Mathematics Education as a Discipline: A Critical Analysis of Metaphysical and Epistemological Foundations

Yenealem Ayalew Degu
E-mail: yene927@gmail.com

PhD student, Department of Science and Mathematics Education
Addis Ababa University

Abstract: Mathematics education is situated at the nexus of mathematics and education. As to my information, graduate programs in mathematics education were on and off at Bahir Dar, Haramaya, and Addis Ababa universities over the last ten years. This indicates a reputation problem; the dilemma is whether mathematics education is a field in its own right or not. The objective of this review is, therefore, to critically analyze the metaphysical and epistemological foundations of mathematics education. A mathematics educator's orientation and awareness about the mathematics teaching, the aim of mathematics education, the theories of learning process, the students (or environment) and the nature of mathematics are determined by general mathematical and educational philosophies. It is believed that mathematics education has diverging theoretical perspectives. However, there are arguments about whether there is a need to develop own philosophies and theories or adapt from other disciplines. Adapted theories, such as semiotics and mathematical discourse, could be applied. On the other hand, the development of grand philosophy (and theory) of mathematics education could be useful in providing warrants for the field's autonomy. Theorizing and philosophizing could be opportunities for mathematics education researchers; it is a good opportunity for frontiers to play lots of roles. This review is hoped to bring discussion forum on the issue.

Keywords: *mathematics education, metaphysical and epistemological foundations, educational philosophies*

Introduction

As to my information, postgraduate programs in mathematics education were on and off at Bahir Dar, Haramaya, and Addis Ababa universities over the last ten years. Currently, Addis Ababa University is running the program under the Department of Science and Mathematics Education. This signifies a reputation problem. In my view, among other factors, the contest between applied mathematics (and science) and educational mathematics (and science) has destabilized the field. The question "what does mathematics education provide for pure mathematics and applied mathematics practices?" would be decisive for the mutual development of the three fields. An important aim of mathematics education should be to make students properly aware of the value of mathematical applications in a wide range of situations. The benefits that will accumulate are essential for the survival and future growth of commerce, industry, and science, and there are opportunities for them to be realized at every level of employment.

Vigorous new perspectives are pervading mathematics education from disciplines as diverse as psychology, philosophy, logic, sociology, anthropology, history, women's studies, cognitive science, semiotics, hermeneutics, post-structuralism and post-modernism (Hoyles, 1999). To adequately define and describe its scope, we need to look into the philosophical and theoretical orientations of mathematics education. Epistemology, as a branch of philosophy, is concerned with scientific knowledge (Sierpinska & Lerman, 1996) whereas metaphysics is about truth and reality. Hence, the objective of this review is to critically analyze the metaphysical and epistemological foundations of mathematics education.

Metaphysical Foundations

The theme under this section is discussing the metaphysical aspects of philosophies in mathematics and education so that implication could be inferred. Many literatures use the broad term philosophy for dialogues about truth and reality.

Philosophy of Mathematics

Philosophical arguments have had a significant impact on the advancement of mathematics throughout its history, and especially in recent times (Jacquette, 2002). Since mathematicians seek out patterns, formulate new conjectures, and establish truth by rigorous deduction from appropriately chosen axioms and definitions (Schleicher & Lackmann, 2011), there are diversification and unification in the development of mathematics (Ruelle, 2007). For example, the fifth postulate stating "that if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles" (Ostermann & Wanner, 2012, p.29) has created disagreement among geometers and, as a result, different forms of geometry had emerged: Euclidean, Absolute, Elliptical and Hyperbolic geometries. On the other hand, Algebraic Geometry, Differential Geometry and Combinatorial Geometry (to mention just a few) tell us that mathematical development has a unification characteristic.

These achievements are not pertinent to absolutist paradigm of mathematics, which views it as a body of infallible and objective truth (Ernest, 1991). Ernest classified this broad perspective into Logicism, Formalism, Intuitionism and Platonism. The classical laws of logic are foundations of all mathematics (George & Velleman, 2002). That is briefly about Logicism. In the case of Formalism, we study systems of postulates that are treated as uninterpreted and we avoid controversial claims (Bostock, 2009). For the advocators of this thought, mathematical exactness exists on the paper not on the mind (Jacquette, 2002); it consists merely in the method of developing series of relations by fixed laws, say, from axioms. A better conception of formalism, given by Prediger (2007), is that mathematics is a meaning making activity of a society of practitioners. In contrast with the formalist view, intuitionists believe that mathematical objects exist in human intellect (Jacquette, 2002; Bostock, 2009). An intuitive idea should be expressed in a format which is sufficiently precise to enable deduction, calculation and proof (Seirpinska & Kilpatrick, 1998). Platonism is another similar view but it assumes that the objects of mathematics have real, objective existence in some ideal realm; hence mathematics is discovered, not invented (Schim, 1998).

Nowadays, large parts of the philosophy of mathematics are concerned not only with questions about the foundations (Bostock, 2009) like in Absolutism, but with descriptions and analyses of mathematical practices, the relationship between mathematics and human beings, and socio-philosophical reflections on the role of mathematics for society (Prediger, 2007). This means that mathematics should not be seen as an absolute body of knowledge, but as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context. It is a discipline in its own right (Hoyles et al., 1999); therefore, it needs to be viewed as much more than the basic functionality associated with numeracy and algebra; it is a powerful language for sharing and systematizing knowledge as part of human culture.

The absolutist view of mathematical knowledge has such criticism that its rejection leads to the acceptance of the view that mathematical truth is fallible and corrigible, and can never be regarded as beyond revision and correction (Ernest, 1991).

Educational Philosophies

Although aspects of educational philosophy can be derived from the roots of idealism, realism, pragmatism, and existentialism, just to provide a pattern, there are four agreed-on philosophies of education (Ornstein & Hunkins, 2004; Oliva, 2009): (1) Perennialism–aims to educate the rational person and cultivate the intellect; (2) Essentialism–tends to promote the intellectual growth of the individual and to adjust men and women to the society with essential skills and essential subjects; (3) Progressivism–inclines to promote democratic and social living; (4) Reconstructionism–favors improvement and reconstruction of society.

Implications to Mathematics Education

The philosophy of Mathematics education has backgrounds from the general philosophy, the philosophy of mathematics and the educational philosophies. One of the central issues for the philosophy of mathematics education is the link between philosophies of mathematics and mathematical practices. The mathematical development of students posits issues of teaching, learning the mathematical contents, and school environment. The practice on these areas would be impacted by one or a combination of philosophical assumptions. Prediger in Francois and Van Bendegem (2007) argued that "we cannot exclude philosophical reflections from mathematics classrooms, especially in a community of reflectiveness, dialogue, and mutual respect for diverging thoughts" (p.56).

From our earlier discussion of 'Philosophy of Mathematics', we can imply the following points. A mathematics educator influenced by an ideology of Logicism might focus on classical laws of logic in communicating mathematics. For instance, an affine geometry teacher of Formalism belief may follow axiomatic approach than matrix representation and analytical ways in delivering the subject. Student's intuitive idea could be elicited to enable deduction, calculation and proof. A Platonist teacher or curriculum expert might present theorem for students to discover the proof.

Moreover, a mathematics teacher having Essentialism ideology might give students' postulates, concepts, theorems and formulae; and provide many opportunities to practice or react accordingly. On the other hand, a progressivist mathematics curriculum expert may design the curriculum base on student's interests and involves the application of human problems and affairs. Hence, interdisciplinary subject matter, activities and projects, problem solving, and scientific method inquiry became both goal and a technique. A reconstructionist mathematics educator regards the young people to consider pressing social, economic and political problems; its major premise to make the school as agency of social change.

Therefore, a mathematics educator's orientation and awareness of the mathematics teaching, the aim of mathematics education, the theories of learning process, the students (or environment) and the nature of mathematics is determined by the philosophical roots. In turn, our awareness shapes our activity.

Epistemological Foundations

Epistemology quests for the origins of knowledge, the criteria of validity of knowledge and the process of development of knowledge (Sierpinska & Lerman, 1996). To inquire about mathematics, Schim (1998) raised such questions as: what does mathematical knowledge consist in? Does mathematics have a subject-matter? Is it concerned with a reality independent of human mind? Is it a creation of mind or convention? Does it derive its certainty? However, some of these questions are related to metaphysics. Hence, it can be said that epistemology and metaphysics are interrelated.

Mathematical Knowledge

Mathematical knowledge is the set of truths, in the form of propositions with proofs, and the function of the philosophy of mathematics is to establish the certainty of this knowledge (Ernest, 1991). It constitutes a coherent whole (Steinbring, 2005) and relates to given objects; relations, structures, patterns are expressed in its formulae, equations, and computation (Paulos, 1980). We may characterize the historical development of mathematical thought to reformulate the 'principle of continuity', which is the fundamental principle of mathematical generalization. Mathematics is relational thinking (Kilpatrick et al., 2005). The objective of mathematics is to use as few hypotheses as possible to get as strong a conclusion as possible (Houston, 2009).

Epistemological Issues in Mathematics Education

The mathematical argumentation of young students is constrained by the epistemological conditions of mathematical knowledge. Communication between teacher and students in the context of mathematical instruction is determined by the intention of mediating and learning mathematical knowledge. An important dimension of the relational quality of knowing is dialogue, especially as it applies concretely to language as social activity (Gill, 1993). The influence of instructional goals must be particularly taken into consideration in understanding and analyzing the specificity of interactive mathematical teaching and learning processes (Steinbring, 2005).

Sriraman and English (2010) noted Burton's (2004) epistemological model of "coming to know mathematics" as consisting of five interconnecting categories namely: the person and the social/cultural system; aesthetics, intuition/insight, multiple approaches, and connections; grounded in the extensive literature base of mathematics education; sociology of knowledge and feminist science; and challenges of objectivity, homogeneity, impersonality, and incoherence. This is just one model, Burton's model. To gain the perspective, we need to view mathematics as a socio-cultural artifact, part of a larger cultural system. Gates & Jorgensen (2014) put the epistemological controversies, in mathematics education, in terms of subjective-objective characteristics of mathematical knowledge, the role in the cognition of social and cultural context, and relations between language and knowledge.

Models and Theories of Mathematics Education

Since epistemology quests for the origins of knowledge (Sierpinska & Lerman, 1996), to inquire about mathematics education Schim's (1998) concerns can be raised in context: what does mathematical education knowledge consist in? Does Mathematics Education have a subject-matter?

These in turn call for assessing models and theories being practiced in the field. We can discuss the differences between model and theory. Whereas a model describes a phenomenon, embodies a theory, and therefore deals with some of the criteria, a theory needs to cover more territory, addresses a body of phenomena, and satisfies all criteria more fully.

In pure mathematics one encounters the theory of equations, the theory of numbers, group theory, Galois Theory and so on. Such theories exhibit a number of theorems (or results) derived from certain basic premises or postulates and the kind of derivation is usually described as deduction. "Mathematical theory is not what he [the mathematician] has in mind when he inveighs against theory, and he would not regard the pure mathematician's disregard for practical application as an example of his thesis that theory and practice are poles apart and utterly unrelated" (Barrow & Woods, 2006, p.186).

The development of "universal" theoretical frameworks has been problematic for mathematics education. Phenomenology, critical theory, post-structuralism, hermeneutics and semiotics are modern developments in philosophy which influenced mathematics education (Gates & Jorgensen, 2014). According to Sriraman & English (2010), the expansion of theories in use within the mathematics education research community can be categorized as:

- Cultural psychology, including work based on Vygotsky, activity theory, situated cognition, communities of practice, social interactions, semiotic mediation
- Ethno mathematics
- Sociology, sociology of education, post structuralism, hermeneutics, critical theory

- Discourse, to include psychoanalytic perspectives, social linguistics, semiotics.

These are adapted theories into the field mathematics education.

Discussion and Commentary

We have seen different philosophies of mathematics. The pure mathematics focuses on the logic of argument and joy of the investigation itself, the beauty of mathematics. On the other hand, the interest of mathematics should rest on its applications. And the philosophies of mathematics education can be derived a lot from these groups and as well from the general philosophies or from particular educational school of thoughts. A mathematics educator influenced by any of the philosophies would reflect in his/her actions.

Theories in mathematics education are decisive, too, like the philosophies, to enhance the development of the field. How far have mathematics educators moved toward a theory, whether grand or otherwise? To say that something is a theory of mathematics education—rather than, say, an approach, theoretical framework, theoretical perspective, or model—is to make an exceedingly strong claim.

“Internationally, mathematics education research is shaped by a diversity of theories” (Sriraman & English, 2010, p. 483). But, the diversity of theories must be considered as a resource and not as a handicap. In the last twenty years or so, mathematics education has struggled with (at least) three broad theoretical approaches to language: language as a code, language as representation, and language as discourse (Kilpatrick, 2005). This challenge would require a concerted effort of a large group of mathematics educators. Japan Society of Mathematical Education research tended toward expanding the abilities and potential of each student striving to enhance education (Isoda et al., 2007). The implications of roles of mathematics for mathematics education depend on our aims of the curriculum. Paul Ernest raised important questions and issues to consider in the preface section of the book *Rethinking the Mathematics Curriculum* (Hoyles et al., 1999):

- Is it for preparing skilled workers, for fostering critical citizenship in democratic societies, or for other purposes?
- How should mathematics, mathematics teaching and mathematics teacher education respond to these changes?

Mathematics is distinguished by coherence and consensus (Steinbring, 2005). Mathematicians build on each other’s work using results and techniques and following up variations and generalizations (Ernest, 1994).

In mathematics education, it is much harder to find any firm ground on which to stand. Mathematics education, unlike ‘pure’ disciplines in the sciences, is heavily influenced by cultural, social, and political forces. Hence, specialties in mathematics education need to be made up of diverse areas. Table 1 shows the researcher's proposed courses and contents for specialties in mathematics education.

Table 1

Proposed Courses and Contents for specialties in Mathematics Education

Mathematics Education Courses	Contents
Philosophy of Mathematics Education	Epistemology in Mathematics and Mathematics Education
	Educational Values
	Philosophy of Mathematics; General Philosophy
Ethnomathematics	Set of Moral Values
	Mathematics and Culture
	Theory of the Society
Psychology of Mathematics Education	Theory of the Child
	Occupational consultancy
	Becoming a Mathematician
Mathematics Curriculum	Aims of Mathematics Education
	Course and Program Design and Development
	Educational Quality and Standard
Mathematicology *(a research where mathematics is the object of the study)*	Theory of School Mathematics Knowledge
	Conceptual Understanding and Misconception
	Fundamentals of Mathematics
Mathematics Pedagogy	Theory of Learning Mathematics
	Theory of Teaching Mathematics
	Theory of Assessment of Mathematics Learning
Technology in Mathematics Education	Theory of Resources for Mathematics Education
	Matlab, Mathematica, Sketch pad softwares
	Laboratories in Mathematics Education
Mathematical Discourse and Communication	Theory of Mathematics Ability
	Leadership in Mathematics Olympiads, Clubs, Forums & Associations
Equity in Mathematics Education	Theory of Social Diversity in Mathematics Education
	Inclusive Mathematics Education
	Mathematics and Gender
Research in Mathematics Education	Mixed Research Methods
	Action Research

If one looks at the profile of a mathematics education professor or researcher, one might get him/her being expert in teaching-learning and teacher education. Sriraman & English (2010) credited Lincoln and Guba for their idea that mathematics education needs to clarify for itself the following questions:

(1) What is reality? Or what is the nature of the world around us?
(2) How do we go about knowing the world around us?
(3) How can we be certain in the "truth" of what we know?

Besides, Sriraman & English adopted eight criteria that models and theories in mathematics education ought to satisfy: descriptive power, explanatory power, scope, predictive power, rigor and specificity, falsifiability, replicability, and multiple sources of evidence ("triangulation").

Conclusion

The status of mathematics education with respect to the proper mathematics is at its infancy stage in Ethiopian context. Implicitly or explicitly, it serves for the continuation and promotion of the mathematics subject. Mathematics education has diverging theoretical perspectives with multiple and widely diverging views. There are arguments about the need to develop own philosophies and theories or to borrow or adapt from other disciplines. The development of grand philosophy (and theory) of mathematics education could be useful in providing warrants for the field's identity and intellectual autonomy within apparently broader fields such as education, psychology, or mathematics. Since the field is situated at the nexus of two fields of inquiry, mathematics and education, mathematics and education philosophies and epistemologies surely imply to mathematics education. Therefore, adapted theories such as semiotics and mathematical discourse could be sufficient. Still, theorizing and philosophizing should be opportunities for mathematics education researchers; it is a good opportunity for frontiers to conduct research and play lots of roles.

References

Barrow, B. & Woods, R. (2006). *An Introduction to philosophy of education.* (4th ed). London: Routledge.

Bostock, D. (2009). *Philosophy of mathematics: An introduction*. London: Wiley- Blackwell.

Ernest, P. (2004). What is Philosophy of Mathematics Education? *Philosophy of Mathematics Education Journal*, 18 http://people.exeter.ac.uk/PErnest/pome18/PhoM_%20for_ICME_04.h

Ernest, P. (1991). *The Philosophy of mathematics education*. New York: The Falmer Press.

Jacquette, D. (Ed) (2002). *Philosophy of mathematics: An anthology*. Malden: Blackwell Publishers

Gates, P. & Jorgensen, R. (Eds.) (2014). Shifts in the field of mathematics education: Stephen Lerman and the turn to the social. Singapore: Springer Science + Business Media.

George, A. & Velleman, D.J. (2002). *Philosophies of mathematics*. London: Blackwell Publishers Inc.

Gill, J.H. (1993). *Learning to learn: Toward a philosophy of education*. New Jersey: Humanities Press International, Inc.

Holmes, M.H. (2009). *Introduction to the foundations of applied mathematics*. New York: Springer.

Houston, K. (2009). *How to think like a mathematician: A companion to undergraduate mathematics.* New York: Cambridge University Press.

Hoyles , C., Morgan, C., & Woodhouse, G. (Eds.) (1999). *Rethinking the mathematics curriculum*. London: Falmer Press.

Isoda et al. (Eds.) (2007). Japanese lesson study in mathematics: Its impact, diversity and potential for educational improvement. Singapore: World Scientific Publishing Co. P. Ltd.

Kilpatrick, J. et al (Eds.) (2005). Meaning in mathematics education, mathematics education library (Vol. 37). Springer Science + Business, USA.

Oliva, P.F. (2009). Developing the Curriculum (7th ed). Pearson Education, Inc., USA.

Ornstein, A. & Hunkins, F. (2004). *Curriculum: Foundations, principles & Issues.* (4th Ed). Boston: Pearson Education, Inc.

Ostermann, A. & Wanner, G. (2012). *Geometry by its history*. Berlin: Springer-Verlag.

Paulos, J.A. (1980). *Mathematics and humor*. Chicago: The University of Chicago Press.

Prediger, S. (2007). Philosophical reflections in mathematics classrooms. In K. François & P.J. Van Bendegem (Eds.). *Philosophical dimensions in mathematics education* (pp.43-58). Springer US.

Ruelle, D. (2007). *The mathematician's brain*. New Jersey: Princeton University Press.

Schim, M. (Ed.) (1998). *The philosophy of mathematics today*. New York: Oxford University Press.

Schleicher, D. & Lackmann, M. (Eds.) (2011). *An invitation to mathematics: From competitions to research*. Berlin: Springer-Verlag.

Sierpinska, A. & Kilpatrick, J. (Eds.). (1998). *Mathematics education as a research domain: A search for identity*. Dordrecht: Kluwer Academic Publishers.

Sierpinska, A., & Lerman, S. (1996). Epistemologies of mathematics and of mathematics education. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 827–876). Dordrecht: Kluwer Academic Publishers.

Sriraman B., & English L. (2010). *Theories of mathematics education: Seeking new frontiers.* Berlin: Springer-Verlag.

Steinbring, H. (2005). The construction of new mathematical knowledge in classroom interaction: An epistemological perspective. *Mathematics Education Library*, 38.