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SOLUTION OF THIRD ORDER NONLINEAR NON HOMOGENEOUS FUZZY ORDINARY DIFFERENTIAL EQUATIONS USING FUZZY LAPLACE TRANSFORM METHOD

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ABSTRACT
In this study, we will consider nonlinear non homogeneous third order fuzzy ordinary differential equation and solve using fuzzy Laplace transform under the generalized Hukuhara differentiability concept. The results of the solution of non-linear nonhomogeneous third order FODE is obtain using FLT by consider the equation $y''' + yy' = k\,y + y_c$ ***with*** $y(t) \neq 0$ ***. The results obtained is tested on the existing growth model in the literature using*** C^{++} ***code.***
Keywords: Nonlinear Non Homogeneous, Fuzzy Ordinary Differential equations, Fuzzy Laplace Transform,

INTRODUCTION

In recent years, the theory of FDEs has attracted widespread attention and has been rapidly growing. It was massively studied by several authors (Abbasbandy, et al. 2004). Allahviranloo et al. (2007). A novel method for solving fuzzy nonlinear differential equations, which its construction based on the equivalent integral forms of original problems under the assumption of strongly generalized differentiability by fuzzy Laplace transform was considered by many Authors such as Melliani et al. (2015), Nieto et al. (2006) and Regan et al. (2003).

The approximate solution of first order nonlinear fuzzy initial value problems (FIVP) was considered by formulating and analyzing the use of the Optimal Homotopy Asymptotic Method (OHAM). OHAM allow the solution of the FDE to be calculated in the form of an infinite series in which the components was easily computed. The method provides a convenient way to control the convergence of approximation series. Numerical examples using the well-known nonlinear FIVP were presented to show the capability of the method in that regard and the results satisfied the convex triangular fuzzy number. Jameel (2018)

FDEs for nonlinear system modeling with bernstein neural networks with fuzzy set theory was considered. The uncertainty of nonlinear systems was modeled using FDEs. The solutions of those equations was obtained. Moreover, the first transform fuzzy differential equations into four ODEs, was constructed by neural models with the structure of those equations. Theory analysis and simulation results showed that the new models were effective for modeling uncertain in nonlinear systems. Raheleh, et al. (2017)

This study therefore will consider Nonlinear non homogeneous third order fuzzy ordinary differential equation which will be solve using fuzzy Laplace transform method under the generalized Hukuhara differentiability concept and the results obtained will be tested on existing growth model presented in Sankar and Tapan (2013) and also a new C^{++} code will be developed to implement the result.

Methods

Results of the solution of non linear nonhomogeneous third order FODE by FLT. Consider equation below with $y(t) \neq 0$

$$y''' + yy' = k\,y + y_c \tag{1}$$

Taking the Laplace transform of equation (1) and simplify the equation we have

$$L\left[y'''\right] + yL\left[y'\right] = k\,L\left[y\right] + L\left[y_c\right] \tag{2}$$

$$s^3L\left[y(t,\alpha)\right]-s^2x_0(\alpha)-sy_0(\alpha)-z_0(\alpha)+\frac{s}{p^2}L\left[y(t,\alpha)\right]-\frac{1}{p^2}x_0(\alpha)=\frac{k}{s^2}+\frac{1}{p^2} \quad (3)$$

Equating and simplify equations (2) and (3) we have

$$s^3L\left[y(t,\alpha)\right]-s^2x_0(\alpha)-sy_0(\alpha)-z_0(\alpha)+\frac{s}{p^2}L\left[y(t,\alpha)\right]-\frac{1}{p^2}x_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}= \\ s^2\left(a+bs\right)L\left[y(t,\alpha)\right]-s(a+s)x_0(\alpha)-(a+s)y_0(\alpha)-z_0(\alpha) \quad (4)$$

From equation (4)

$$\left(s^3+\frac{s}{p^2}\right)L\left[y(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)x_0(\alpha)-sy_0(\alpha)-z_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}= \\ s^2\left(a+bs\right)L\left[y(t,\alpha)\right]-s(a+s)x_0(\alpha)-(a+s)y_0(\alpha)-z_0(\alpha) \quad (5)$$

Results

Now, applying the following cases to equation (5) we have

Case 1. $a\geq 0$ and $b\geq 0$. Now, applying case 1 to equation (5), we have

$$\left(s^3+\frac{s}{p^2}\right)L\left[\underline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\underline{x}_0(\alpha)-s\underline{y}_0(\alpha)-\underline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}= \\ s^2\left(a+bs\right)L\left[\underline{y}(t,\alpha)\right]-s(a+s)\underline{x}_0(\alpha)-(a+s)\underline{y}_0(\alpha)-\underline{z}_0(\alpha) \quad (6)$$

and

$$\left(s^3+\frac{s}{p^2}\right)L\left[\overline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\overline{x}_0(\alpha)-s\overline{y}_0(\alpha)-\overline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}= \\ s^2\left(a+bs\right)L\left[\overline{y}(t,\alpha)\right]-s(a+s)\overline{x}_0(\alpha)-(a+s)\overline{y}_0(\alpha)-\overline{z}_0(\alpha) \quad (7)$$

Solving equation (6) we have

$$L\left[\underline{y}(t,\alpha)\right]=\frac{\left[\left(s-s\left(a+s\right)+\frac{1}{p^2}\right)\right]\underline{x}_0(\alpha)+\left[s-(a+s)\right]\underline{y}_0(\alpha)+\frac{k}{s^2}+\frac{1}{p^2}}{\left(s^3+\frac{s}{p^2}\right)-s^2(a+bs)} \quad (8)$$

Taking the inverse Laplace transform of equation (8) we have

$$L^{-1}\left[\underline{y}(t,\alpha)\right]=\underline{x}_0\frac{t}{b-1}-\frac{1}{a^2}t(k-bk)-\frac{1}{a^3p^2}(-\underline{y}_0a^3p^2+a^2+kb^2p^2-2kbp^2+kp^2) \\ -\frac{1}{2a}kt^2-\frac{e^{-a\frac{t}{b-1}}}{(a^3p^2-a^3bp^2)(b-1)}(kp^2-2a^2b+a^2b^2+a^3p^2+a^2-4bkp^2-a^3bp^2 \\ +6b^2kp^2-4b^3kp^2+b^4kp^2-2a^4p^2\underline{x}_0-a^3p^2\underline{y}_0-a^3b^2p^2\underline{y}_0+a^4bp^2\underline{x}_0+2a^3bp^2\underline{y}_0) \quad (9)$$

Also, solving equation (7) we have

$$L\left[\overline{y}(t,\alpha)\right]=\frac{\left[\left(s-s\left(a+s\right)+\frac{1}{p^2}\right)\right]\overline{x}_0(\alpha)+\left[s-(a+s)\right]\overline{y}_0(\alpha)+\frac{k}{s^2}+\frac{1}{p^2}}{\left(s^3+\frac{s}{p^2}\right)-s^2(a+bs)} \quad (10)$$

Taking the inverse Laplace transform of equation (10) we have

$$L^{-1}\left[\overline{y}(t,\alpha)\right]=\overline{x}_0\,\frac{t}{b-1}-\frac{1}{a^2}t(k-bk)-\frac{1}{a^3p^2}(-\overline{y}_0a^3p^2+a^2+kb^2p^2-2kbp^2+kp^2)$$

$$-\frac{1}{2a}kt^2-\frac{e^{-a\frac{t}{b-1}}}{(a^3p^2-a^3bp^2)(b-1)}(kp^2-2a^2b+a^2b^2+a^3p^2+a^2-4bkp^2-a^3bp^2 \tag{11}$$

$$+6b^2kp^2-4b^3kp^2+b^4kp^2-2a^4p^2\overline{x}_0-a^3p^2\overline{y}_0-a^3b^2p^2\overline{y}_0+a^4bp^2\overline{x}_0+2a^3bp^2\overline{y}_0)$$

Case 2. If $a\geq 0$ and $b<0$. Applying case 2 into equation (5) we have

$$\left(s^3+\frac{s}{p^2}\right)L\left[\overline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\underline{x}_0(\alpha)-s\underline{y}_0(\alpha)-\underline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}=$$

$$s^2\left(a+bs\right)L\left[\underline{y}(t,\alpha)\right]-s(a+s)\underline{x}_0(\alpha)-(a+s)\underline{y}_0(\alpha)-\underline{z}_0(\alpha) \tag{12}$$

and

$$\left(s^3+\frac{s}{p^2}\right)L\left[\underline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\overline{x}_0(\alpha)-s\overline{y}_0(\alpha)-\overline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}=$$

$$s^2\left(a+bs\right)L\left[\overline{y}(t,\alpha)\right]-s(a+s)\overline{x}_0(\alpha)-(a+s)\overline{y}_0(\alpha)-\overline{z}_0(\alpha) \tag{13}$$

From equation (12) we have

$$\left(s^3+\frac{s}{p^2}\right)L\left[\overline{y}(t,\alpha)\right]-s^2\left(a+bs\right)L\left[\underline{y}(t,\alpha)\right]=K_1(t,\alpha) \tag{14}$$

Denote

$$K_1(t,\alpha)=\left(s^3+\frac{1}{p^2}\right)\underline{x}_0(\alpha)-s(a+s)\underline{x}_0(\alpha)+s\underline{y}_0(\alpha)-(a+s)\underline{y}_0(\alpha)+\frac{k}{s^2}+\frac{1}{p^2}$$

Similarly, from equation (13) we have

$$\left(s^3+\frac{s}{p^2}\right)L\left[\underline{y}(t,\alpha)\right]-s^2\left(a+bs\right)L\left[\overline{y}(t,\alpha)\right]=K_2(t,\alpha) \tag{15}$$

Denote

$$K_2(t,\alpha)=\left(s^2+\frac{1}{p^2}\right)\overline{x}_0(\alpha)-s(a+s)\overline{x}_0(\alpha)+s\overline{y}_0(\alpha)-(a+s)\overline{y}_0(\alpha)+\frac{k}{s^2}+\frac{1}{p^2}$$

Now, solving equations (14) and (15) we have

$$L\left[\overline{y}(t,\alpha)\right]=\frac{\left(s^3+\frac{s}{p^2}\right)K_1(t,\alpha)+s^2(a+bs)K_2(t,\alpha)}{\left(s^3+\frac{s}{p^2}\right)^2-s^4(a+bs)^2} \tag{16}$$

Taking the inverse Laplace transform of equation (16) we have

$$L^{-1}\left[\overline{y}(t,\alpha)\right]=$$

$$k_1\left(\frac{1}{p^2s^2-ap^2s^3-bp^2s^4+1}\right)+p^2\left(\frac{s^2}{p^2s^2-ap^2s^3-bp^2s^4+1}\right) \tag{17}$$

$$ap^2k_2\left(\frac{s}{p^2s^2-ap^2s^3-bp^2s^4+1}\right)+bp^2k_2\left(\frac{s^2}{p^2s^2-ap^2s^3-bp^2s^4+1}\right)$$

Similarly, from equations (14) and (15) we have

$$L\left[\underline{y}(t,\alpha)\right]=\frac{s^2(a+bs)K_1(t,\alpha)+\left(s^3+\frac{s}{p^2}\right)K_2(t,\alpha)}{\left(s^3+\frac{s}{p^2}\right)^2-s^4(a+bs)^2} \tag{18}$$

Taking the inverse Laplace transform of equation (18) we have

$$L^{-1}\left[\underline{y}(t,\alpha)\right] =$$

$$k_2\left(\frac{1}{p^2s^2-ap^2s^3-bp^2s^4+1}\right)+p^2\left(\frac{s^2}{p^2s^2-ap^2s^3-bp^2s^4+1}\right) \quad (19)$$

$$ap^2k_1\left(\frac{s}{p^2s^2-ap^2s^3-bp^2s^4+1}\right)+bp^2k_1\left(\frac{s^2}{p^2s^2-ap^2s^3-bp^2s^4+1}\right)$$

Case 3. If $a<0$ and $b\geq 0$. Now, applying case 1 to equation (5), we have

$$\left(s^3+\frac{s}{p^2}\right)L\left[\underline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\underline{x}_0(\alpha)-s\underline{y}_0(\alpha)-\underline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}=$$

$$s^2\left(a+bs\right)L\left[\underline{y}(t,\alpha)\right]-s(a+s)\underline{x}_0(\alpha)-(a+s)\underline{y}_0(\alpha)-\underline{z}_0(\alpha) \quad (20)$$

and

$$\left(s^3+\frac{s}{p^2}\right)L\left[\overline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\overline{x}_0(\alpha)-s\overline{y}_0(\alpha)-\overline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}= \quad (21)$$

$$s^2\left(a+bs\right)L\left[\overline{y}(t,\alpha)\right]-s(a+s)\overline{x}_0(\alpha)-(a+s)\overline{y}_0(\alpha)-\overline{z}_0(\alpha)$$

Now, solving equation (20) we have

$$L\left[\underline{y}(t,\alpha)\right]=\frac{\left[s+\frac{1}{p^2}-s\left(a+s\right)\right]\underline{x}_0(\alpha)-a\underline{y}_0(\alpha)+\frac{k}{s^2}+\frac{1}{p^2}}{\left(s^3+\frac{s}{p^2}\right)-s^2(a+bs)} \quad (22)$$

Taking the inverse Laplace transform of equation (22) we have

$$L\left[\underline{y}(t,\alpha)\right]=akp^5+kp^3t-\exp\left(ap^2\frac{t}{2p^2-2bp^2}\right)$$

$$\cosh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(2p^2-2bp^2)^2}}-\frac{\sinh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(2p^2-2bp^2)^2}}}{\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(2p^2-2bp^2)^2}}} \quad (23)$$

$$\left(\frac{kp^4-bkp^4+ap^2\underline{y}_0-a^2kp^6}{2p^2s-akp^6-p^2\underline{y}_0+bp^2\underline{x}_0+abkp^6}-a\frac{p^2}{2p^2-2bp^2}\right)(ap^3\underline{x}_0-akp^6-p^2\underline{y}_0$$

$$+bp^3\underline{x}_0+abkp^6)$$

Also, solving equation (21) we have

$$L\left[\overline{y}(t,\alpha)\right]=\frac{\left[s+\frac{1}{p^2}-s\left(a+s\right)\right]\overline{x}_0(\alpha)-a\overline{y}_0(\alpha)+\frac{k}{s^2}+\frac{1}{p^2}}{\left(s^3+\frac{s}{p^2}\right)-s^2(a+bs)} \quad (24)$$

Taking the inverse Laplace transform of equation (24) we have

$$L\left[\overline{y}(t,\alpha)\right] = akp^5 + kp^3t - \exp\left(ap^2\frac{t}{2p^2-2bp^2}\right)$$

$$\cosh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(2p^2-2bp^2)^2}} - \frac{\sinh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(2p^2-2bp^2)^2}}}{\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(2p^2-2bp^2)^2}}} \quad (25)$$

$$\left(\frac{kp^4-bkp^4+ap^2\overline{y}_0-a^2kp^6}{2p^2s-akp^6-p^2\overline{y}_0+bp^2\overline{x}_0+abkp^6}-a\frac{p^2}{2p^2-2bp^2}\right)(ap^3\overline{x}_0-akp^6-p^2\overline{y}_0$$

$$+bp^3\overline{x}_0+abkp^6)$$

Case 4. If $a<0$ and $b<0$. Applying case 2 into equation (5) we have

$$\left(s^3+\frac{s}{p^2}\right)L\left[\underline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\overline{x}_0(\alpha)-s\overline{y}_0(\alpha)-\overline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}=$$

$$s^2\left(a+bs\right)L\left[\underline{y}(t,\alpha)\right]-s(a+s)\underline{x}_0(\alpha)-(a+s)\underline{y}_0(\alpha)-\overline{z}_0(\alpha) \quad (26)$$

and

$$\left(s^3+\frac{s}{p^2}\right)L\left[\overline{y}(t,\alpha)\right]-\left(s^2+\frac{1}{p^2}\right)\underline{x}_0(\alpha)-s\underline{y}_0(\alpha)-\underline{z}_0(\alpha)-\frac{k}{s^2}-\frac{1}{p^2}=$$

$$s^2\left(a+bs\right)L\left[\overline{y}(t,\alpha)\right]-s(a+s)\overline{x}_0(\alpha)-(a+s)\overline{y}_0(\alpha)-\underline{z}_0(\alpha) \quad (27)$$

Now, solving equation (26) we have

$$L\left[\underline{y}(t,\alpha)\right]=$$

$$\frac{\left(s+\frac{1}{p^2}\right)\overline{x}_0(\alpha)-s(a+b)\underline{x}_0(\alpha)+s\overline{y}_0(\alpha)-(a+s)\underline{y}_0(\alpha)+\overline{z}_0(\alpha)-\underline{z}_0(\alpha)-\frac{k}{s^2}+\frac{1}{p^2}}{\left(s^3+\frac{s}{p^2}\right)-s^2(a+bs)} \quad (28)$$

Taking the inverse Laplace transform of equation (28) we have

$$L^{-1}\left[\underline{y}(t,\alpha)\right]=$$

$$\underline{x}_0 p^2+kp^4+p^2\overline{z}_0+p\overline{x}_0-\overline{y}_0 ap^2-\underline{z}_0 bkp^4-\underline{y}_0 a^2kp^6-\frac{1}{2}kp^2t^2+\exp\left(ap^2\frac{t}{ap^2-2bp^2}\right)$$

$$\cosh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(bp^2-2bp^2)^2}}-\frac{\sinh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(bp^2-2bp^2)^2}}}{\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(bp^2-2bp^2)^2}}}$$

$$\frac{\left(\frac{ap^2-\underline{z}_0p^2+p^2\overline{y}_0+p^2+\overline{x}_0ap^4-bp^2+2akp^6+ap^3\underline{x}_0+ap^4-\overline{z}_0a^2p^4-a^3kp^8-2abkp^6}{\overline{x}_0p^4-bp^2+kp^6+p^3\overline{x}_0+p^4\underline{x}_0+p^2-\overline{x}_0bp^4-\underline{z}_0ap^4-2bkp^6-bp^3\overline{x}_0-bp^4\underline{x}_0-a^2kp^8+b^2kp^6+\underline{z}_0abp^4+a^2bkp^8}-a\frac{p^2}{2p^2-2bp^2}\right)}{bp^2-p^2}$$

$$(\underline{x}_0p^4-bp^2+kp^6+p^3\overline{x}_0+p^4\overline{z}_0+p^2-\underline{x}_0bp^4-\underline{z}_0ap^4-2bkp^6-bp^3\overline{x}_0-bp^4\overline{z}_0-a^2kp^8+\underline{y}_0b^2kp^6+\underline{z}_0abp^4+a^2bkp^8)-akp\underline{z}_0+1 \quad (29)$$

Also, solving equation (27) we have

$$L\left[\overline{y}(t,\alpha)\right]=$$

$$\frac{\left(s+\frac{1}{p^2}\right)\underline{x}_0(\alpha)-s(a+b)\overline{x}_0(\alpha)+s\underline{y}_0(\alpha)-(a+s)\overline{y}_0(\alpha)+\underline{z}_0(\alpha)-\overline{z}_0(\alpha)-\frac{k}{s^2}+\frac{1}{p^2}}{\left(s^3+\frac{s}{p^2}\right)-s^2(a+bs)} \quad (30)$$

Taking the inverse Laplace transform of equation (30) we have

$$L^{-1}\left[\overline{y}(t,\alpha)\right]=$$

$$\underline{x}_0 p^2+kp^4+p^2\overline{z}_0+p\overline{x}_0-\overline{y}_0ap^2-\underline{z}_0bkp^4-\underline{y}_0a^2kp^6-\frac{1}{2}kp^2t^2+\exp\left(ap^2\frac{t}{ap^2-2bp^2}\right)$$

$$\cosh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(bp^2-2bp^2)^2}}-\frac{\sinh t\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(bp^2-2bp^2)^2}}}{\sqrt{\frac{1}{bp^2-p^2}+a^2\frac{p^4}{(bp^2-2bp^2)^2}}}$$

$$\frac{\left(\frac{ap^2-\underline{z}_0p^2+p^2\overline{y}_0+p^2+\overline{x}_0ap^4-bp^2+2akp^6+ap^3\underline{x}_0+ap^4-\overline{z}_0a^2p^4-a^3kp^8-2abkp^6}{\overline{x}_0p^4-bp^2+kp^6+p^3\overline{x}_0+p^4\underline{x}_0+p^2-\overline{x}_0bp^4-\underline{z}_0ap^4-2bkp^6-bp^3\overline{x}_0-bp^4\underline{x}_0-a^2kp^8+b^2kp^6+\underline{z}_0abp^4+a^2bkp^8}-a\frac{p^2}{2p^2-2bp^2}\right)}{bp^2-p^2}$$

$$(\underline{x}_0p^4-bp^2+kp^6+p^3\overline{x}_0+p^4\overline{z}_0+p^2-\underline{x}_0bp^4-\underline{z}_0ap^4-2bkp^6-bp^3\overline{x}_0-bp^4\overline{z}_0$$
$$-a^2kp^8+\underline{y}_0b^2kp^6+\underline{z}_0abp^4+a^2bkp^8)-akp\underline{z}_0+1 \quad (31)$$

Example

(Due to Sankar and Tapan 2013)**:** Considering the model presented by Sankar and Tapan (2013), growth model is presented in the work of Sankar and Tapan (2013) with $N=\left(3\times10^6,5\times10^6,7\times10^6,0.8\right)$ and $k=0.4055$ where k is constant. In this study, we will use the information presented in Sankar and Tapan (2013). Therefore, substituting $a=3\times10^6, b=5\times10^6, p=7\times10^6, s=0.8$ and $k=0.4055$ in equation (9) and (11) we have

$$L\left[\underline{y}(t,\alpha)\right]=$$
$$2.00\times10^{-03}(1-\alpha)t+2.23\times10^{14}t-7.6\times10^{32}[-1.32\times10^{33}(1-\alpha)$$
$$-1.99\times10^{20}]-6.76\times10^{-05}t^2-\frac{e^{-6012.02}}{5.2\times10^{16}}[-7.9\times10^{39}(1-\alpha)-1.3(1-\alpha) \quad (32)$$
$$3.3\times10^{46}(1-\alpha)+1.98\times10^{46}(1-\alpha)-3.9\times10^{39}]$$

$$L\left[\overline{y}(t,\alpha)\right]=$$
$$2.00\times10^{-03}(\alpha-1)t+2.23\times10^{14}t-7.6\times10^{32}[-1.32\times10^{33}(\alpha-1)$$
$$-1.99\times10^{20}]-6.76\times10^{-05}t^2-\frac{e^{-6012.02}}{5.2\times10^{16}}[-7.9\times10^{39}(\alpha-1)-1.3(\alpha-1) \quad (33)$$
$$3.3\times10^{46}(\alpha-1)+1.98\times10^{46}(\alpha-1)-3.9\times10^{39}]$$

Simplifying equations (32) and (33) we have we have the equations presented as $\underline{y}(t,\alpha)$ and $\overline{y}(t,\alpha)$ respectively.

$$\underline{y}(t,\alpha)=2.00\times10^{-03}(1-\alpha)t+2.23\times10^{-14}t+1.00\times10^{-57}(1-\alpha)+1.5\times10^{20}-6.76\times10^{-05}t^2$$
$$+1.32\times10^{40}+2.2(1-\alpha)+5.5\times10^{46}(1-\alpha)+3.3\times10^{46}(1-\alpha)-6.5\times10^{39}$$

$$\overline{y}(t,\alpha)=2.00\times10^{-03}(\alpha-1)t+2.23\times10^{-14}t+1.00\times10^{-57}(\alpha-1)+1.5\times10^{20}-6.76\times10^{-05}t^2$$
$$+1.32\times10^{40}+2.2(\alpha-1)+5.5\times10^{46}(\alpha-1)+3.3\times10^{46}(\alpha-1)-6.5\times10^{39}$$

Algorithm for C^{++}

Start

Read $\underline{y}$, $\overline{y}$, t, α ;

for (int i= 0; i<9; i++)

for (int j=0; j<1; j++)

Compute

$L\left[\underline{y}(t,\alpha)\right]$

$L\left[\overline{y}(t,\alpha)\right]$

Print $\underline{y}$

Compute

α = α + 0.1

Print $\overline{y}$

Stop.

Table 1: Solution of non-linear non-homogeneous case for $t = 0.05$

α	$\underline{y}(t,\alpha)$	$\overline{y}(t,\alpha)$
0	21.2	-41.4
0.1	18.1	-38.3
0.2	14.9	-35.1
0.3	11.8	-32
0.4	8.7	-28.9
0.5	5.5	-25.8
0.6	2.4	-22.6
0.7	-0.7	-19.5
0.8	-3.8	-16.4

From table 1, we see that $\underline{y}(t,\alpha)$ is a decreasing function, $\overline{y}(t,\alpha)$ is an increasing function and $\underline{y}(t,\alpha) > \overline{y}(t,\alpha) < 0$. Therefore, the solution of the Sankar and Tapan (2013) model for the particular value of t is a week solution

Table 2: Solution of Sankar and Tapan (2013) model

α	$\underline{y}(t,\alpha)$	$\overline{y}(t,\alpha)$
0	1022.7357	1377.5550
0.1	1047.5458	1357.0748
0.2	1072.4580	1336.7224
0.3	1097.4726	1316.4973
0.4	1122.5899	1296.3987
0.5	1147.8103	1276.4260
0.6	1173.1340	1256.5786
0.7	1198.5614	1236.8558
0.8	1218.5341	1256.8285

From table 2, we see that $\underline{y}(t,\alpha)$ is an increasing function, $\overline{y}(t,\alpha)$ is a decreasing function and $\underline{y}(t,\alpha) < \overline{y}(t,\alpha)$. Therefore, the solution is a strong solution.

From the results presented in table 2, the nonlinear nonhomogeneous case, we observed that the lower bound variable $\underline{y}(t,\alpha)$ is a decreasing function and the upper bound variable $\overline{y}(t,\alpha)$ is also a increasing function which indicated week solution both also still maintained very good interval of convergens when compared with the work of Sankar and Tapan (2013) presented as table 2

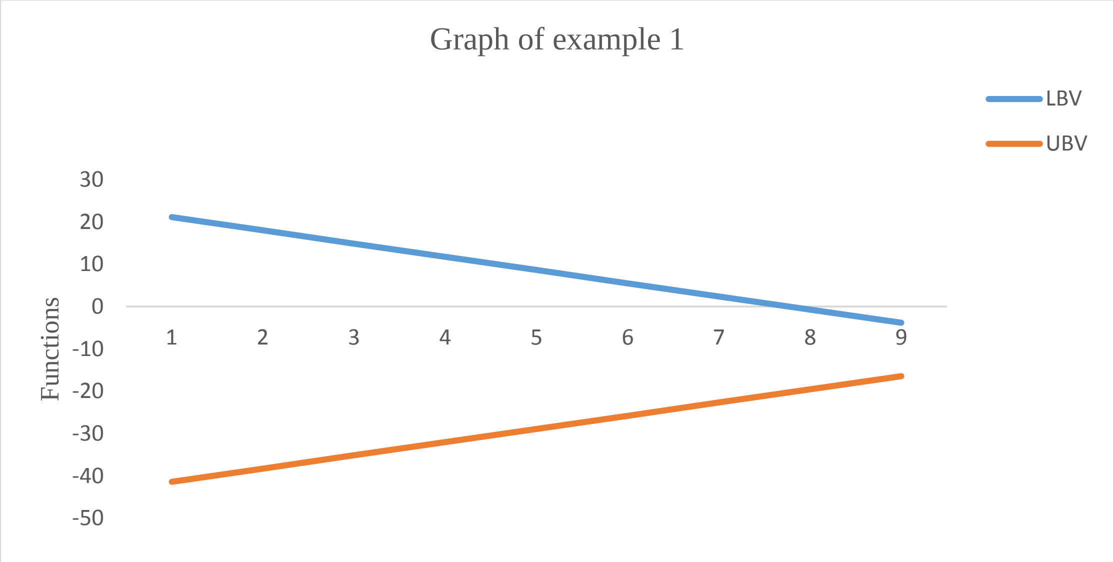

Figure 1: Graph of nonlinear non homogeneous FODES

In figure (4.4), it is also observed that the gap between the upper-bound variable $\overline{y}(t,\alpha)$ and lower- bound variable $\underline{y}(t,\alpha)$ decreases as α increases which indicate convergence.

DISCUSSION

The results for nonlinear nonhomogeneous FODE is presented by considering equation (5). The four cases discussed are thus; case 1 when $a \geq 0$ and $b \geq 0$ are applied to equation (5), the results established are found in equations (9) and (11). When $a \geq 0$ and $b < 0$ are applied to equation (5), the results established are indicated in equations (17) and (19) respectively. Case 3 presented $a < 0$ and $b \geq 0$, when applied to equation (5), the results established formed equations (23) and (25) respectively. Finally, with $a < 0$ and $b < 0$ and applying to equation (5), the results established are equations (29) and (31).

Based on the results presented above for linear homogeneous, linear non-homogeneous, nonlinear homogeneous and nonlinear nonhomogeneous FODEs have been established by FLTM which indicated that FLTM is an essential method for the analytical solution of FODES.

CONCLUSION

This study considers nonlinear homogeneous third order fuzzy ordinary differential equation using fuzzy Laplace transform under generalized Hukuhara differentiability concept. The results obtained compete favourably when compared with the exact solution which formed the basis of extension for further research in this area.

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