Exploring and fostering Second Graders' mathematical patterning

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Abstract

A teaching experiment on patterning was conducted with grade two pupils using pattering activities to identify their abilities and difficulties, and to develop their thinking as much as possible. Thus, the objective was to explore and foster pupils' mathematical patterning: creating and extending patterns, identifying units of repeat and its length, and predicting terms of given patterns. The teaching experiment was conducted at Dona Berber Primary School found in Bahir Dar City Administration. Twenty-five students (from seven to thirteen years of age) who got parental permission to come to school in the opposite school shifts were taken as participants of the study. These pupils sat in groups of five and each group was provided with matchsticks, beads with different sizes, bottle tops, and counters with different shapes and colors. The teaching experiment was conducted in two periods each lasting two and half an hour. Data were gathered using task-based group interviews, field notes, observations, and pupils' sample works that were captured through mobile camera. Although students indicated that they had learned patterning in their previous grade, they constructed only constant patterns. However, after provided with an analogy for repeating patterns, they created many and even complex repeating patterns. They could easily extend given patterns and determine the unit of repeat and its length. However, predicting terms was difficult for them except for patterns with length of core unit two.

Keywords: Core Unit, Mathematics, Patterning

Background of the Study

Pattern refers to "any replicable regularity" (Papic et al., 2011, p. 235). Papic et al. (2011) have indicated that basically there are two types of patterns: repeating and growing patterns. Repeating patterns are described as having a cyclic structure that can be generated by the repeated application of a smaller portion of the pattern whereas growing patterns consist of sequences of elements that increase (or decrease) systematically. Repeating patterns contain a segment that continuously recurs where the segment can vary in size and level of complexity, but the simplest includes just two items (Taylor-Cox, 2003). The segment of a repeating pattern that continuously repeats itself is called cycle of elements or unit of repeat (Zazkis & Liljedahlp, 2002). For example, in the pattern ABABABAB, AB is the element that repeats itself four times. Here, the element of the pattern is made of letters A and B. However, it can consist of number, geometric figures, other symbols or a mix of these. The number of the symbols, figures or a mix of them in a unit of repeat is called length. Thus, in the above pattern, AB is a unit of repeat with length 2. In the complex pattern A®□µA®□µA®□µ, A®□µ is a unit of repeat with length four.

According to Rivera (2013), mathematics deals with searching or seeking for patterns (regularities) and making generalizations about them. Similarly, Van de Walle, Karp and Bay-Williams (2010) indicated that "finding and exploring regularity or order, and then making sense of it, is what doing mathematics in the real world is all about" (p.38-39). To advance students in the field of mathematics, Van de Walle et al. (2010) also suggested that it requires engaging learners in doing it themselves which includes searching for patterns.

Bay-Williams (2001) established that children's explorations with patterns and their expressions of generalization can lead to algebraic reasoning. The ability to recognize patterns is fundamental to the development of algebraic thinking and consequently to the processes and knowledge of mathematics (Warren & Cooper, 2008; Mulligan & Mitchelmore, 2009; Mulligan et al., 2005; Papic, 2007). For instance, Mulligan et al. (2005) indicated that early patterning experiences assist students to engage in considering the structure of mathematics. Thus, "patterns serve as the cornerstone of algebraic thinking" (Taylor-Cox, 2003, p.15). Moreover, Zazkis and Liljedahlp (2002) asserted that "patterns are the heart and soul of mathematics" (p.379).

Pattern is one of the major mathematical themes that allows students to investigate their initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. In the primary grades, students learn to identify patterns in shapes, designs, and in sets of numbers. They study both repeating and growing patterns and develop ways to extend them. Students use concrete materials and pictorial displays to create patterns and recognize relationships (NCTM, 2000). Through the observation of different representations of a pattern, students begin to identify some of the properties of the pattern. Thus, primary school patterning tasks should have structures that enable children to extend the stages and predict outcomes in a convenient manner (Rivera, 2013).

Statement of the Problem

As indicated in the Principles and Standards for School Mathematics (NCTM, 2000), "systematic experience with patterns can build up to an understanding of the idea of function, and experience with numbers and their properties lays a foundation for later work with symbols and algebraic expressions" (p. 37). That is, the study of patterns has embedded algebraic reasoning in the elementary grades (Blaton & Kaput, 2005). As a result, Van de Walle et al. (2010) indicated that there is a general agreement that the development of algebraic thinking must begin "from the very beginning of school so that students will learn to think productively with the powerful ideas of mathematics" (p.254). According to Rivera (2013), around age 6, pupils begin to interpret, recognize, and construct a core unit of repeat for a given pattern, which also enable them in some cases to create patterns of their own choice; at age 7, they are able to describe figural growth patterns numerically and translate between their figural and numerical representations.

Generally, children from prekindergarten to grade two are expected to (a) sort, classify, and order objects by size, number, color and other properties; (b) recognize, describe, and extend patterns; and (c) analyze how patterns are generated (NCTM, 2000). Moreover, Bay-Williams (2001) indicated that "first graders enjoy working with patters" (p.196). He also pointed out that "students in primary grades must learn to identify the start and end of the unit of a pattern and be aware that they may need to see many numbers before they can conclusively determine a pattern" (p.196). Similarly, Warren and Cooper (2005, 2008) have shown that 8-year-old children in grade 2 can successfully learn functional thinking through the analysis of growing patterns.

Mathematical patterning has not previously been central to mathematics curricula. Patterns as separate topics using figures or shapes as visual or geometric representations are included only in the current grade 1 and 2 mathematics curricula. Still in these curricula, patterning activities are provided only on two pages of the two grades textbooks by the end of the learning units. Although such inclusion reflects the intention of the curriculum developers fostering students' algebraic thinking at the early ages, the patterning contents and activities displayed as only linear repeating patterns are not enough even to understand patterns.

Thus, there is a need to include enough activities so that students could create their own patterns, learn about the core elements, and predict both near and far terms. Although the syllabi recommend the use of manipulatives for students, being in groups, to play pattern creation games, the textbooks provide and require students just to simply extend given patterns or complete missing terms or elements in given sequences. Students are not required to create and inquire with their own patterns. As such, there is a need to support students so that they can develop patterning activities to the maximum. Thus, this teaching experiment is guided by the following research questions. How do grade two students develop repeating patterns? To what extent do grade two students progress in their mathematical patterning with teacher scaffolding?

Method

Design

The design of the study was a teaching experiment, a special type of design in which the researcher takes the role of the teacher. This design is preferred for investigating pupils' mathematical patterning and their development in learning.

This teaching experiment was conducted in Dona Berber Primary School, an urban school in Bahir Dar City. The purpose for selecting the school was that there had already been a relationship established with the school since the researcher was conducting a project work to explore and foster pupils' and teachers' mathematical learning. The school was selected because of convenience for its proximity to the researcher. In this school, on one hand the number of students in a class is large, and on the other hand the teacher needed her scheduled class. Thus, 25 students (from seven to thirteen years old) who got permission from parents took part in the experiment in the opposite school shifts.

Instrument

Children were provided with a variety of concrete objects such as matchsticks, beads, counters and bottle tops. Data were collected using different tools such as task-based group interviews, observations with field notes, and pupils' sample patterns captured through camera. As an observer-as participant, the researcher recorded the pupils' group activities and their exchange of ideas, and his scaffoldings, reflections and interactions with the pupils using field notes. Photographs (visual images of geometric patterns constructed by groups of children) were used to supplement field notes and prompt the observations.

Data Analysis Techniques

A narrative discourse was used to analyze data gathered through the observations. A narrative text was constructed from field notes taken during observations of the classroom interactions (group discussions and the whole class discourses). Meanings were explored from constructed texts and the photographs of students' patterns were used to remind meaning.

Data Gathering Procedures

The classroom culture is important to develop algebraic thinking. Regarding this, Holmes (1995, p.1) suggested that "instruction should help children construct knowledge by experiencing, investigating, solving problems and thinking critically. Learners should often work in cooperative groups to exchange ideas and learn from one another". As for Holmes (1995), in order to understand mathematics, learners must induce patterns from examples, see relations and connections, and draw conclusions. Moreover, Kaput et al. (2009) indicated that "students learn most effectively when they engage in worthwhile tasks and are allowed to explore, discuss, and unpack the mathematics in each task with support from their teacher" (p.446).

Thus, taking the position that learning is an investigative process, for students to develop and create their own knowledge, children participated in a series of activities which aimed to provide them with varying opportunities to learn. Pupils were encouraged to confront, construct and develop new knowledge by actively taking part in the teaching and learning process through social interactions. Thus, a set of teaching in small groups engaging students in creating and extending patterns, identifying the core units and their lengths, and predicting terms of sequences was the theoretical framework in this exploration.

Since grade 2 students are at concrete operational stage, the investigation started with classifying, comparing and ordering-using junk materials, mixing different objects (matchsticks, counters, bottle tops, and beads), which mark children's number readiness. That is, repeating patterns are created using counters, beads, matchsticks and bottle tops.

Informed Consent

First of all, the children were informed and got permission from their parents. Moreover, to gather data for the final analysis, the teaching experiment was started by telling the participant pupils that their responses would be used only for research purposes, and, thus, they had to inform and get permission from their parents. Being informed that their classroom activities, reflections, presentations and group interviews would be collected for analysis by the researcher-researcher through field notes, and photos, the participant pupils got permission from parents to participate in the teaching experiment and consequently in the research.

Results

Patterning starts from categorizing and classifying concrete objects. Thus, children should be provided with numerous opportunities to sort, classify, and describe collections of many different types of objects. Consequently, the patterning activities were started with sorting the varieties of concrete objects. Almost all of them put beads and matchsticks together, and the bottle tops and the counters separately on different places, but they could not tell why they classified the objects like that. After illustration with an example, students understood that they could classify on the basis of color and size though they could not list all possible criteria such as shape and the uses of objects.

After whole class discussion about classification and sorting, the researcher requested students to make patterns using the concrete objects. One group piled up and another group horizontally ordered similar objects together. That is, out of five groups, two constructed constant linear patterns. However, they could create neither repeating nor growing patterns. After they were provided and inquired with an analogy and discussed about the differing attributes that could help to form repeating patterns, one group created six different repeating patterns with a unit of repeat length two and another group formed two complex repeating patterns.

The rest two groups made errors. But with the teacher-researcher’s scaffolding, they corrected their mistakes on their own and created repeating patterns. The fifth group created one correct alternating pattern using the color attribute ignoring shape (white green) having five elements and were unable to settle their argument about a second unfinished pattern using bottle tops and beads. One student was arguing against the rest four about constructing correct but different patterns regardless of their failure of listening to and convincing each other. Concerning prediction, they could predict near elements either by extending or counting. However, it was difficult for them to predict far elements of repeating patterns.

Discussion

The first objective provided in the syllabus is about sorting. It suggests the manipulatives to be used and how to carry on the classroom instruction. As children look for criteria for sorting and classifying objects, they think analytically. While sorting, students understand what is similar and what is different about the things to be sorted. And, identifying what is similar (invariant) and different (changing) helps students to recognize patterns and make generalizations. Consequently, the teaching experiment about patterns started with sorting activities.

Sorting Concrete Objects

Students were requested to sit in groups of five and each group was provided with the concrete objects. They were asked what the objects are, and they answered in their local language that those were '**ማስያ** which means calculating objects or manipulatives. Then, they were asked to sort (classify into types) the concrete objects using their own criteria of sorting. Almost all of them put beads and matchsticks together and the bottle tops and the counters separately on different places, but they could not tell why they did so. Though they were not able to explain, their classification was on the bases of materials which the objects were made of (the beads and matchsticks were made of wood, the counters from plastic and the bottle tops from tin) as shown by Figure 1.

Figure 1. Concrete objects sorted by students.

They were encouraged and given time for group discussion to classify based on other properties or attributes, yet they could not. To help them consider other attributes for classification, the researcher-teacher gave them an analogy. He said to them, "*suppose I was requested to classify this class of students, that is you; a criterion to do so may be sex. I can classify you into two groups as males and females. I can also assume another criterion such as age. As you told me at the beginning, your age ranges from seven to thirteen years. Thus, I can classify you into seven groups where seven year olds form one group; eight year olds form another group and so on up to age 13. Based on this example, try to think of other criteria to classify the objects given to you.*" After this whole class discussion, students worked again in their small groups. This time, students understood that they could classify the objects based on color and size although they could not list all possible criteria such as shape and the uses of objects.

Figure 2. Constant patterns created by

Creating, Extending and Predicting Repeating Patterns

After discussing the criteria of classification (based on what is similar and different about the objects),

the researcher-teacher asked students what they had learned about patterns (called 'ድርድር' in grade 1 textbook using the local language). Some of the students said they had learned about it while others never remembered whether they had learned about it or not. After they debated, the students were requested to make patterns using the concrete objects anyway. One of the groups piled up and another group horizontally ordered similar objects together. Such pile ups or horizontally lined objects with only a single attribute are considered patters in mathematics. White and black strips on zebras are patterns, and there are similar natural and manmade patterns. Rivera (2013) called these kinds of mathematical patterns constant patterns. On the other hand, the rest three groups were still trying to sort out not to form patterns. While they were constructing patterns, the researcher-teacher went round to observe and learned that the beads were not good to work with on tables since they were being easily scrolled down and were difficult to pile up.

After encouraging students to create patterns by imagining the decorations on cloths or houses, or some designs on ceilings, one group formed a human picture while another fashioned a flower (see Fig.3). The group that fashioned a flower also drew a flower on a piece of paper and put it beside their pattern to show that what they fashioned was a flower. However, a pattern is a series of things not just one object (one element, term, or stage). This is because, let alone with only one object, even with more objects, it will not be predictable unless more elements are added.

Figure 3. A human picture and a flower designed by students.

As such, students' designs were neither repeating nor growing ones. When asked to add the next stage so as to make it a pattern, they simply repeated the first, and thus there was no difference between consecutive terms; they created constant patterns as before. The researcher-teacher encouraged them to keep something similar and something different at the same time between terms in their sequence. The intention was if they could make the length of the core element more than one to create repeating patterns. When any one of the groups could not create repeating patterns with more than one length, the

Figure 4. A male-female pattern

researcher-teacher asked two groups to come out and line up and created the pattern shown in Figure 4. Then, he asked the rest of the groups to discuss what pattern they could observe from the queue of students. One group said that the line-up was made based on heights, but the other two groups asked how. They convinced each other that the queue did not make a pattern based on heights. Later on, a girl called Amani answered that the queue was formed as male-female, male-female pattern and the remaining students soon agreed with her.

Now the researcher-teacher discussed that patterns have definite regularities which can be predicted. He explained that it could be predicted who would be next to the boy at the back or at the front: a male or female. Since males and females were sequenced in an alternative position, a boy would be followed by a girl if we had to maintain the regularity. The researcher asked students about the decorations on cloths, bed sheets, floors, and on many other things. He extended the discussion that they could create so many types of patterns based on different attributes (color, shape, size, type of object). After that they were directed to create different patterns in their groups. Thereafter, they created many patterns including complex ones.

One group constructed the pattern shown in Figure 5. It is somehow circular with green and white counters using beads around the center as decorations. Two of the green elements were made from double counters. Students were asked whether they had constructed a pattern with predictable elements. They soon picked up the extra green counters from each of the two elements and said now it was a correct pattern. When the group members were asked why they liked that they replied that they knew they did not have to use double green counters, but they did since they had more green counters and thought that a counter should not be left out. They believed that they had to use all the counters. The researcher teacher asked them if they had to use all the objects and why they used only some of the beads at the center. The students giggled being embarrassed and the researcher-teacher passed to the next group telling them to continue to create a different pattern. Including extra green counters was not an essential mistake since the group modified as prompted by the researcher-teacher.

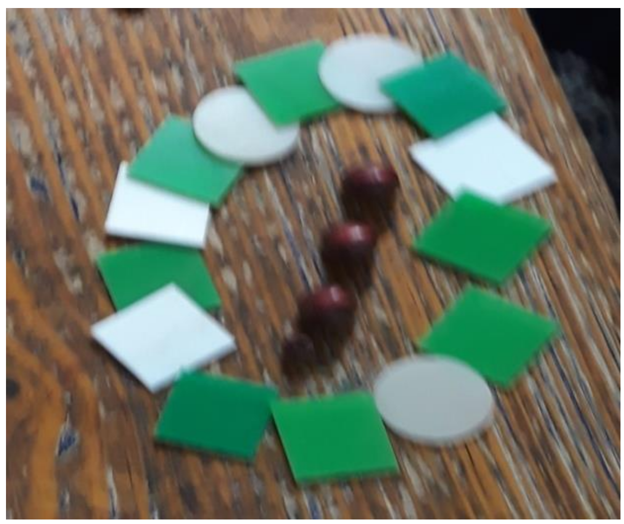

Figure 5. A green-white pattern of counters

Another group formed a complex pattern using beads with different sizes and matchsticks. It is a kind of repeating pattern using one big and two small beads as core units (repeating units), but the beginning and last terms overturned the regularities. They used one small bead on one end and two bigger beads on the other end. Moreover, the big matchstick-inserted beads placed in front were in a haphazard or random way, not in a predictable regularity.

The researcher-teacher asked students if they could improve to make a regular arrangement and as a consequence they took away the small bead at one end and the two bigger beads on the other end as well as the matchstick-inserted beads at the front. The researcher-teacher advised them to order in a predictable manner than avoid the matchstick-inserted beads. They could, for example, place the matchstick-inserted beads exactly in front of the big beads without inserted matchsticks or in front of one of the smaller beads. They could also place the stick-inserted beads parallel to the middle of the small and big beads or the middle of the two small beads (See Figure 6).

Figure 6. A complex pattern of beads and matchstick-inserted beads

The third group constructed correct repeating patterns considering four attributes. As shown in Figure 7 below, students constructed six patterns; three based on the materials from which the concrete objects were made of (bottle top and matchstick, bead and counter, and bottle top and bead), one based on color (using white and green counters), one based on size (big and small beads), and one more based on both shape (circle and rectangle) and color (green and white). They also correctly explained how they created those patterns.

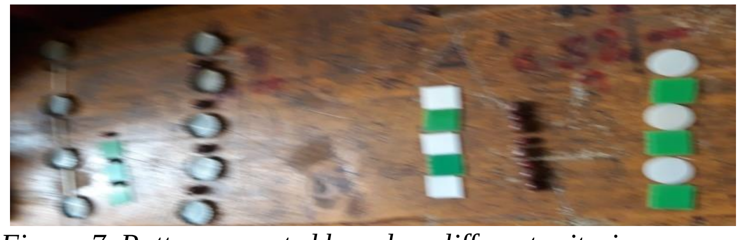

Figure 7. Patterns created based on different criteria.

The fourth group constructed two patterns: one using the color attribute (white and green counters) and the second using bottle tops and beads. They disregarded shape and considered only color. The group members were debating concerning their second pattern. One of them, Workineh, was thinking about his own pattern and the rest four members were thinking about another pattern. They were right except failing to listen and understand one another's ideas. They disagreed on considering size and number of the beads as attributes of their pattern. Workineh was arguing that they had to disregard the size of beads and use the whole what they made as a core unit and repeat it over and over again. Workineh explained that he wanted to construct a pattern with a core unit of length 16 (one bottle top four beads one bottle top four beads one bottle top two beads one bottle top two beads). He also explained that the sizes of the beads does not matter, he took all just as beads. From his explanation, it was clear that he understood the concept of unit of repeat.

On the other hand, the other four group members were arguing about using two beads constantly throughout the pattern instead of using two and four alternatively. They also believed that the two beads had to be constantly small ones or bigger ones. They considered size as an attribute. They asked the researcher-teacher to confirm who was right and who was wrong. However, he asked them to justify their thinking turn by turn and saw if it worked. Workineh soon agreed with others' idea and explained that the other members' argument was correct since they could use either one bottle top and four bigger beads or one bottle top and two smaller beads as core units. On the other hand, the four students of that group struggled to understand Workineh's idea of using the whole as a complex element of a repeating pattern. The researcher encouraged Workineh to explain his thinking or do anything and convince his groupmates that his beginning was a repeating pattern.

Figure 8. Above, a pattern of white-green counters; below, a pattern with a core unit of length 16

The researcher-teacher made visible Workineh's unfinished pattern to the whole class and requested Workineh to explain his idea. After Workineh's explanation, the researcher-teacher gave time for the groups to discuss about it. However, they could not understand it. He was expected to extend his pattern to make more visible, but he focused only on explaining.

Lastly, the researcher-teacher extended the pattern and showed the elements. Even then, some students were not sure whether it was a proper pattern.

The last group (group 5) did two patterns. Unlike the other groups, students of this group were working in two smaller groups. They created complex patterns with different core units and decorations. The pattern on the left-hand side of Figure 9 is a complex pattern with 2 units of core unit and decorations. They used bottle tops at the bottom, counters with alternating colors at the middle and beads of alternating sizes at the top. On the other hand, the pattern in the right hand side is a complex pattern with a core unit of length 4 and a kind of circuit design.

Figure 9. Left, a complex pattern with a core unit of two; right, complex pattern with a core unit of four.

In the first episode, students created repeating patterns after an example was illustrated to them. In the next period, a short revision was done about sorting and repeating patterns. Then, pupils were asked to extend repeating patterns, and they easily identified what came next. The researcher-teacher formed a linear alternating pattern using white and red counters on the front desk and wrote WRWRWR on the chalkboard as the symbolic representation of the counters. He asked them what came next to the right. They shouted W, and as they were asked to justify, they again shouted that the pattern was white red white red and therefore white should come next to red.

Figure 10. WRWR pattern

He then asked students what is similar (remain invariant) and what is different (changes) in the given pattern. As they became confused to tell what was similar and different, he asked them to discuss in their groups and identify an element that repeated itself over and over again. They recognized WR as an element that constantly repeated itself. The researcher told them that WR is called the core unit or the unit of repeat, and since it is made of two symbols to represent the two white and red counters, it has a length of two. Then they were asked what they really did while extending patterns at the beginning. Thus, they came to understand that as they were extending patterns they were just repeating the core units. That is, students understood the core units of repeating patterns and how to repeat those core units over and over again to extend the pattern. Then, students were provided with repeating patterns having different lengths of the core units and asked to determine the core units and their lengths. They never took time to think or discuss but raised hands to tell the unit of repeat and its length; they successfully answered.

Then the researcher-teacher cleaned the board and wrote the pattern WRWRWR again, and asked students to predict what would be the 10th letter, the 50th letter and the 100th letter. Some were extending the pattern extensively to go up to 50th and 100th terms, but the researcher-teacher interrupted and advised students to try it without extending. As they became challenged, the researcher-teacher asked them to discuss in their groups and discover other strategies. Then, two groups started skip counting as the first is white, the third is white and so on. When advised to predict without exhaustive skip counting, the two groups soon understood that W and R were represented by odd and even numbers respectively (although they confused even numbers as whole numbers in the local language), and they found out that the 50th and 100th letters are R. They represented the letters W and R by numbers based on their positions in the series.

After a whole class discussion, by allowing the two groups to justify their solutions, pupils were given the pattern GYRGYRGYR. The researcher-teacher described the sequence of letters as a representation of green, yellow and red. He then asked them to predict the 50th, 109th, and 1005th terms. All the groups started to give position numbers and they were extending the pattern to arrive at the required terms. However, the researcher-teachers informed them that the extending strategy would not make them successful since they could be asked very far away terms like the millionth. Then, they stopped extending and discussed for longer time, but no group could successfully predict it. Some pupils said the 50th term is G, others argue it is Y and still some others claim it to be R, but none of them could justify their answer. Since predicting far elements without extending requires deriving rules, it was difficult for them.

Papic et al. (2011) conducted a research on preschoolers patterning. They selected two comparable preschools in student enrollments (38 children in each preschool), staffing and resources, and one preschool was assigned as the experimental intervention preschool and the other as the comparison nonintervention preschool. Three assessments conducted during the intervention indicated that all children in the intervention group identified the unit of repeat independently of the number, type, and complexity of items and of attributes such as size, shape, dimension, and orientation and most used the unit of repeat to extend the pattern and complete more complex tasks. Similarly, Rivera (2013) indicated that, around age 6, pupils begin to interpret, recognize, and construct a core unit of repeat for a given pattern, which also enable them in some cases create patterns of their own choice. Thus, the current findings are consistent with former research reports.

Conclusions and Implications

With teacher scaffolding, grade two pupils can create varying repeating patterns including complex ones, extend and predict near terms easily. Predicting far elements of repeating patterns with more than two length of unit of repeat is difficult for them. Although integrating mathematical patterns into the early grades is fundamental for developing algebraic thinking, to provide more patterning activities in grade two and next classes, it requires teacher's scaffolding. Consequently, teacher professional development should consider teachers' understanding of mathematical pattering.

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