sigma}P(t) &= \Lambda - (\tau+\mu)P
\end{aligned}\right\} \quad (5)$$

Basic Properties of the Model

Positivity of the Solution

Theorem 1: Suppose that the initial data for the model (5) be $S(0)>0,\ E(0)\geq 0,\ D(0)\geq 0,\quad J(0)\geq 0,\ R(0)\geq 0, P(0)\geq 0,$, then the solution $S(t),\ E(t),\ D(t),\ P(t),\ C(t),\ J(t)$ and $R(t)$ of the model with positive initial data will remain positive for all $t>0$.

Proof.

From the first equation of the model (5), we have

$$ {}^{abc}_{\ 0}D_t^{\sigma}S(t) \geq -\left(\frac{\beta(1-\eta\xi)D}{N} + \mu\right)S \quad (6)$$

Following the approach in Atangana, (2023), yield

$$S(t) \geq S(0)E_{\sigma}\left(\frac{-t^{\sigma}\sigma\left(\beta(1-\chi\xi)\|D\|_{\infty}+\mu\right)}{B(\sigma)+(1-\sigma)\left(\beta(1-\chi\xi)\|D\|_{\infty}+\mu\right)}\right)$$

Similarly it can be shown that $S(t)>0,\ E(t)\geq 0,\ D(t)\geq 0,\ P(t)\geq 0,\ C(t)\geq 0$,$J(t)\geq 0$ and $R(t)\geq 0$.

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$$\left.\begin{aligned}E(t)&\geq E(0)E_{\sigma}\left(\frac{-t^{\sigma}\sigma(\alpha+\mu)}{B(\sigma)+(1-\sigma)(\alpha+\mu)}\right)\\D(t)&\geq D(0)E_{\sigma}\left(\frac{-t^{\sigma}\sigma(d+\mu+\vartheta P)}{B(\sigma)+(1-\sigma)(d+\mu+\vartheta P)}\right)\\J(t)&\geq J(0)E_{\sigma}\left(\frac{-t^{\sigma}\sigma(\gamma+\mu)}{B(\sigma)+(1-\sigma)(\gamma+\mu)}\right)\\R(t)&\geq R(0)E_{\sigma}\left(\frac{-t^{\sigma}\sigma(1+\mu)}{B(\sigma)+(1-\sigma)(1+\mu)}\right)\\P(t)&\geq P(0)E_{\sigma}\left(\frac{-t^{\sigma}\sigma(\tau+\mu)}{B(\sigma)+(1-\sigma)(\tau+\mu)}\right)\end{aligned}\right\} \quad (7)$$

Hence all the solutions of the model remain positive for all $t > 0$.

The Invariant Region

Lemma 2: The closed set

$$\mathrm{B}=\mathrm{B}_H\cup\mathrm{B}_P=\left\{(S,E,D,J,R)\in \ _{+}^{5};\ N_H\leq\frac{\pi}{\mu},\ P\in \ _{+};\ N_P\leq\frac{\Lambda}{(\tau+\mu)}\right\}.$$

is positively- invariant and attract all the positive solutions of the model

Proof

Adding equations in system (5), we have

$$ {}_{0}^{abc}D_t^{\sigma}N\leq\pi-\mu N. \quad (8)$$

Taking the Laplace transform of (8) and further simplification yield

$$N_C\leq\frac{\pi}{\mu}\left(1-E_{\sigma}\left(-t^{\sigma}\frac{\alpha\mu}{H}\right)\right)+\frac{1}{\mathrm{H}}E_{\sigma}\left(-t^{\sigma}\frac{\alpha\mu}{H}\right)\left[\pi(1-\alpha)+B(\sigma)N_C(0)\right].$$

Similarly

$$N_P\leq\frac{\Lambda}{(\tau+\mu)}\left(1-E_{\sigma}\left(-t^{\sigma}\frac{(\tau+\mu)\alpha}{M}\right)\right)+\frac{1}{M}E_{\sigma}\left(-t^{\sigma}\frac{(\tau+\mu)\alpha}{M}\right)\left[\Lambda(1-\alpha)+B(\sigma)N_P(0)\right].$$

Where $H=B(\sigma)+\mu(1-\alpha)$ and $M=B(\sigma)+(\tau+\mu)(1-\alpha)$

Since $E_{\alpha}\left(-\frac{\alpha\mu}{H}t^{\alpha}\right)$, and $E_{\alpha}\left(-\frac{\alpha(\tau+\mu)}{M}t^{\alpha}\right)$ tends to zero monotonically as $t\to\infty$ (Choi *et al.,* 2014). In particular $N_C\leq\frac{\pi}{\mu}$ and $N_P\leq\frac{\Lambda}{(\tau+\mu)}$. Hence, B is positively invariant and an attractor so that no solution path leaves through any boundary of B. Since the region B is positively-invariant, the usual existence and uniqueness hold for the system (5). Hence it is sufficient to consider the dynamic of the flow generated by the crime model (5) in the region B.

Crime Free Equilibrium Point (E_0) and Stability Analysis

The crime free equilibrium point (E_0), is obtained as

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$$E_0 = \left\{ \frac{\pi}{\mu}, 0, 0, 0, 0, \frac{\Lambda}{\tau + \mu} \right\}$$

The local stability analysis can be established using the next generation matrix approach (Van den Driessche & Watmough, 2002) and using the symbols F and v in (Van den Driessche & Watmough, 2002), given by

$$F = \begin{pmatrix} 0 & \beta(1-\chi\xi) \\ 0 & 0 \end{pmatrix} \text{ and } v = \begin{pmatrix} \alpha+\mu & 0 \\ -\alpha & d+\mu+\vartheta P^* \end{pmatrix}$$

It follows that the crime basic reproduction number (R_0) is given as

$$R_0 = \rho\left(F v^{-1}\right) = \frac{\beta\alpha(1-\chi\xi)(\tau+\mu)}{(\alpha+\mu)\left[(d+\mu)(\tau+\mu)+\vartheta\Lambda\right]}.$$

Lemma 3. The Crime Free Equilibrium point (E_0), of the crime model (5) is locally asymptotically stable (LAS) whenever $R_0 < 1$ and unstable whenever $R_0 > 1$.

Proof

The Jacobian matrix of the model (5.0) is obtain as below

$$J(E_0) = \begin{bmatrix} -\mu & 0 & -\beta(1-\xi\chi) & 0 & (1-\eta)\varepsilon & 0 \\ 0 & -h_1 & \beta(1-\xi\chi) & 0 & 0 & 0 \\ 0 & \alpha & -h_2 & 0 & \eta\varepsilon & 0 \\ 0 & 0 & 0 & -h_3 & 0 & 0 \\ 0 & 0 & 0 & \gamma & -h_4 & 0 \\ 0 & 0 & \Lambda & 0 & 0 & -h_5 \end{bmatrix}.$$

Thus,

$\chi_1 = -\mu$, while the remaining eigenvalues are obtained from the polynomial (9)

$$P(\chi) = \chi^5 + a_4\chi^4 + a_3\chi^3 + a_2\chi^2 + a_1\chi + a_0 = 0. \tag{9}$$

where

$h_1 = (\alpha+\mu)$, $h_2 = (d+\mu+\vartheta P)$, $h_3 = (\gamma+\mu)$, $h_4 = (1+\mu)$, $h_5 = (\tau+\mu)$.

$a_4 = h_1 + h_2 + h_3 + h_4 + h_5,$

$a_3 = h_1\left(h_2(1-R_0)+h_3\right)+(h_4+h_5)(h_1+h_2+h_3),$

$a_2 = h_1h_2(1-R_0)(h_3+h_4+h_5)+(h_4+h_5)(h_1h_3+h_2h_3)+(h_2+h_1)h_4h_5,$

$a_1 = h_1h_2(1-R_0)(h_3h_4+h_3h_5+h_4h_5)+h_2h_4h_5(h_1+h_2),$

$a_0 = h_1h_2h_3h_4h_5(1-R_0).$

Now, applying the Routh-Hurwitz criterion (Hassan *et al.,* 2022) which implies that the eigenvalues of (9) are all negative if and only if $a_i > 0,\ i = 1,2,3,4,5$,

$a_2a_3 + 2a_1a_4 + a_1a_2a_3a_4 > a_1^2a_4^2 + a_4a_3^2 + a_5^2$ and $R_0 < 1$. Hence, $\chi_i < 0\ \ (i = 1,2,...,6)$,

$\arg(\chi_i) = \pi\ \ \arg(\chi_i) = \pi \geq \sigma\frac{\pi}{2}$, which implies that E_0, is locally asymptotically stable.

Theorem 2: The Crime Free Equilibrium Point (E_0) with $\eta = 0$, of the model (5) is Globally Asymptotically Stable (GAS) whenever $\mathsf{R}_0 < 1$, and unstable if otherwise.

Proof

Consider the Lyponuv function

$$\mathsf{L} = \mathsf{R}_0 E + \frac{\beta(1-\chi\xi)}{h_2} D. \tag{10}$$

whose fractional derivative is

$$\begin{aligned} {}_0^{abc}D_t^{\sigma}\mathsf{L} &= \mathsf{R}_0\left(\frac{\beta(1-\chi\xi)DS}{N} - h_1 E\right) + \frac{\beta(1-\chi\xi)}{h_2}(\alpha E - h_2 D) \\ &= \mathsf{R}_0 \frac{\beta(1-\chi\xi)DS}{N} - \frac{\beta(1-\chi\xi)\alpha E}{h_2} + \frac{\beta(1-\chi\xi)\alpha E}{h_2} - \beta(1-\chi\xi)D \\ &= \mathsf{R}_0 \frac{\beta(1-\chi\xi)DS}{N} - \beta(1-\chi\xi)D \\ &\le \beta D(1-\eta\xi)(\mathsf{R}_0 - 1). \end{aligned}$$

Hence ${}_0^{abc}D_t^{\sigma}\mathsf{L} \le 0$ if and only if $\mathsf{R}_0 < 1$ and ${}_0^{abc}D_t^{\sigma}\mathsf{L} = 0$ if and only if $D = 0$ with $\eta = 0$. Since all the parameters are non-negative it follows that, $\mathscr{L}$ is a Lyapunov function on B.
Furthermore the largest compact invariant set in
$\left\{\left(S^*(t), E^*(t), D^*(t), J^*(t), R^*(t), P^*(t)\right) \in \mathsf{B} : {}_0^{abc}D_t^{\sigma}\mathsf{L} = 0\right\}$ is the singleton set (E_0).
Therefore by LaSalle's Invariant Principles every solution to the model with initial condition in B, approaches E_0 as $t \to \infty$ whenever $\mathsf{R}_0 < 1$ so that E_0 is GAS in B if $\mathsf{R}_0 < 1$.

Crime Present Equilibrium Point (E_1)

Let $\mathsf{E}_1 = \left\{S^{**}, E^{**}, D^{**}, J^{**}, R^{**}, P^{**}\right\}$ and $\lambda^{**} = \dfrac{\beta(1-\chi\xi)D^{**}}{N^{**}}$ be the crime present equilibrium point and recruitment rate respectively, where

$$S^{**} = \frac{\pi h_3 h_4 h_7 + (1-\eta)\varepsilon\vartheta\alpha\pi h_3 h_4 P^{**}\lambda^{**}}{h_3 h_4 h_7\left(\mu + \lambda^{**}\right)}, \quad E^{**} = \frac{\pi h_3 h_4 h_7 \lambda^{**} + (1-\eta)\varepsilon\vartheta\alpha\pi h_3 h_4 P^{**}\left(\lambda^{**}\right)^2}{h_1 h_3 h_4 h_7\left(\mu + \lambda^{**}\right)}$$

$$D^{**} = \frac{\alpha\pi h_3 h_4 \lambda^{**}}{h_7}, \quad P^{**} = \frac{\Lambda}{h_5}, \quad J^{**} = \frac{\vartheta\alpha\pi h_4 \lambda^{**}}{h_7}, \quad R^{**} = \frac{\vartheta\alpha\pi\Lambda\lambda^{**}}{h_7},$$

$$h_7 = \left(\lambda^{**} + \mu\right)\left(h_1 h_2 h_3 h_4 - \eta\varepsilon\gamma\vartheta P^{**} h_1\right) - (1-\varepsilon)\eta\gamma P^{**}.$$

Evaluating

$$\lambda^{**} = \frac{\beta(1-\chi\xi)D^{**}}{N^{**}}. \tag{11}$$

yield

$$\left(\lambda^{**}\right)^2 q_1 + \lambda^{**} q_2 + q_3 = 0. \tag{12}$$

where,

$$q_1 = \pi h_3 h_4\left(\eta\varepsilon\gamma\vartheta P^{**} h_1 - h_1 h_2 h_3 h_4\right) - (1-\varepsilon)\eta\gamma\vartheta\alpha\pi h_3 h_4 P^{**} - \pi\alpha h_3 h_4\left(h_3 h_4 + \vartheta h_4 + \gamma\vartheta\right)$$

$$q_2 = \pi h_1^2 h_3 h_4 \eta\varepsilon\gamma\vartheta P^{**} + \pi h_3 h_4 \mu\left(\eta\varepsilon\gamma\vartheta P^{**} h_1 - h_1 h_2 h_3 h_4\right) + (1-\varepsilon)\eta\gamma\vartheta\alpha\pi h_3 h_4 P^{**}(1-h_1)$$
$$+\pi\alpha h_1 h_2 h_3 h_4 \mu(R_0 - 1) - \pi\alpha h_1 h_2 h_3 h_4 \mu(h_3 h_4 + \vartheta h_4 + \gamma\vartheta)$$
$$q_3 = \pi\vartheta\eta h_1 h_2 h_3 h_4 \mu P^{**}(\mu\varepsilon h_1 + 1 - \varepsilon) + \pi h_1^2 h_2 h_3^2 h_4^2 \mu(\mathsf{R}_0 - 1)$$

whose solution is

$$\lambda^{**} = \frac{-q_2 \pm \sqrt{q_2^2 - 4q_1 q_3}}{2q_1} \quad . \tag{13}$$

Thus, the following results.

Theorem 5. The model (5) has

i. a unique endemic equilibrium if $q_3 < 0$

ii. a unique endemic equilibrium if $q_2 < 0$ and $q_3 = 0$

iii. two endemic equilibrium $q_2 < 0, q_3 > 0$ and $\mathsf{R}_0 > 1$

iv. no endemic equilibrium if otherwise.

Theorem 6. The model (5) has a backward bifurcation at $\mathsf{R}_0 = 1$ if and only if $a > 0$ where $a = 2w_3\left[\beta v_2 \mu(1-\chi\xi)(w_2 + w_3) + \pi w_6 \vartheta(v_4 - v_3)\right]$.

Proof

Let $\mathsf{E}_1 = \left\{S^{**}, E^{**}, D^{**}, J^{**}, R^{**}, P^{**}\right\}$ denote the arbitrary endemic equilibrium point of the model (5). Now, let $S = x_1, E = x_2, D = x_3, J = x_4, R = x_5, P = x_6$. The model (5) can be re-presented in the following form.

$$\left.\begin{aligned}
{}^{abc}_{0}D_t^{\sigma} x_1(t) &= \pi + (1-\eta)\varepsilon x_5 - \left(\frac{\beta(1-\chi\xi)x_3}{N} + \mu\right)x_1 = f_1 \\
{}^{abc}_{0}D_t^{\sigma} x_2(t) &= \frac{\beta(1-\chi\xi)x_3}{N} - (\alpha+\mu)x_2 = f_2 \\
{}^{abc}_{0}D_t^{\sigma} x_3(t) &= \alpha x_2 + \eta\varepsilon x_5 - (d + \mu + \vartheta x_6)x_3 = f_3 \\
{}^{abc}_{0}D_t^{\sigma} x_4(t) &= \vartheta x_3 x_6 - (\gamma + \mu)x_4 = f_4 \\
{}^{abc}_{0}D_t^{\sigma} x_5(t) &= \gamma x_4 - (1+\mu)x_5 = f_5 \\
{}^{abc}_{0}D_t^{\sigma} x_6(t) &= \Lambda - (\tau + \mu)x_6 = f_6
\end{aligned}\right\} \tag{14}$$

The Jacobian matrix of the systems (5) evaluates at the crime free equilibrium point is obtained as

$$\mathsf{J}\,(\mathsf{E}_0) = \begin{bmatrix} -\mu & 0 & -\beta(1-\xi\kappa) & 0 & (1-\eta)\varepsilon & 0 \\ 0 & -h_1 & \beta(1-\xi\kappa) & 0 & 0 & 0 \\ 0 & \alpha & -h_2 & 0 & \eta\varepsilon & 0 \\ 0 & 0 & 0 & -h_3 & 0 & 0 \\ 0 & 0 & 0 & \gamma & -h_4 & 0 \\ 0 & 0 & \Lambda & 0 & 0 & -h_5 \end{bmatrix}. \tag{15}$$

Now, $R_0 = 1$ and suppose β is the bifurcation parameter, hence

$$\beta^* = \frac{(\alpha+\mu)\left[(d+\mu)(\tau+\mu)+\vartheta\Lambda\right]}{\alpha(1-\chi\xi)(\tau+\mu)}.$$

with $\beta = \beta^*$, the transformed system (14) has a simple eigenvalue with zero real part and all the other eigenvalues are negative. Thus, by Centre Manifold Theorem (Castillo-Chavez & Song, 2004), we investigate the transform matrix (15), near $\beta = \beta^*$. The right eigenvectors of $J(E_0)\big|_{\beta=\beta^*}$, denoted as $w = (w_1, w_2, w_3, w_4, w_5, w_6)$ are obtain as

$$w_1 = \beta(1-\chi\xi)w_3, \quad w_2 = \beta(1-\chi\xi)w_3, \quad w_3 = w_3 > 0,\ w_4 = 0,\ w_5 = 0,\ w_6 = \frac{\Lambda}{\mu+\tau}w_3.$$

Similarly the left eigenvectors denoted as $v = (v_1, v_2, v_3, v_4, v_5, v_6)$ are obtain as

$$v_1 = 0, \quad v_2 = \frac{\alpha}{h_1}v_3,\ v_3 = v_3 > 0,\ v_4 = \frac{\gamma}{h_3}v_5,\ v_5 = \frac{(1-\eta)\varepsilon v_1 + \eta\varepsilon v_3}{h_4},\ v_6 = 0.$$

The bifurcation coefficients a and b are obtained as described in Theorem 4.1 (Castillo-Chavez & Song, 2004) as,

$$a = 2w_3\left[\beta v_2\mu(1-\chi\xi)(w_2+w_3) + \pi w_6\vartheta(v_4 - v_3)\right]$$

$$b = v_2 w_3(1-\xi\chi) > 0$$

Theorem 5. The crime present equilibrium point (E_1), with $\eta = 0$, of the model (5) is Globally Asymptotically Stable (GAS) if $R_0 > 1$.

Proof

Consider the function

$$M = \left(S - S^{**} - S\ln\frac{S^{**}}{S}\right) + \left(E - E^{**} - E^{**}\ln\frac{E^{**}}{E}\right) + \frac{\beta D^{**}S^{**}}{\alpha E^{**}}\left(D - D^{**} - D^{**}\ln\frac{D^{**}}{D}\right).$$

whose derivative is

$${}_{0}^{abc}D_t^{\sigma}M = \left(1-\frac{S^{**}}{S}\right){}_{0}^{abc}D_t^{\sigma}S + \left(1-\frac{E^{**}}{E}\right){}_{0}^{abc}D_t^{\sigma}E + \frac{\beta D^{**}S^{**}}{\alpha E^{**}}\left(1-\frac{D^{**}}{D}\right){}_{0}^{abc}D_t^{\sigma}D \tag{16}$$

Substituting the values ${}_{0}^{abc}D_t^{\sigma}S$, ${}_{0}^{abc}D_t^{\sigma}E$, ${}_{0}^{abc}D_t^{\sigma}D$ in (14.0), gives

$${}_{0}^{abc}D_t^{\sigma}M = \left(1-\frac{S^{**}}{S}\right)\left(\pi - \frac{\beta(1-\eta\xi)DS}{N} - \mu S\right) + \left(1-\frac{E^{**}}{E}\right)\left(\frac{\beta(1-\eta\xi)DS}{N} - h_1E\right)$$
$$+ \frac{\beta D^{**}S^{**}}{\alpha E^{**}}\left(1-\frac{D^{**}}{D}\right)(\alpha E - h_2 D)$$

Further Simplification, yield

$${}_{0}^{abc}D_t^{\sigma}M \leq +\mu S^{**}\left(2 - \frac{S}{S^{**}} - \frac{S^{**}}{S}\right) + \tilde{\beta}S^{**}D^{**}\left(3 - \frac{S^{**}}{S} - \frac{DE^{**}S}{D^{**}ES^{**}} - \frac{D^{**}E}{DE^{**}}\right)$$

Since the arithmetic mean exceeds the geometric mean the following inequality hold:

$$2 - \frac{S}{S^{**}} - \frac{S^{**}}{S} \leq 0,\ 3 - \frac{S^{**}}{S} - \frac{DE^{**}S}{D^{**}ES^{**}} - \frac{D^{**}E}{DE^{**}} \leq 0.$$

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Furthermore, since all the model are non-negative, thus ${}^{abc}_{0}D_t^{\sigma}\mathrm{M} \le 0$, if $D^{**} = D$, $E^{**} = E$ and $\eta = 0$, then the largest compact invariant set in B such that ${}^{abc}_{0}D_t^{\sigma}\mathrm{M} \le 0$ is the singleton set (E_1) then by LaSalle Invariant Principle it implies E_1, is globally asymptotically stable (GAS) in the interior of B .

Model Fitting

To validate the model (5), we fitted it using yearly data from Macrotrends, (2023), covering the years 2002 to 2021. The time series visualization of the (5) least square fit model is displayed in figure 4. The developed model was fitted with the traditional nonlinear least squares method, which involved using the built-in Matlab R2023a and the optimizer function "lsqcurvefit" to determine the values of the unknown parameter, ϑ .

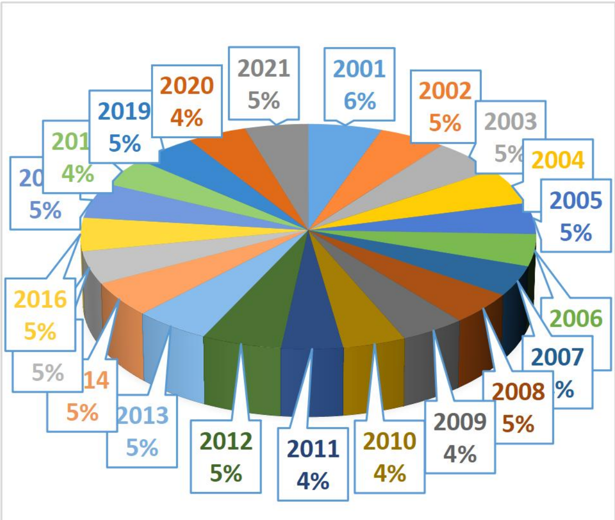

Figure 2. Time series data on reported cases of crime in Ghana. Source: Macrotrends, 2023

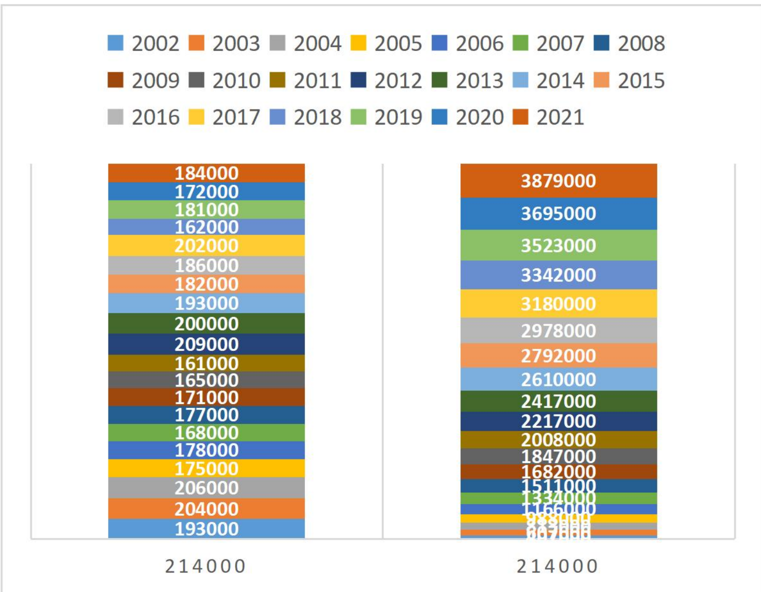

Figure 3. Time series data on reported cases of crime in Ghana. Source: Macrotrends, 2023.

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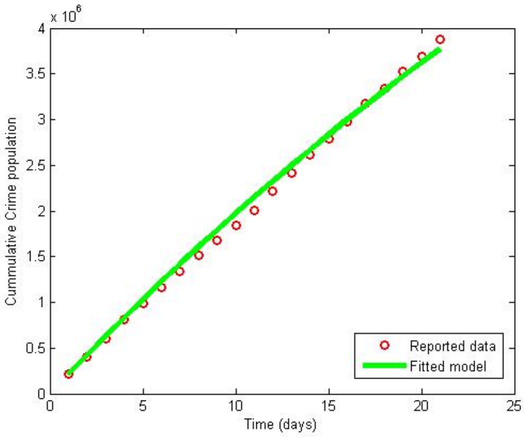

Figure 4. Comparison of observed crime cumulative data from Ghana (dotted lines) and prediction (solid curve).

Sensitivity Analysis

Sensitivity analysis shows how important every parameter is for the transmission of illness. It is employed to determine which factors, because to their substantial influence, should be the focus of intervention efforts. Sensitivity indices allow us to evaluate the proportionate change in a variable when a parameter changes. The normalized forward sensitivity index of a variable with regard to a parameter is the ratio of the relative change in the variable to the relative change in the parameter. Table 2, showed that all of the factors have positive indices. This indicates that if one of the parameters (τ, β, α) is increased while the others remain same, the effective reproduction number will increase, increasing the chance of a crime outbreak. On the other hand, the parameters $(\chi, \xi, d, \mu, \theta, \Lambda, v)$, indices are negative, meaning that raising one of them while keeping the others same reduces the effective reproduction number and, in turn, the burden of crime on human populations.

Table 1: Parameter description.

Parameter	Description
β	1
α	0.0489
τ	0.0301
μ	-0.1126
d	-0.9058
Λ	-0.0303
ξ	-0.2880
χ	-0.2880
v	-0.0303

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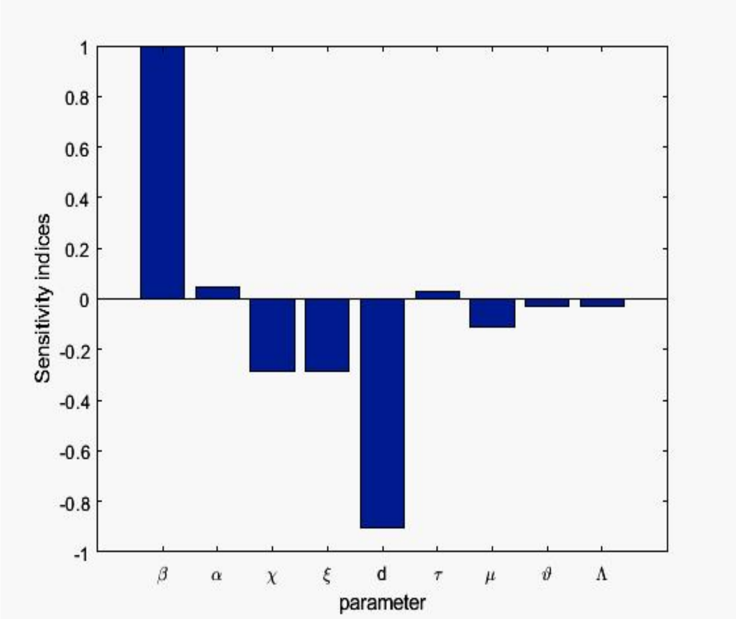

Figure 5: Sensitivity indices of the reproduction number with respect to parameter values in the reproduction number.

Table 2: Parameter Values

Parameter	Description	Value	Source
π	Recruitment rate in the susceptible class	101.0302/day	Kwafie *et al,* 2023
β	Effective contact rate	0.022/day	Kwafie *et al*, 2023
α	Progression rate from exposed class to the crime class	0.000714/day	Kwafie *et al*, 2023
η	Proportion of removed criminals that go back to committing crime	0.000429/day	Kwafie *et al*, 2023
τ	Retired rate of the police force	0.005/day	Misra, 2014
μ	Natural death rate	0.0000367/day	Kwafie *et al*, 2023
d	Death rate as a result of the crime	0.00052/day	Misra, 2014
Λ	Police force recruitment rate	0.032/day	Assumed
γ	Progression rate from J to R	0.00227/day	Kwafie *et al*, 2023
ξ	Education program efficacy	0.43/dimensionless	Kwafie *et al*, 2023
χ	Proportion of the population that are educated	0.53/dimensionless	Kwafie *et al,* 2023
ε	Progression rate from removed class to crime population	0.7/dimensionless	Kwafie *et al,* 2023
ϑ	Progression rate from I to J	0.00000127/day	Fitted

Numeral Scheme and Simulation

We employ approached in Toufit & Atangana (2017), to develop the numerical scheme for the model (5). Consider the first equation of model (5)

$$\begin{aligned} {}^{ABC}_{\ 0}D_t^{\sigma} S(t) &= F_1(t, S(t)) \\ S(0) &= S_0 \geq 0 \end{aligned} \quad (17)$$

where, $F_1(t, S(t)) = \pi + (1-\varepsilon)\eta R - \left(\dfrac{\beta(1-\chi\xi)D}{N} + \mu\right)S$

Taking the Laplace transform of (17) and Simplifying, yield

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$$L\left\{S(t)\right\}=\frac{1}{s}S(0)+\frac{(1-\sigma)}{M(\sigma)}L\left\{F_1\left(t,S(t)\right)\right\}+\frac{\sigma}{M(\sigma)}\frac{1}{s^{\sigma}}L\left\{F_1\left(t,S(t)\right)\right\}. \tag{18}$$

Taking the inverse Laplace of (18) and using the Convolution Theorem, yield

$$S(t)-S(0)=\frac{1-\sigma}{M(\sigma)}F_1\left(t,S(t)\right)+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\int_0^t F_1\left(\tau,S(\tau)\right)(t-\tau)^{\sigma-1}d\tau. \tag{19}$$

At a given point t_{n+1}, $n=0,1,2,...$, then equation (19) can be written as

$$S(t_{n+1})=S(0)+\frac{1-\sigma}{M(\sigma)}F_1\left(t,S(t_n)\right)+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\int_0^{t_{n+1}} F_1\left(\tau,S(\tau)\right)(t_{n+1}-\tau)^{\sigma-1}d\tau,$$

$$S(t_{n+1})=S(0)+\frac{1-\sigma}{M(\sigma)}F_1\left(t,S(t_n)\right)+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{k=1}^{n}\int_{t_k}^{t_{k+1}} F_1\left(\tau,S(\tau)\right)(t_{n+1}-\tau)^{\sigma-1}d\tau \tag{20}$$

Applying the Lagrange polynomial interpolation on the interval $\left[t_{k+1},t_k\right]$, that is

$$\begin{aligned}S_k(\tau)&=\frac{\tau-t_{k-1}}{t_k-t_{k-1}}F_1\left(t_k,S(t_k)\right)-\frac{\tau-t_{k-1}}{t_k-t_{k-1}}F_1\left(t_{k-1},S(t_{k-1})\right)\\&\approx\frac{\tau-t_k}{h}F_1\left(t_k,S(t_k)\right)-\frac{\tau-t_k}{h}F_1\left(t_{k-1},S(t_{k-1})\right),\end{aligned} \tag{21}$$

where $h=t_k-t_{k-1}$

Hence, the approximation (21) is included in (20), to have

$$\begin{aligned}S(t_{n+1})&=S(0)\\&+\frac{1-\sigma}{M(\sigma)}F_1\left(t,S(t_n)\right)+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{k=1}^{n}\left[\frac{F_1\left(t_k,S(t_k)\right)}{h}\int_{t_k}^{t_{k+1}}(\tau-t_{k-1})(t_{n+1}-\tau)^{\sigma-1}d\tau\right]\\&-\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{k=1}^{n}\left(\frac{F_1\left(t_{k-1},S(t_{k-1})\right)}{h}\int_{t_k}^{t_{k+1}}(\tau-t_k)(t_{n+1}-\tau)^{\sigma-1}d\tau\right).\end{aligned}$$

Let

$$\begin{aligned}W_{\sigma,1}&=\int_{t_k}^{t_{k+1}}(\tau-t_{k-1})(t_{n+1}-\tau)^{\sigma-1}d\tau=-\frac{1}{\sigma}\left[(t_{k+1}-t_{k-1})(t_{n+1}-t_{k+1})^{\sigma}-(t_k-t_{k-1})(t_{n+1}-t_k)^{\sigma}\right]\\&-\frac{1}{\sigma(\sigma+1)}\left[(t_{k+1}-t_{k-1})^{\sigma+1}(t_{n+1}-t_{k-1})^{\sigma}-(t_{n+1}-t_k)^{\sigma+1}\right].\end{aligned}$$

and

$$\begin{aligned}W_{\sigma,2}&=\int_{t_k}^{t_{k+1}}(\tau-t_k)(t_{n+1}-\tau)^{\sigma-1}d\tau=-\frac{1}{\sigma}\left[(t_{k+1}-t_{k-1})(t_{n+1}-t_{k+1})^{\sigma}-(t_k-t_{k-1})(t_{n+1}-t_k)^{\sigma}\right]\\&-\frac{1}{\sigma(\sigma+1)}\left[(t_{n+1}-t_{k-1})^{\sigma+1}-(t_{n+1}-t_k)^{\sigma+1}\right].\end{aligned}$$

Substituting $t_k=kh$, in $W_{\sigma,1}$, $W_{\sigma,2}$, we have

$$W_{\sigma,1}=\frac{h^{\sigma+1}}{\sigma(\sigma+1)}\left[(n-k+2+\sigma)(n+1-k)^{\sigma}-(n-k+2+2k)(n-k)^{\sigma}\right],$$

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$$W_{\sigma,2}=\frac{h^{\sigma+1}}{\sigma(\sigma+1)}\left[(n+1-k)^{\sigma+1}-(n-k+1+k)(n-k)^{\sigma}\right].$$

Hence,

$$S(t_{n+1})=S(t_0)+\frac{1-\sigma}{M(\sigma)}\Theta_1(t_n,S(t_n))$$
$$+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[\frac{\Theta_1(t_j,S(t_j))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n-j+2+\sigma)(n+1-j)^{\sigma}-(n-j+2+2j)(n-j)^{\sigma}\right]\right]$$
$$-\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[-\frac{\Theta_1(t_{j-1},S(t_{j-1}))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n+1-j)^{\sigma+1}-(n-j+1+j)(n-j)^{\sigma}\right]\right].$$

Similarly for E, D, J, R and P, we obtained

$$E(t_{n+1})=E(t_0)+\frac{1-\sigma}{M(\sigma)}\Theta_2(t_n,E(t_n))$$
$$+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[\frac{\Theta_2(t_j,E(t_j))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n-j+2+\sigma)(n+1-j)^{\sigma}-(n-j+2+2j)(n-j)^{\sigma}\right]\right]$$
$$-\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[-\frac{\Theta_2(t_{j-1},E(t_{j-1}))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n+1-j)^{\sigma+1}-(n-j+1+j)(n-j)^{\sigma}\right]\right].$$

$$D(t_{n+1})=D(t_0)+\frac{1-\sigma}{M(\sigma)}\Theta_3(t_n,D(t_n))$$
$$+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[\frac{\Theta_3(t_j,D(t_j))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n-j+2+\sigma)(n+1-j)^{\sigma}-(n-j+2+2j)(n-j)^{\sigma}\right]\right]$$
$$-\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[-\frac{\Theta_3(t_{j-1},D(t_{j-1}))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n+1-j)^{\sigma+1}-(n-j+1+j)(n-j)^{\sigma}\right]\right].$$

$$J(t_{n+1})=J(t_0)+\frac{1-\sigma}{M(\sigma)}\Theta_4(t_n,J(t_n))$$
$$+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[\frac{\Theta_4(t_j,J(t_j))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n-j+2+\sigma)(n+1-j)^{\sigma}-(n-j+2+2j)(n-j)^{\sigma}\right]\right]$$
$$-\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[-\frac{\Theta_4(t_{j-1},J(t_{j-1}))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n+1-j)^{\sigma+1}-(n-j+1+j)(n-j)^{\sigma}\right]\right].$$

$$R(t_{n+1}) = R(t_0) + \frac{1-\sigma}{M(\sigma)}\Theta_5(t_n, R(t_n))$$

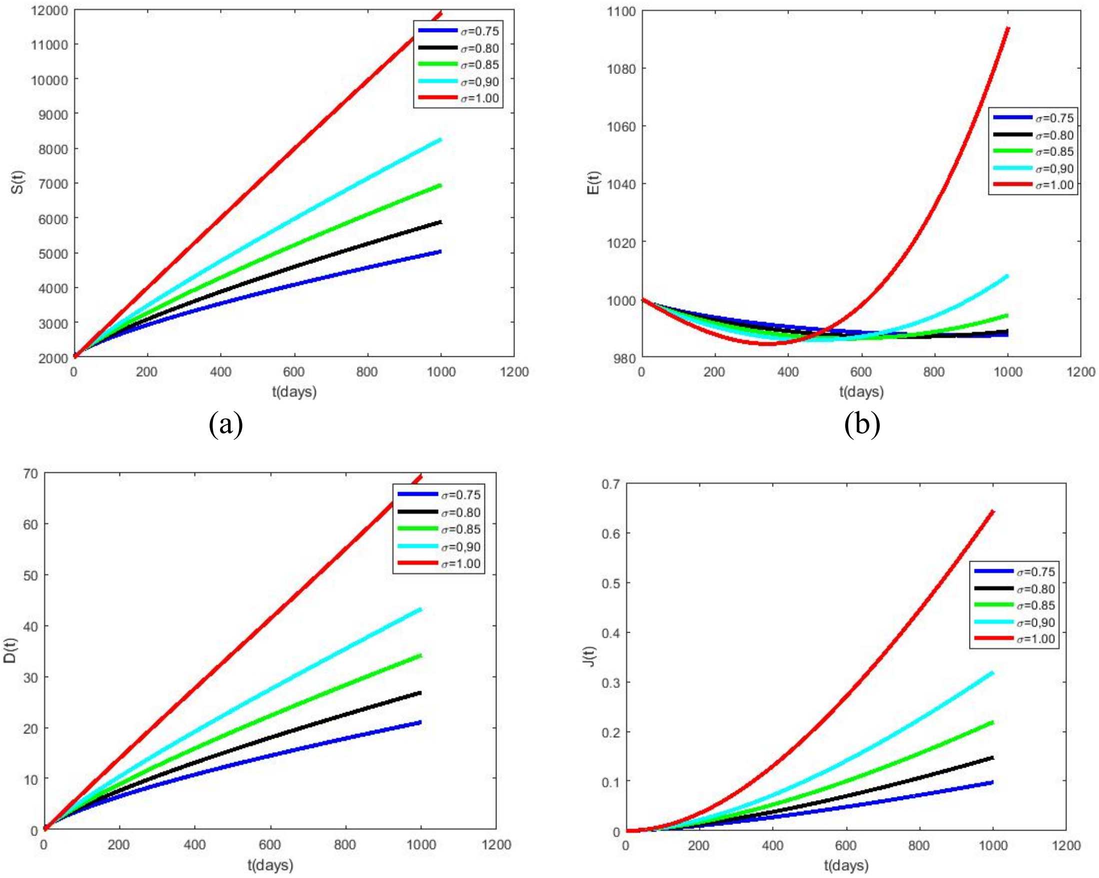
$$+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[\frac{\Theta_5(t_j, R(t_j))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n-j+2+\sigma)(n+1-j)^{\sigma}-(n-j+2+2j)(n-j)^{\sigma}\right]\right]$$

$$-\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[-\frac{\Theta_5(t_{j-1}, R(t_{j-1}))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n+1-j)^{\sigma+1}-(n-j+1+j)(n-j)^{\sigma}\right]\right].$$

$$P(t_{n+1}) = P(t_0) + \frac{1-\sigma}{M(\sigma)}\Theta_6(t_n, P(t_n))$$

$$+\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[\frac{\Theta_6(t_j, P(t_j))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n-j+2+\sigma)(n+1-j)^{\sigma}-(n-j+2+2j)(n-j)^{\sigma}\right]\right]$$

$$-\frac{\sigma}{M(\sigma)\Gamma(\sigma)}\sum_{j=1}^{n}\left[-\frac{\Theta_6(t_{j-1}, P(t_{j-1}))}{\Gamma(\sigma+2)}h^{\sigma}\left[(n+1-j)^{\sigma+1}-(n-j+1+j)(n-j)^{\sigma}\right]\right].$$



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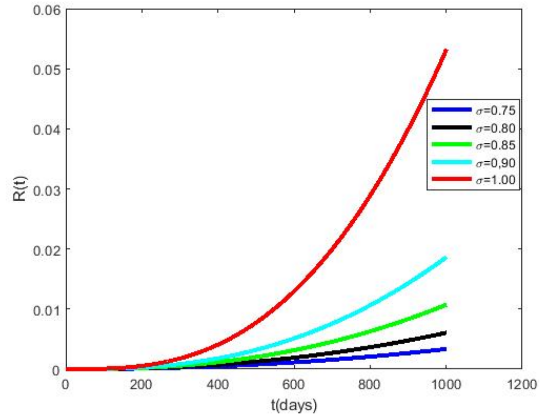
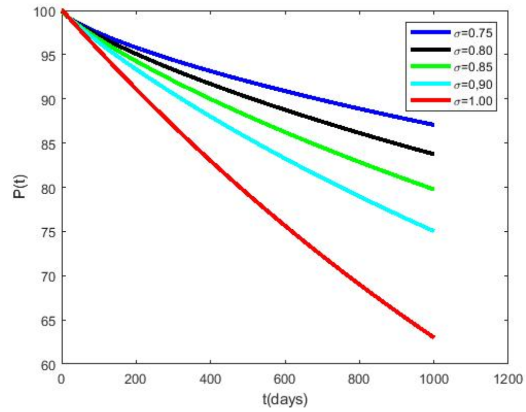
(c) (d)

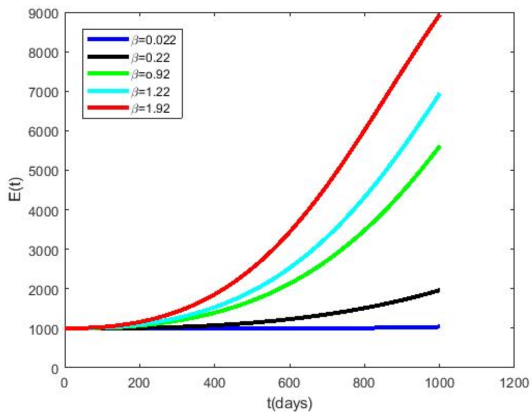


(e)

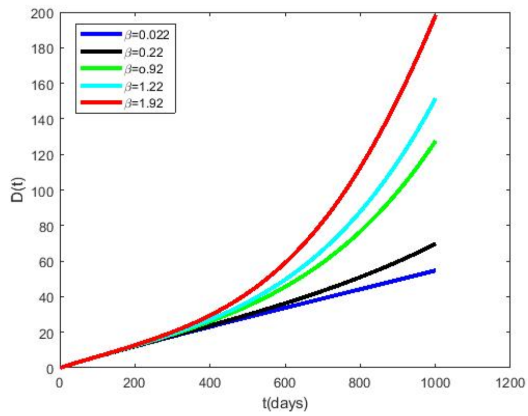


(f)

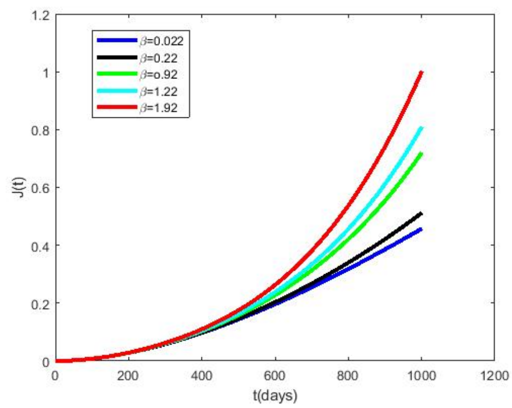
Figure 5: The Numerical visualization for model (5) with different fractional order.



(a)



(b)



(c)

Figure 6: Numerical visualization showing the impact of contact rate β on the model (5), for $\sigma = 0.85$.

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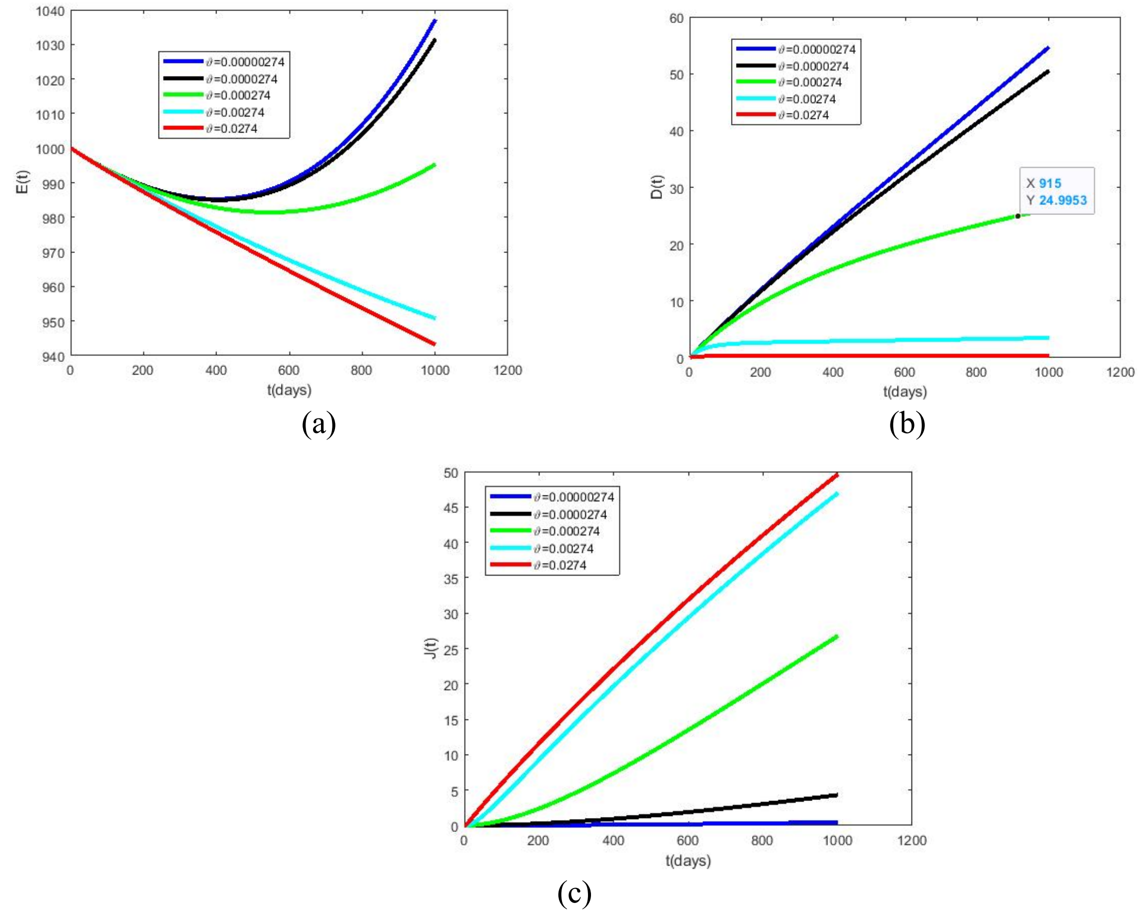
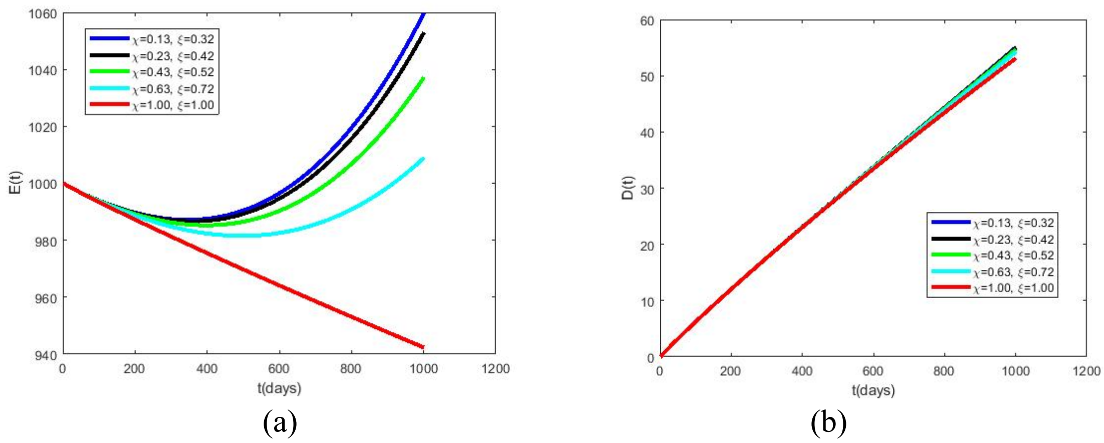


Figure 7: Numerical visualization showing the impact of ϑ (police intervention rate), on the model (5), for $\sigma = 0.85$.



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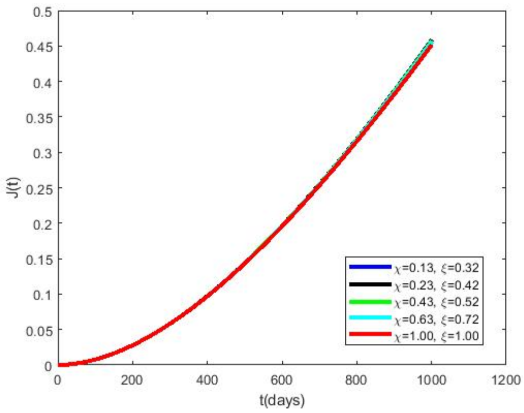

(C)

Figure 8: Numerical visualization showing the impact of χ, ξ (public education) on the model (5) for $\sigma = 0.85$.

DISCUSSION

The outcomes of a numerical simulation of the model (5) are shown in figure 4(a) to figure 4(f), which also shows how memory influenced each compartment as the order increased. As a result of migration into the criminal class, figure 4b, shows a decline in the exposed class population and Figure 4a shows an increase in the susceptible population. In contrast to the observed increase in the population of the crime class when the order is increased, figure 4 (d), (e), and (f) shows a drop in population when the order decreases. Figure 5(a) to figure 5(b) showed the occurrence of endemic locations with different fractional orders. Figure 6, shows that he dynamics of the exposed population, crime population and the jail population may all be significantly impacted by an increase in the effective contact rate from 0.022 to 1.92. When the effective contact rate increases, people are more likely to come into contact with one another, which could result in a higher likelihood of crime transmission within the exposed community. This may lead to a rise in the number of people who are exposed to crime, which lead to increase in the size of the exposed population. Furthermore, social relationships and patterns in communities may be impacted by the higher effective contact rate, and this could have an impact on crime rates. A higher contact rate may lead to more chances for criminal conduct, which can enhance the crime rate among the populace. Since an increase in crime rates brought on by the higher effective contact rate result in an increased in the number of persons who are detained and arrested, the consequences on the jail population could also be significant.

The dynamics of the exposed population, crime rates, and the jail population are all affected in different ways by an increase in the police intervention rate from 0.00000274 to 0.0274, as shown in figure 7. An increase in the police intervention rate denotes a greater degree of law enforcement engagement in addressing and averting criminal activity in a neighbourhood. There were fewer people in the exposed population as a result of this increase's effects. This is because increased police presence could discourage would-be criminals, decreasing the chance that crimes would be committed and, consequently, the number of people who are exposed. The proactive steps taken by law enforcement to catch and prevent criminals can also lead to a fall in crime rates, as evidenced by the rise in police intervention rates. Stricter enforcement of the law and a greater presence of police personnel may help to curb criminal activity, which in turn lowers the population's overall crime rates. Nevertheless, a rise in the rate of police intervention may result in an increase

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in the jail population even in the face of declining exposed and criminal populations. This is due to the fact that increased police involvement may lead to more arrests and convictions, which in turn may result in a rise in the prison population.

Figure 8 shows that if more members of the vulnerable population receive knowledge on crime, the exposed class may decline. This is because education serves to shield people from being exposed to criminal activity. This could therefore result in a decline in the number of people incarcerated and crimes committed. Raising awareness and teaching people about crime prevention may have a good effect on decreasing criminal behaviour, which in turn may reduce the number of people who commit crimes and wind up behind bars.

CONCLUSION

This study presents a fractional mathematical model of crime dynamics with police and public education, demonstrating its positivity and boundedness. The model's asymptotically stable local and global states are established using the basic reproduction number, validated using yearly data from 2002-2021. The fractional crime mode model reveals complex relationships between elements influencing criminal behaviour, facilitating more informed decision-making and policy formulation in crime prevention and law enforcement strategies. Crime model simulations, when combined with interventions like public education and increased police presence, have been shown to be effective in lowering crime rates. A higher effective contact rate increases the probability of criminal activity, leading to more interactions and criminal behaviour within the community. However, deploying police intervention and public education programs can minimize this increase by addressing and lowering the number of exposed people, infected people, and eventually incarcerated individuals. Public education can also foster healthy habits, raise awareness of crime prevention strategies, and provide individuals with resources to make better decisions. By addressing the root causes of crime, a safer, more secure environment can be created for the community.

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